

Alligations

INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time wherever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1 for group	1	containing	n_1	elements
A_2 for group	2	containing	n_2	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that the weighted average, Aw is given by:

$$Aw = \frac{(n_1A_1 + n_2A_2 + n_3A_3 + \dots + n_kA_k)}{(n_1 + n_2 + n_3 + \dots + n_k)}$$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$$Aw = \frac{(n_1A_1 + n_2A_2)}{(n_1 + n_2)}$$

Rewriting this equation we get: $(n_1 + n_2)Aw = n_1A_1 + n_2A_2$

$$n_1(Aw - A_1) = n_2(A_2 - Aw)$$

or $n_1/n_2 = (A_2 - Aw)/(Aw - A_1) \rightarrow$ The alligation equation.

The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2 , then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages, we get $A_1 < Aw < A_2$.

Illustration 1

Two varieties of rice at ₹ 10 per kg and ₹ 12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.

Solution $1/2 = (12 - Aw)/(Aw - 10) \rightarrow Aw - 10 = 24 - 2Aw$

$$\Rightarrow 3Aw = 34 \quad \Rightarrow Aw = 11.33 \text{ ₹/kg.}$$

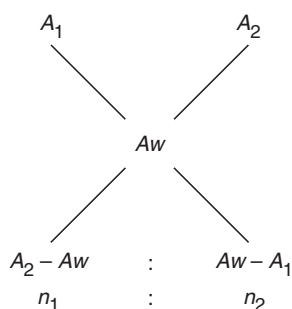
Illustration 2

On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution $n_1/n_2 = (40 - 34)/(34 - 30) = 6/4 = 3/2$

Graphical Representation of Alligation

The formula illustrated above can be represented by the following cross diagram:



[Note that the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.]

As we have seen, there are five variables embedded inside the alligation equation. These being:

the three averages $\rightarrow A_1, A_2$ and Aw

and the two weights $\rightarrow n_1$ and n_2

Based on the problem situation, one of the following cases may occur with respect to the knowns and the unknown, in the problem.

Case	Known	Unknown
I	(a) A_1, A_2, Aw	(a) $n_1 : n_2$
	(b) A_1, A_2, Aw, n_1	(b) n_2 and $n_1 : n_2$
II	A_1, A_2, n_1, n_2	Aw
III	A_1, Aw, n_1, n_2	A_2

Now, let us try to evaluate the effectiveness of the cross method for each of the three cases illustrated above:

Case 1: A_1, A_2, Aw are known; may be one of n_1 or n_2 is known.

To find: $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

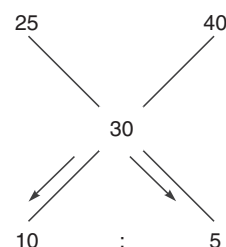
Let us illustrate through an example:

Illustration 3

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

- The ratio of students in the classes
- The number of students in the first class if the second class had 30 students.

Solution



- Hence, solution is 2 : 1.
- If the ratio is 2 : 1 and the second class has 30 students, then the first class has 60 students.

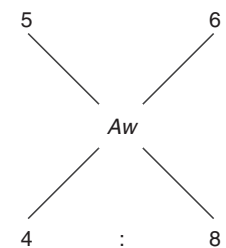
Note: The cross method becomes pretty effective in this situation when all the three averages are known and the ratio is to be found out.

Case 2: A_1, A_2, n_1 and n_2 are known, Aw is unknown.

Illustration 4

4 kg of rice at ₹ 5 per kg is mixed with 8 kg of rice at ₹ 6 per kg. Find the average price of the mixture.

Solution



$$= (6 - Aw) : (Aw - 5)$$

$$\Rightarrow (6 - Aw)/(Aw - 5) = 4/8 \rightarrow 12 - 2Aw = Aw - 5$$

$$3Aw = 17$$

$$\therefore Aw = 5.66 \text{ ₹/kg. (Answer)}$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to

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get the solution. Notice, the amount of time required in doing the same.

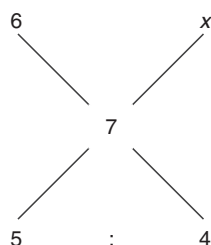
Note: The cross method becomes quite cumbersome in this case, as this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

Case 3: A_1, Aw, n_1 and n_2 are known; A_2 is unknown.

Illustration 5

5 kg of rice at ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.

Solution Using the cross method:



$$= (x - 7) : 1$$

$$\therefore (x - 7)/1 = 5/4 \rightarrow 4x - 28 = 5$$

$$\therefore x = ₹ 8.25.$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

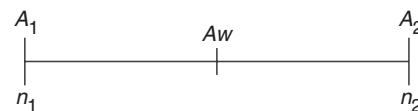
Note: The cross method becomes quite cumbersome in this case since this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

The above problems can be dealt quite effectively by using the straight line approach, which is explained below.

The Straight Line Approach

As we have seen, the cross method becomes quite cumbersome in Case 2 and Case 3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases.

Consider the following diagram, which results from closing the cross like a pair of scissors. Then the positions of A_1, A_2, Aw, n_1 and n_2 are as shown.



