Trigonometry

Problems on Height and Distance

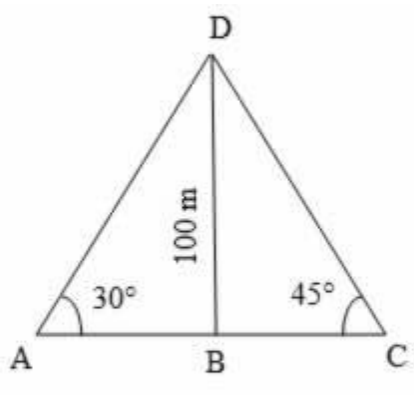
1. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30º and 45º respectively. If the lighthouse is 100 m high, the distance between the two ships is:

A. 300 m B. 173 m
C. 273 m D. 200 m

Answer with explanation

Answer: Option C

Explanation:



Let BD be the lighthouse and A and C be the positions of the ships.
Then, BD = 100 m, ∠BAD = 30°, ∠BCD = 45°

tan30°=BDBA⇒1√3=100BA⇒BA=100√3tan30°=BDBA⇒13=100BA⇒BA=1003

tan45°=BDBC⇒1=100BC⇒BC=100tan45°=BDBC⇒1=100BC⇒BC=100

Distance between the two ships
= AC = BA + BC
=100√3+100=100(√3+1)=1003+100=100(3+1)
= 100(1.73+1) = 100 × 2.73 = 273 m

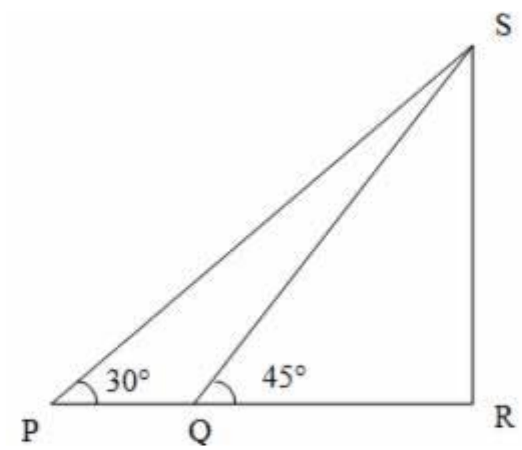
2. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30º with the man's eye. The man walks some distance towards the tower to watch its top and the angle of the elevation becomes 45º. What is the distance between the base of the tower and the point P?

A. 9 units B. 3√3 33 units
C. Data inadequate D. 12 units

answer with explanation

Answer: Option C

Explanation:



tan 45°=SRQRtan 30°=SRPR=SR(PQ + QR)tan 45°=SRQRtan 30°=SRPR=SR(PQ + QR)

Two equations and 3 variables. Hence we can not find the required value with the given data.

(Note that if one of SR, PQ, QR is known, this becomes two equations and two variables and if that was the case, we could have found out the required value.)

3. From a point P on a level ground, the angle of elevation of the top tower is 30º. If the tower is

200 m high, the distance of point P from the foot of the tower is:

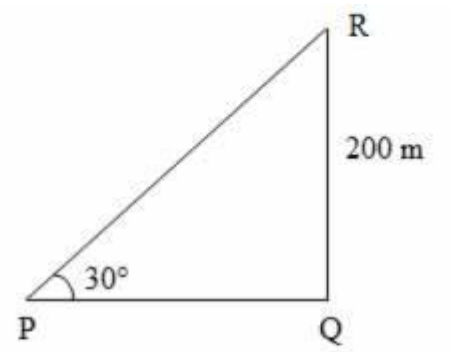
A. 346 m B. 400 m

C. 312 m D. 298 m

answer with explanation

Answer: Option A

Explanation:



$\tan 30°=RQPQ1\sqrt{\boxed{3}}=200PQPQ=200\sqrt{\boxed{3}}=200\times1.73=346$ m$\tan 30°=RQPQ13=200PQPQ=2003=200\times1.73=346$ m