Visualise this as a fragment of the number line with points A_1, Aw and A_2 in that order from left to right.

Then,

- (a) n_2 is responsible for the distance between A_1 and Aw or n_2 corresponds to $Aw - A_1$
- (b) n_1 is responsible for the distance between Aw and A_2 . or n_1 corresponds to $A_2 - Aw$
- (c) $(n_1 + n_2)$ is responsible for the distance between A_1 and A_2 . or $(n_1 + n_2)$ corresponds to $A_2 - A_1$.

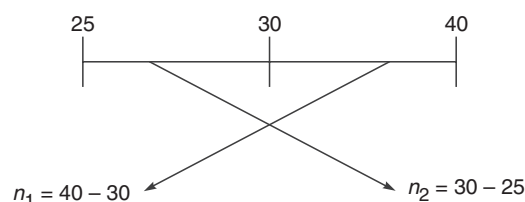
The processes for the 3 cases illustrated above can then be illustrated below:

Illustration 6

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

- (a) the ratio in which the classes were mixed.
- (b) the number of students in the first class if the second class had 30 students.

Solution



Hence, ratio is 2 : 1, and the second class has 60 students.

Case 2 A_1, A_2, n_1 and n_2 are known; Aw is unknown.

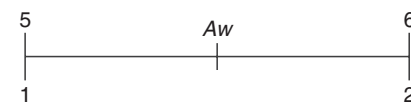
Illustration 7

4 kg of rice at ₹ 5 per kg is mixed with 8 kg of rice at ₹ 6 per kg. Find the average price of the mixture.

Solution



is the same as



Then, by unitary method:

$$\begin{aligned} n_1 + n_2 \text{ corresponds to } A_2 - A_1 \\ \rightarrow 1 + 2 \text{ corresponds to } 6 - 5 \end{aligned}$$

That is, 3 corresponds to 1

$$\therefore n_2 \text{ will correspond to } \frac{(A_2 - A_1) \times n_2}{(n_1 + n_2)}$$

In this case $(1/3) \times 2 = 0.66$.

Hence, the required answer is 5.66.

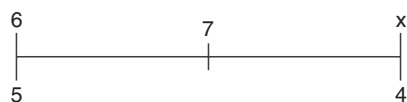
Note: In this case, the problem associated with the cross method is overcome and the solution becomes graphical.

Case 3: A_1, A_2, n_1 and n_2 are known; A_w is unknown.

Illustration 8

5 kg of rice at ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.

Using straight line method:



4 corresponds to $7 - 6$ and 5 corresponds to $x - 7$.

The thought process should go like:

$$4 \rightarrow 1$$

$$\therefore 5 \rightarrow 1.25$$

$$\text{Hence, } x - 7 = 1.25$$

$$\text{and } x = 8.25$$

SOME TYPICAL SITUATIONS WHERE ALLIGATIONS CAN BE USED

Given below are typical alligation situations, which students should be able to recognize. This will help them improve upon the time required in solving questions. Although in this chapter we have illustrated problems based on alligation at level 1 only, alligation is used in more complex problems where the weighted average is an intermediate step in the solution process.

The following situations should help the student identify alligation problems better as well as spot the way A_1, A_2, n_1 and n_2 and A_w are mentioned in a problem.

In each of the following problems the following magnitudes represent these variables:

$$A_1 = 20, \quad A_2 = 30, \quad n_1 = 40, \quad n_2 = 60$$

Each of these problems will yield an answer of 26 as the value of A_w .

1. A man buys 40 kg of rice at ₹ 20/kg and 60 kg of rice at ₹ 30/kg. Find his average price. (26/kg)
2. Pradeep mixes two mixtures of milk and water. He mixes 40 litres of the first containing 20% water and 60 litres of the second containing 30% water. Find the percentage of water in the final mixture. (26%)
3. Two classes are combined to form a larger class. The first class having 40 students scored an average of 20 marks on a test while the second having 60 students scored an average of 30 marks on the same test. What was the average score of the combined class on the test. (26 marks)
4. A trader earns a profit of 20% on 40% of his goods sold, while he earns a profit of 30% on 60% of his goods sold. Find his percentage profit on the whole. (26%)
5. A car travels at 20 km/h for 40 minutes and at 30 km/h for 60 minutes. Find the average speed of the car for the journey. (26 km/hr)
6. 40% of the revenues of a school came from the junior classes while 60% of the revenues of the school came from the senior classes. If the school raises its fees by 20% for the junior classes and by 30% for the senior classes, find the percentage increase in the revenues of the school. (26%)

Some Keys to spot A_1, A_2 and A_w and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This isn't foolproof though, since at times the question might confuse the student by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and A_w are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n_1 and n_2).
4. All percentage values represent the average values.

A Typical Problem

A typical problem related to the topic of alligation goes as follows:

4 litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated 3 times

- (a) What is the final quantity of wine left in the cask.
- (b) What is the ratio of wine to water finally.

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

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Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus at the end of the second step we get: 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 3.24 litres wine and 0.76 litres water leaving 29.16 litres of wine and 10.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can get the amount of wine left by:

$$40 \times 36/40 \times 36/40 \times 36/40 = 40 \times (36/40)^3 \\ \Rightarrow 40 \times (1 - 4/40)^3$$



This yields the formula:
Wine left : Capacity \times (1 – fraction of wine withdrawn)ⁿ
for n operations.