4. The angle of elevation of the sun, when the length of the shadow of a tree is equal to the height of the tree, is:

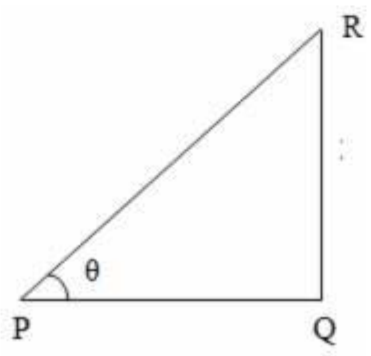
A. None of these B. 60°

C. 45° D. 30°

answer with explanation

Answer: Option C

Explanation:



Consider the diagram shown above

where QR represents the tree and PQ represents its shadow

We have, QR = PQ

Let $\angle QPR = \theta$

$\tan\theta=QRPQ=1$ $\tan\theta=QRPQ=1$ *(since QR = PQ)*

$\Rightarrow \theta = 45°$

i.e., required angle of elevation = 45°

5. An observer 2 m tall is $10\sqrt{\boxed{3}}103$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is:

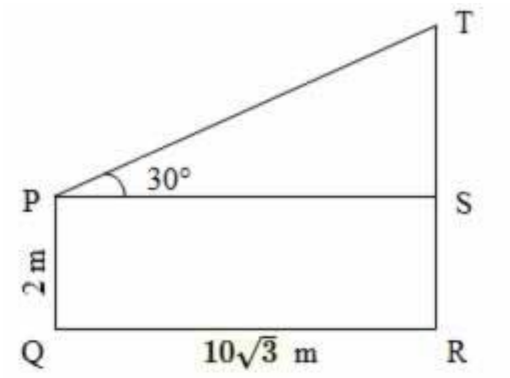
A. None of these B. 12 m

C. 14 m D. 10 m

answer with explanation

Answer: Option B

Explanation:



SR = PQ = 2 m

PS = QR $=10\sqrt{\boxed{3}}=103$ m

$\tan 30°=TSPS1\sqrt{\boxed{3}}=TS10\sqrt{\boxed{3}}TS=10\sqrt{\boxed{3}}\sqrt{\boxed{3}}=10$ m$\tan 30°=TSPS13=TS103TS=1033=10$ m

TR = TS + SR = 10 + 2 = 12 m

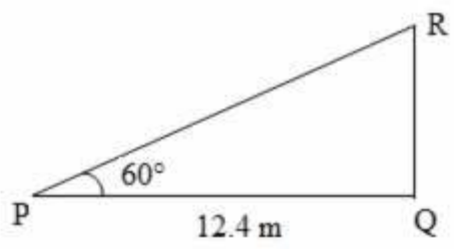
6. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 12.4 m away from the wall. The length of the ladder is:

A. 14.8 m	B. 6.2 m
C. 12.4 m	D. 24.8 m

answer with explanation

Answer: Option D

Explanation:



Consider the diagram shown above where PR represents the ladder and RQ represents the wall.

cos 60°
= PQPR12=12.4PRPR=2×12.4=24.8 mcos 60°
= PQPR12=12.4PRPR=2×12.4=24.8 m

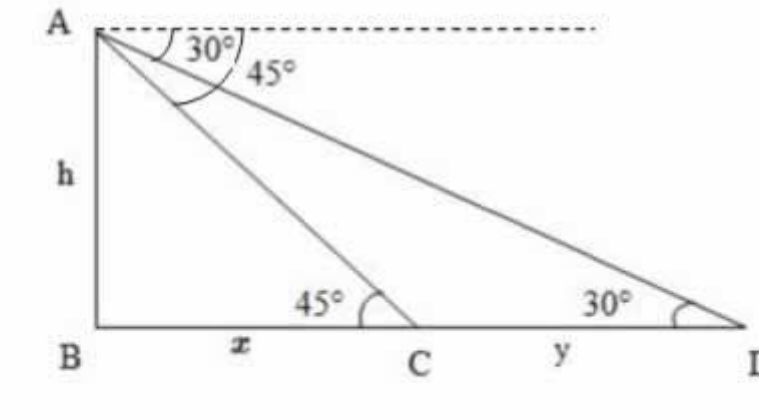
7. A man on the top of a vertical observation tower observers a car moving at a uniform speed coming directly towards it. If it takes 8 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower?

A. 8 min 17 second	B. 10 min 57 second
C. 14 min 34 second	D. 12 min 23 second

answer with explanation

Answer: Option B

Explanation:



Consider the diagram shown above. Let AB be the tower. Let D and C be the positions of the car.

Then, ∠ADC = 30° , ∠ACB = 45°

Let AB = h, BC = x, CD = y

tan45°=ABBC=hx=>1=hx=>h=x⋯(1)tan45°=ABBC=hx=>1=hx=>h=x⋯(1)

tan30°=ABBD=AB(BC + CD)=hx+y=>1√$\boxed{3}$=hx+y=>x + y = √$\boxed{3}$h=>y = √$\boxed{3}$h - xtan30°=ABBD=AB(BC + CD)=hx+y=>13=hx+y=>x + y = 3h=>y = 3h - x
=>y = √$\boxed{3}$h−h =>y = 3h−h (∵ Substituted the value of x from equation 1)
=>y = h(√$\boxed{3}$−1)=>y = h(3−1)

Given that distance y is covered in 8 minutes.
i.e, distance h(√$\boxed{3}$−1)h(3−1) is covered in 8 minutes.

Time to travel distance x
= Time to travel distance h (∵ Since x = h as per equation 1).

Let distance h is covered in t minutes.

since distance is proportional to the time when the speed is constant, we have

h(√[3]–1)∝8⋯(A)h∝t⋯(B)(A)(B)=>h(√[3]–1)h=8t=>(√[3]–1)=8t=>t=8(√[3]–1)=8(1.73–1)=8.73=80073minutes=107073minutes≈10 minutes 57 secondsh(3–1)∝8⋯(A)h∝t⋯(B)(A)(B)=>h(3–1)h=8t=>(3–1)=8t=>t=8(3–1)=8(1.73–1)=8.73=80073minutes=107073minutes≈10 minutes 57 seconds

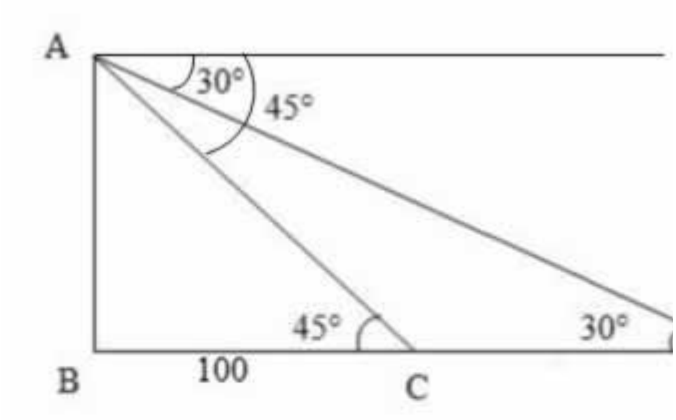
8. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 100 metres from the tower. After 10 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

A. 26.28 km/hr B. 32.42 km/hr
C. 24.22 km/hr D. 31.25 km/hr

answer with explanation

Answer: Option A

Explanation:



Consider the diagram shown above.