Thus, you could have multiplied:

$$40 \times (0.9)^3 \text{ to get the answer}$$

That is, reduce 40 by 10% successively thrice to get the required answer.

Thus, the thought process could be:

$$40 - 10\% \rightarrow 36 - 10\% \rightarrow 32.4 - 10\% \rightarrow 29.16$$

Space for Notes

LEVEL OF DIFFICULTY (I)

- A mixture of 160 gallons of wine and water contains 25% water. How much water must be added to the mixture in order to increase the percentage of water to 40% of the new mixture?
 - 40 gals
 - 50 gals
 - 80 gals
 - 33 gals
- 800 students took the CAT exam in Delhi. 50% of the boys and 90% of the girls cleared the cut off in the examination. If the total percentage of qualifying students is 60%, how many girls appeared in the examination?
 - 100
 - 120
 - 150
 - 200
- If 10 kg of sugar costing ₹15/kg and 20 kg of salt costing ₹10/kg are mixed, find the average cost of the mixture in ₹ per kilogram.
 - 11.67
 - 12.33
 - 12.67
 - 11.33
- The average salary per head of all workers (Grade A and Grade B) of a company is ₹ 400. The average salary of 100 grade A workers is ₹ 1000. If the average salary per head of the rest of the Grade B workers is ₹ 300, find the total number of workers in the company.
 - 1000
 - 800
 - 500
 - 700
- Ashok purchased two qualities of grains at the rate of ₹ 100 per quintal and ₹ 160 per quintal. In 50 quintals of the second quality, how much grain of the first quality should be mixed so that by selling the resulting mixture at ₹ 195 per quintal, he gains a profit of 30%?
 - 10 quintals
 - 14 quintals
 - 20 quintals
 - None of these
- Two types of milk having the rates of ₹8/kg and ₹10/kg respectively are mixed in order to produce a mixture having the rate of ₹9.20/kg. What should be the amount of the second type of milk if the amount of the first type of milk in the mixture is 20 kg?
 - 25 kg
 - 30 kg
 - 40 kg
 - 20 kg
- How many kilograms of salt worth ₹ 360 per kg should be mixed with 10 kg of salt worth ₹420 per kg, such that by selling the mixture at ₹ 480 per kg, there may be a gain of 20%?
 - 5 kg
 - 3 kg
 - 2 kg
 - 4 kg
- Kiran lends ₹ 1000 on simple interest to Harsh for a period of 5 years. She lends a part of the amount at 2% interest and the rest at 8% and receives ₹ 300 as the amount of interest. How much money (in ₹) did she lend on 2% interest rate?
 - 333.33
 - 666.67
 - 400
 - 500
- A tank contains 500 liters of wine. 50 liters of wine is taken out of it and replaced by water. The process is repeated again. Find the proportion of water and wine in the resulting mixture.
 - 1 : 4
 - 41 : 50
 - 19 : 81
 - 81 : 19
- A man purchased a table and a chair for ₹2000. He sold the table at a profit of 20% and the chair at a profit of 40%. In this way, his total profit was 25%. Find the cost price (in ₹) of the table.
 - 1500
 - 900
 - 1000
 - 800
- A dishonest shopkeeper purchased milk at ₹100 per litre and mixed 10 liters of water in it. By selling the mixture at the rate of ₹ 100 per litre he earns a profit of 25%. The quantity of the amount of the mixture that he had was:
 - 50 liters
 - 40 liters
 - 25 liters
 - 60 liters
- A tank has a capacity of 10 gallons and is full of alcohol. 2 gallons of alcohol are drawn out and the tank is again filled with water. This process is repeated 5 times. Find out how much alcohol is left in the resulting mixture finally?
 - 2048/625 gallons
 - 3346/625 gallons
 - 2048/3125 gallons
 - 625 gallons
- A vessel is full of milk 1/4 of the milk is taken out and the vessel is filled with water. If the process is repeated 4 times and 100 liters of milk is finally left in the vessel, what is the capacity of the vessel?
 - 25600/243 liters
 - $\frac{2461}{81}$ liters
 - 25600/81 liters
 - 30 liters
- In what ratio should two qualities of tea having the rates of ₹ 40 per kg and ₹ 30 per kg be mixed in order to get a mixture that would have a rate of ₹ 35 per kg?
 - 1 : 2
 - 1 : 1
 - 1 : 3
 - 3 : 1
- Raman steals four gallons of liquid soap kept in a train compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he

- steals 4 gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36: 13. What was the initial amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?
- (a) 7 gallons (b) 14 gallons
(c) 21 gallons (d) 28 gallons
16. In what ratio should water be mixed with soda costing ₹12 per litre so as to make a profit of 50% by selling the diluted liquid at ₹15 per litre?
(a) 10 : 1 (b) 5 : 1
(c) 1 : 5 (d) 6 : 1
17. A sum of ₹ 4 is made up of 20 coins that are either 10 paise coins or 60 paise coins. Find out how many 20 paise coins are there in the total amount.
(a) 10 (b) 13
(c) 16 (d) 15
18. Pinku a dishonest grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of adulterated fat in the mixture assuming that adulterated fat is freely available.
(a) 20% (b) 25%
(c) 33.33% (d) 40%
19. A mixture of 75 liters of alcohol and water contains 20% of water. How much water must be added to the above mixture to make the water 25% of the resulting mixture?
(a) 5 liters (b) 1.5 litre
(c) 2 liters (d) 2.5 liters
20. A mixture of 40 liters of milk and water contains 10% water. How much water should be added to it to increase the percentage of water to 25%?
(a) 5 liters (b) 6 liters
(c) 2.5 liters (d) 8 liters
21. Two vessels contain a mixture of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many liters are taken from the first vessel?
(a) 11 liters (b) 22 liters
(c) 16.5 liters (d) 17.5 liters
22. There are two mixtures of milk and water, the quantity of milk in them being 20% and 80% of the mixture. If 2 liters of the first are mixed with three liters of the second, what will be the ratio of milk to water in the new mixture?
(a) 11 : 12 (b) 11 : 9
(c) 19 : 11 (d) 14 : 11
23. There are two kinds of alloys of silver and copper. The first alloy contains silver and copper such that 93.33% of it is silver. In the second alloy there is 86.66% silver. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 100 kg mass containing 90% of silver?
(a) 55 kg (b) 50 kg
(c) 70 kg (d) 25 kg
24. Two buckets of equal capacity are full of a mixture of milk and water. In the first, the ratio of milk to water is 1 : 7 and in the second it is 3 : 8. Now both the mixtures are mixed in a bigger container. What is the resulting ratio of milk to water?
(a) 35 : 141 (b) 42 : 49
(c) 43 : 41 (d) 41 : 53
25. A bag contains a total of 105 coins of ₹1, 50 p and 25 p denominations. Find the total number of coins of ₹ 1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.
(a) 56 (b) 25
(c) 24 (d) None of these
26. Two vessels contain spirit and water mixed respectively in the ratio of 1: 4 and 4: 1 Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 1: 3.
(a) 11 : 1 (b) 13 : 1
(c) 11 : 2 (d) 11 : 3
27. The price of a table and a chair is ₹3000. The table was sold at a 20% profit and the chair at a 10% loss. If in the transaction a man gains ₹ 300, how much is cost price (in ₹) of the table?
(a) 1000 (b) 2500
(c) 2000 (d) None of these
28. A person purchased a pen and a pencil for ₹ 15. He sold the pen at a profit of 20% and the pencil at a profit of 30%. If his total profit was 24%, find the cost price of the pen.
(a) ₹10.50 (b) ₹ 12
(c) ₹ 9 (d) ₹ 10
29. A container is full of a mixture of kerosene and petrol in which there is 18% kerosene. Eight liters are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the container hold?
(a) 40 liters (b) 32 liters
(c) 36 liters (d) 48 liters
30. Two solutions of 80% and 87% purity are mixed resulting in 35 liters of mixture of 84% purity. How much is the quantity of the first solution in the resulting mixture?
(a) 15 liters (b) 12 liters
(c) 9 liters (d) 6 liters

31. In the Delhi zoo, there are lions and there are hens. If the heads are counted, there are 180, while the legs are 448. What will be the number of lions in the zoo?
(a) 36 (b) 88
(c) 44 (d) 136
32. A bonus of ₹ 1,00,000 was divided among 500 workers of a factory. Each male worker gets 500 rupees and each female worker gets 100 rupees. Find the number of male workers in the factory.
(a) 250 (b) 375
(c) 290 (d) 125
33. What will be the ratio of honey and water in the final solution formed by mixing honey and water that are present in three vessels of equal capacity in the ratios 4:1, 5:2 and 6:1 respectively?
(a) 166 : 22 (b) 83 : 22
(c) 83 : 44 (d) None of these
34. A mixture worth ₹ 80 a kg is formed by mixing two types of flour, one costing 50 per kg while the other 110 per kg. In what proportion must they have been mixed?
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 3
35. A 10 percent gain is made by selling the mixture of two types of milk at ₹ 48 per kg. If the type costing ₹ 61 per kg was mixed with 100 kg of the other, how many kilograms of the former was mixed?
(a) 38 kg (b) 30.5 kg
(c) 19 kg (d) Cannot be determined
36. A man buys milk at ₹ 85 per liter and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every liter of milk.
(a) 0.111 liters (b) 0.909 liters
(c) 0.1 litre (d) 0.125 liters
37. In what proportion must water be mixed with honey so as to gain 10% by selling the mixture at the cost price of the honey? (Assume that water is freely available)
(a) 1 : 4 (b) 1 : 5
(c) 1 : 6 (d) 1 : 10
38. A milkman stole milk from a can that contained 50% of milk and he replaced what he had stolen with milk having 20% milk. The bottle then contained only 25% milk. How much of the bottle did he steal?
(a) 80% (b) 83.33%
(c) 85.71% (d) 88.88%
39. Shruti possessing ₹ 10,000, lent a part of it at 5% simple interest and the remaining at 20% simple interest. Her total income after 5 years was ₹7500. Find the sum lent at 20% rate.
(a) ₹1666.67 (b) ₹6666.67
(c) ₹3333.33 (d) None of these
40. Sharman decides to travel 100 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 10 km/h and that on bicycle being 20 km/h, what distance would he travel on foot?
(a) 20 km (b) 30 km
(c) 50 km (d) 60 km