Let AB be the tower. Let C and D be the positions of the boat

Then, ∠ACB = 45° , ∠ADC = 30°, BC = 100 m

tan45°=ABBC=>1=AB100=> AB = 100 ⋯(1)tan45°=ABBC=>1=AB100=> AB = 100 ⋯(1)

tan30°=ABBDtan30°=ABBD =>1√[3]=100BD=>13=100BD(∵ Substituted the value of AB from equation 1)
=> BD =100√[3]=1003

CD = (BD - BC) =(100√[3]–100)=100(√[3]–1)=(1003–100)=100(3–1)

It is given that the distance CD is covered in 10 seconds.
i.e., the distance 100(√[3]–1)100(3–1) is covered in 10 seconds.

Required speed =DistanceTime=DistanceTime =100(√[3]–1)10=10(1.73–1)=100(3–1)10=10(1.73–1)
= 7.3 meter/seconds
= 7.3 × 185185 km/hr = 26.28 km/hr

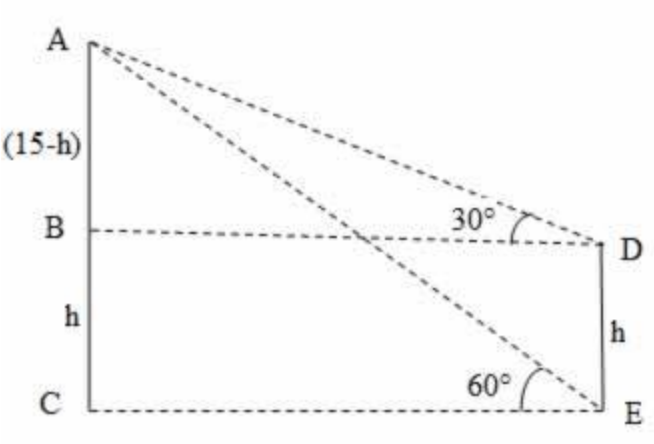
9. The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

A. 5 metres B. 8 metres
C. 10 metres D. 12 metres

answer with explanation

Answer: Option C

Explanation:



Consider the diagram shown above. AC represents the tower and DE represents the pole.

Given that AC = 15 m, ∠ADB = 30°, ∠AEC = 60°

Let DE = h
Then, BC = DE = h,
AB = (15-h) (∵ AC=15 and BC = h),
BD = CE

tan60°=ACCE=>√3=15CE=>CE = 15√3⋯(1)tan⁡60°=ACCE=>3=15CE=>CE = 153⋯(1)

tan30°=ABBD=>1√3=15−hBDtan⁡30°=ABBD=>13=15−hBD
=>1√3=15−h(15√3)=>13=15−h(153)(∵ BD = CE and substituted the value of CE from equation 1)
=>(15−h)=1√3×15√3=153=5=>h=15−5=10 m=>(15−h)=13×153=153=5=>h=15−5=10 m

i.e., height of the electric pole = 10 m

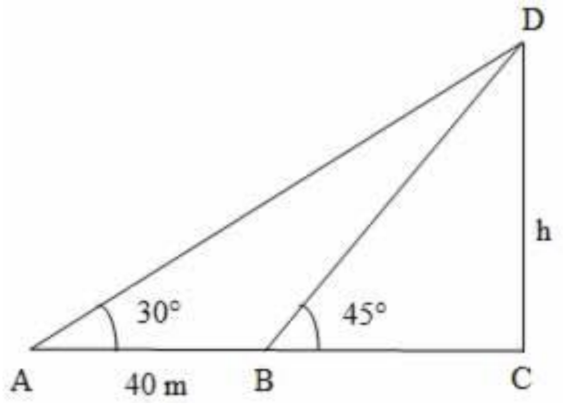
10. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 40 m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is:

A. 64.2 m B. 62.2 m
C. 52.2 m D. 54.6 m

answer with explanation

Answer: Option D

Explanation:



Let DC be the tower and A and B be the positions of the observer such that AB = 40 m