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

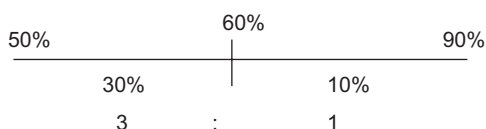
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|---------|---------|---------|---------|
| 1. (a) | 2. (d) | 3. (a) | 4. (d) |
| 5. (a) | 6. (b) | 7. (a) | 8. (a) |
| 9. (c) | 10. (a) | 11. (a) | 12. (a) |
| 13. (c) | 14. (b) | 15. (d) | 16. (c) |
| 17. (c) | 18. (a) | 19. (a) | 20. (d) |
| 21. (a) | 22. (d) | 23. (b) | 24. (a) |
| 25. (c) | 26. (a) | 27. (c) | 28. (c) |
| 29. (d) | 30. (a) | 31. (c) | 32. (d) |
| 33. (b) | 34. (a) | 35. (d) | 36. (a) |
| 37. (d) | 38. (b) | 39. (b) | 40. (d) |

Solutions and Shortcuts

Level of Difficulty (I)

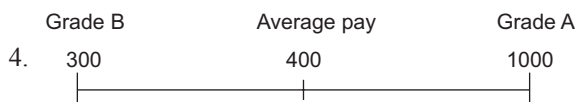
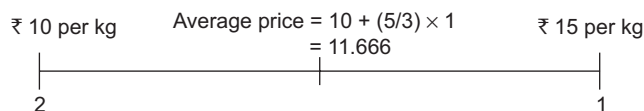
- There are multiple ways of solving this question. In 160 gallons since we have 25% water, the composition would be 120 gallons of wine and 40 gallons of water. After adding more water to this, the water would become $40 + w$, while the wine would remain at 120 gallons. This 120 gallons of wine would correspond to 60% wine in the final mixture. Since, $120 = 60\%$, $200 = 100\%$. So we need to add, 40 gallons of water. Alternately, you can solve this using options to check, the case when the wine to water becomes 60% to 40%. In 120 gallons of wine + 40 gallons of water, if you add 40 gallons of water, you will end up with 60% wine and 40% water. Hence, option (a) is correct.

- The ratio of boys and girls appearing for the exam can be seen to be 3:1 using the following alligation figure.



This means that out of 800 students, there must have been 200 girls who appeared in the exam.

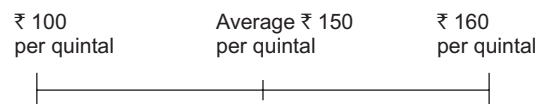
- Solving the following alligation figure:



From the figure we can see that the ratio of Grade A and Grade B workers is 1: 6. Since, there are 100 grade A workers, there would be 600 Grade B workers. Hence, total number of grade B workers = 600

Total workers in the company = $100 + 600 = 700$.

- By selling at 195 if we need to get a profit of 30% it means that the cost price would be $195/1.30 = 150$.



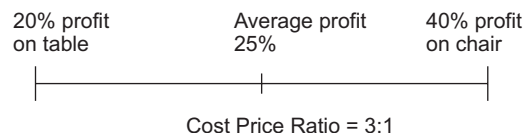
From the alligation figure, you can see that the ratio of the quantities of the two pulses would be 1:5. Since, we have 50 quintals of the second quality, we must have 10 quintals of the first. Hence, option (a) is correct.

- Mixing ₹ 8/kg and ₹ 10/kg to get ₹ 9.20 per kg we get that the ratio of mixing is $(10 - 9.2) : (9.2 - 8) = 2:3$. If the first milk is 20 kg, the second would be 30 kg.
- Since by selling at ₹ 480 we want a profit of 20%, it means that the average cost required is ₹ 400 per kg. Mixing salt worth ₹ 360/kg and ₹ 420/kg to get ₹ 400/kg means a mixture ratio of 1:2. Thus, to 10 kg of the second variety we need to add 5 kg of the first variety to get the required cost price.
- Since Kiran earns ₹ 300 in 5 years, it means that she earns an interest of $300/5 = ₹ 60$ per year. On an investment of 1000, an annual interest of 60 represents an average interest rate of 6%. Then using the alligation figure below:



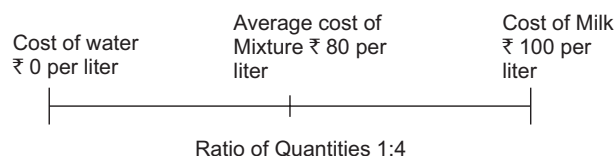
We get the ratio of investments as 1:2. Hence, she lent one third of the amount i.e. $1 \times 1000/3 = 333.33$ at 2% per annum.

- Amount of wine left = $500 \times 9/10 \times 9/10 = 405$ liters. Hence, water = 95 liters. Ratio of water and wine = 19:81. Option (c) is correct.
- The ratio of the cost of the table and the chair would be $(40 - 25) : (25 - 20)$ or 3:1 as can be seen from the following alligation figure:



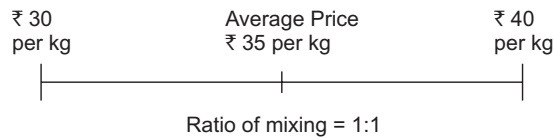
Thus, the cost of the table would be ₹ 1500.

- Cost price of the mixture = ₹ 80 per liter

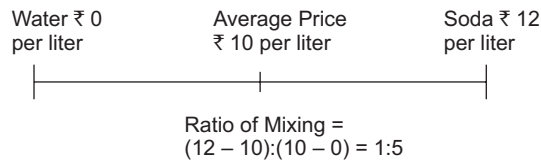


Since, water = 10 liters, Pure milk = 40 liters
 Total quantity of the mixture = 40 + 10 = 50 liters.

12. The amount of alcohol left = $10 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = 10240/3125 = 2048/625$
13. Let the quantity of milk initially be Q . Then we have $Q \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = 100 \rightarrow Q = 25600/81$ liters.
14. The ratio would be 1:1 as seen from the figure:

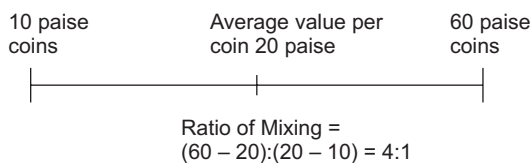


15. It can be seen from the ratio 36:13 that the proportion of liquid soap to water is 36/49 after two mixings. This means that $6/7^{\text{th}}$ of the liquid soap must have been allowed to remain in the container and hence $1/7^{\text{th}}$ of the container's original liquid soap, would have been drawn out by the thief. Since he takes out 4 gallons every time, there must have been 28 gallons in the container. (as 4 is $1/7^{\text{th}}$ of 28).
16. In order to sell at a 50% profit by selling at ₹15 the cost price should be 10. Also since water is freely available, we can say that the ratio of water and soda must be 1:5 as can be seen from the alligation figure.



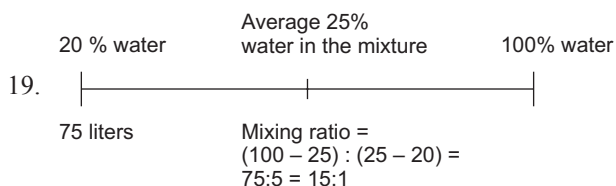
Hence, option (c) is correct.

17. The average value of a coin is 20 paise and there are only 10 paise and 60 paise coins in the sum. Hence, the ratio of the number of 10 paise coins to 60 paise coins would be 4: 1.



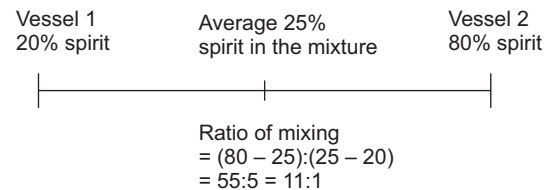
Since there are a total of 20 coins, the number of 10 paise coins would be $4 \times 20/5 = 16$ coins.

18. The ratio of mixing would be 1:4 which means that the percentage of adulterated fat would be 20%.



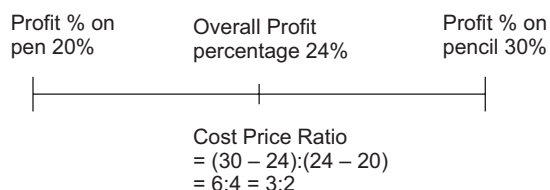
In the 15:1 ratio since, 15 corresponds to 75 liters, 1 would correspond to 5 liters. We should mix 5 liters of water.

20. In 40 liters there is 4 liters water and 36 liters milk. If we mix 8 liters of water to this (from option d); we would get a mixture containing 36 liters milk and 12 liters water – giving us the required 75% milk in the mixture. Hence, option (d) is correct.
21. Solving through options is the best way to tackle this question. Option (a) fits the conditions of the problem as if there are 11 liters in the first vessel, there would be 8 liters of spirit. Also it means that we would be taking 24 liters from the second vessel out of which there would be 20 liters of spirit. Thus, total spirit would be 28 out of 35 liters giving us 7 liters of water. This matches our requirement of a final ratio of 4:1 of spirit and water in the cask.
22. The percentage of milk in the new mixture would be: $(2 \times 20 + 3 \times 80)/5 = 280/5 = 56\%$. The ratio of milk to water in the new mixture would be 56:44 = 14:11.
23. In order to mix two tin alloys containing 86.66% silver and 93.33% silver to get 90% silver, the ratio of mixing should be 1:1. Thus, each variety should be 50 kgs each.
24. Assume the capacity of the two containers is 88 liters each. When we mix 88 liters of the first and 88 liters of the second, the amount of milk would be: $88 \times 1/8 + 88 \times 3/11 = 11 + 24 = 35$ liters. Consequently the amount of water would be $2 \times 88 - 35 = 176 - 35 = 141$ liters. Required ratio = 35: 141. Option (a) is correct.
25. $O + F + T = 105$
 $O + 0.5F + 0.25T = 50.5$
 $T = 2.333 O$.
 Solving we get: 24 coins of ₹1.
26. The first vessel contains 20% spirit while the second vessel contains 80% spirit. To get a 1:3 ratio we need 25% spirit in the mixture. The ratio of mixing can be seen using the following alligation figure:



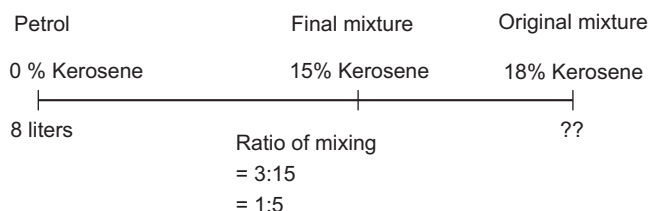
27. Solve using options as that would be the best way to tackle this question. Option (c) fits the situation perfectly as if we take the price of the table as ₹ 2000, the cost of the chair would be ₹ 1000. The profit in selling the table would be ₹ 400 while the loss in selling the chair would be ₹ 100. The total profit would be ₹ 300 as stipulated by the problem.
28. The following alligation visualization would help us solve the problem:

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Cost of pen = $\frac{3}{5}$ of the total cost price = $\frac{3}{5} \times 15 = ₹ 9$.

29. The following visualization would help:



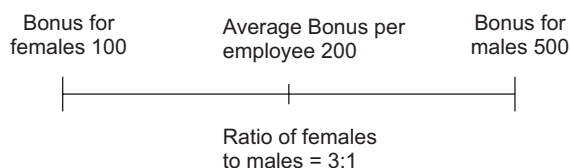
From the figure we can see that the original mixture would be 40 liters and the petrol being mixed is 8 liters. Thus, the container capacity is 48 liters.

30. 80% and 87% mixed to form 84% means that the mixing ratio is 3:4. The first solution would be $\frac{3}{7} \times 35 = 15$ liters.

31. If all the animals were hens we would have 180 heads and 360 legs. If we reduce the number of hens by 1 to 179 and increase the number of lions by 1 to 1, we would get an incremental 2 legs.

Since, the number of legs we need to increment is 88 i.e. $(448 - 360 = 88)$, we need to have 44 lions and 136 hens.

32. Average bonus per worker = $100000/500 = 200$.



Total male workers = $500 \times \frac{1}{4} = 125$.

33. In order to solve this we need to assume a value for the amounts in the vessels. If we assume 35 liters (LCM of 5, 7 and 7) as the quantities in all the three vessels we will get:

28 liters + 25 liters + 30 liters = 83 liters of honey and 22 liters of water in 105 liters of the mixture.

The required ratio is 83:22.

34. The required ratio would be 1:1 as we are mixing flour of ₹ 50 per kg with flour of ₹110 per kg to get flour of ₹80 per kg.

35. We cannot determine the answer to this question as we do not know the price per kg of the other type of milk. Hence, we cannot find the ratio of mixing which would be required in order to move further in this question.

36. The requisite 11.11% profit can be got by mixing 0.111 liters of water in 1 liter of milk. In such a case the total milk quantity would be 1.111 liters and the price would be for 1 liter only. The profit would be $0.111/1 = 11.11\%$.

37. To gain 10% by selling at cost price, water should comprise 10 out of a total of 110. The ratio of mixing that achieves this is 1:10.

38. 20% milk is mixed with 50% milk to get 25% milk. The ratio of mixing would be 5:1. This means he stole $\frac{5}{6}$ of the bottle or 83.33% of the bottle.

39. Annual interest income = $7500 / 5 = 1500$. Interest of ₹ 1500 on a lending of ₹10000 implies a 15% average rate of interest. This 15% is generated by mixing the two loans @ 5% and 20% respectively. The ratio in which the two loans should be allocated would be 1:2. The amount lent at 20% would be $2 \times 10000/3 = 6666.67$.

40. Solve using options. If he travels 60 km on foot he would take 6 hours on foot. Also, in this case he would travel 40 km on bicycle @ 20 kmph – which would take him 2 hours. Thus a total of 8 hours. Option (d) satisfies the conditions of the question.

Space for Rough Work



BLOCK REVIEW TEST

REVIEW TEST

- Rakshit bought 19 erasers for ₹ 10. He paid 20 paise more for each white eraser than for each brown eraser. What could be the price of a white eraser and how many white erasers could he have bought?
 - 60 paise, 8
 - 60 paise, 12
 - 50 paise, 8
 - 50 paise, 10
 - After paying all your bills, you find that you have ₹7.20 in your pocket. You have equal number of 50 paise and 10 paise coins; but no other coins nor any other currency notes. How many coins do you have?
 - 8
 - 24
 - 27
 - 30
 - Suresh Kumar went to the market with ₹ 100. If he buys three pens and six pencils he uses up all his money. On the other hand if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens & pencils, how many pencils can he buy?
 - 4
 - 5
 - 6
 - 7
 - For the above question, what is the amount of money he would save if he were to buy 3 pens and 3 pencils?
 - ₹ 50
 - ₹25
 - ₹75
 - ₹40
 - Abdul goes to the market to buy bananas. If he can bargain and reduce the price per dozen by ₹ 2, he can buy 3 dozen bananas instead of 2 dozen with the money he has. How much money does he have?
 - ₹ 6
 - ₹ 12
 - ₹ 18
 - ₹ 24
 - Two oranges, three bananas and four apples cost ₹15. Three oranges, two bananas and one apple cost ₹10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?
 - ₹10
 - ₹8
 - ₹15
 - cannot be determined
 - John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be
 - ₹ 1
 - ₹ 2
 - ₹ 3
 - ₹ 4
 - Two towns A and B are 100 km apart. A school is to be built for 100 students of Town B and 30 students of Town A. The Expenditure on transport is ₹1.20 per km per person. If the total expenditure on transport by all 130 students is to be as small as possible, then the school should be built at
 - 33 km from Town A
 - 33 km from Town B
 - Town A
 - Town B
 - A person who has a certain amount with him goes to the market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for taxi fare and buys 20 mangoes and of the balance he purchases oranges. Number of oranges he can purchase is
 - 36
 - 40
 - 15
 - 20
 - 72 hens costs ₹_96.7_. Then what does each hen cost, where numbers at “_” are not visible or are written in illegible hand?
 - ₹3.43
 - ₹5.31
 - ₹5.51
 - ₹6.22
- Directions for Questions 10 to 12:** There are 60 students in a class. These students are divided into three groups A, B and C of 15, 20 and 25 students each. The groups A and C are combined to form group D
- What is the average weight of the students in group D?
 - more than the average weight of A.
 - more than the average weight of C.
 - less than the average weight of C.
 - Cannot be determined.
 - If one student from Group A is shifted to group B, which of the following will be true?
 - The average weight of both groups increases
 - The average weight of both groups decreases
 - The average weight of the class remains the same.
 - Cannot be determined.
 - If all the students of the class have the same weight then which of the following is false?
 - The average weight of all the four groups is the same.
 - The total weight of A and C is twice the total weight of B.
 - The average weight of D is greater than the average weight of A.
 - The average weight of all the groups remains the same even if a number of students are shifted from one group to another.
 - The average marks of a student in ten papers are 80. If the highest and the lowest score are not considered

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- the average is 81. If his highest score is 92 find the lowest.
- (a) 55 (b) 60
(c) 62 (d) Cannot be determined
15. A shipping clerk has five boxes of different but unknown weights each weighing less than 100 kg. The clerk weighs the boxes in pairs. The weights obtained are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121 kg. What is the weight of the heaviest box?
- (a) 60 kg (b) 62 kg
(c) 64 kg (d) Cannot be determined
16. The total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is ₹ 700 when there are 25 boarders and ₹ 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?
- (a) 550 (b) 580
(c) 540 (d) 570
17. A yearly payment to a servant is ₹ 90 plus one turban. The servant leaves the job after 9 months and receives ₹65 and a turban, then find the price of the turban.
- (a) ₹10 (b) ₹15
(c) ₹7.50 (d) Cannot be determined
18. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is ₹20 on a standard bag and ₹30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

- (a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags
19. Three math classes: X, Y, and Z, take an algebra test.

The average score of class X is 83.
The average score of class Y is 76.
The average score of class Z is 85.
What is the average score of classes X, Y, Z ?

- (a) 81.5 (b) 80.5
(c) 83 (d) Cannot be determined
20. Prabhat ordered 4 Arrow shirts and some additional Park Avenue shirts. The price of one Arrow shirt was twice that of one Park Avenue shirt. When the order was executed it was found that the number of the two brands had been interchanged. This increased the bill by 40%. The ratio of the number of Arrow shirts to the number of Park Avenue shirts in the original order was:
- (a) 1:3 (b) 1:4
(c) 1:2 (d) 1:5
21. Three groups of companies: Tata, Birla and Reliance announced the average of the annual profit for all years since their establishment.
- The average profit of Tata is ₹ 75,000 lakh
The average profit of Birla is ₹ 64000 lakh
The average profit of Reliance is ₹ 73000 lakh
The average profit of all results of Tata and Birla together is ₹ 70000 lakh.
The average profit of all results of Birla and Reliance together is ₹ 69000 lakh.
Approximately what is the average profit for all the three group of companies?
- (a) ₹ 70800 lakh (b) ₹ 71086 lakh
(c) ₹ 70666 lakh (d) Cannot be determined

ANSWER KEY

Review Test

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|---------|---------|---------|---------|
| 1. (b) | 2. (b) | 3. (a) | 4. (b) |
| 5. (b) | 6. (c) | 7. (b) | 8. (d) |
| 9. (d) | 10. (c) | 11. (d) | 12. (c) |
| 13. (c) | 14. (b) | 15. (b) | 16. (a) |
| 17. (a) | 18. (a) | 19. (d) | 20. (a) |
| 21. (b) | | | |
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