We have ∠DAC = 30°, ∠DBC = 45°

Let DC = h

tan30°=DCAC=>1√3=hAC=>AC = h√3⋯(1)tan⁡30°=DCAC=>13=hAC=>AC = h3⋯(1)

tan45°=DCBC=>1=hBC=>BC=h⋯(2)tan⁡45°=DCBC=>1=hBC=>BC=h⋯(2)

We know that, AB = (AC - BC)
=> 40 = (AC - BC)
=> 40=(h√3−h)40=(h3−h)[∵ from (1) & (2)]
=>40=h(√3−1)=>h=40(√3−1)=40(√3−1)×(√3+1)(√3+1)=40(√3+1)(3−1)=40(√3+1)2=20(√3+1)=20(1.73+1)=20×2.73=54.6 m

11. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of

depression are 45° and 60°. If the height of the tower is 600 m, the distance between the objects is approximately equal to :

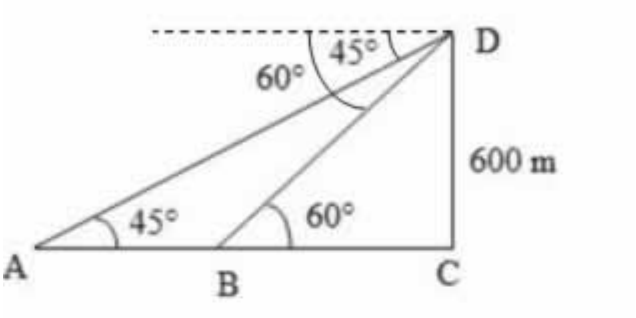
A. 272 m B. 284 m

C. 288 m D. 254 m

answer with explanation

Answer: Option D

Explanation:



Let DC be the tower and A and B be the objects as shown above.
Given that DC = 600 m , ∠DAC = 45°, ∠DBC = 60°

tan
60°=DCBC√3=600BCBC= 600√3⋯(1)tan
60°=DCBC3=600BCBC= 6003⋯(1)

tan 45°=DCAC1=600ACAC= 600⋯(2)tan
45°=DCAC1=600ACAC= 600⋯(2)

Distance between the objects
= AB = (AC - BC)
=600−600√3=600−6003 [∵ from (1) and (2)]
=600(1−1√3)=600(√3−1√3)=600(√3−1√3)×√3√3=600√3(√3−1)3=200√3(√3−1)=200(3−√3)=200(3−1.73)=254 m=600(1−13)=600(3−13)=600(3−13)×33=6003(3−1)3=2003(3−1)=200(3−3)=200(3−1.73)=254 m

12. A ladder 10 m long just reaches the top of a wall and makes an angle of 60° with the wall.Find the distance of the foot of the ladder from the wall (√3=1.73)(3=1.73)

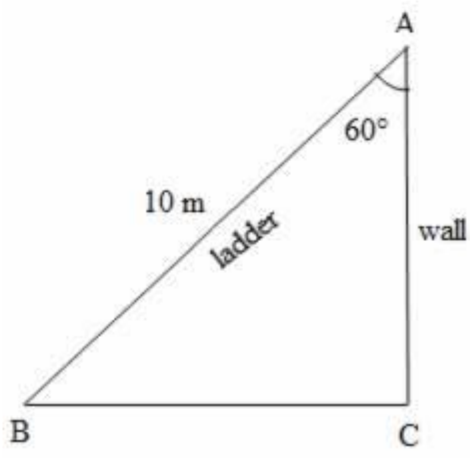
A. 4.32 m B. 17.3 m

C. 5 m D. 8.65 m

answer with explanation

Answer: Option D

Explanation:



Let BA be the ladder and AC be the wall as shown above.
Then the distance of the foot of the ladder from the wall = BC

Given that BA = 10 m , ∠BAC = 60°

sin 60°=BCBA√32=BC10BC
= 10×√32=5×1.73=8.65 msin
60°=BCBA32=BC10BC
= 10×32=5×1.73=8.65 m

13. From a tower of 80 m high, the angle of depression of a bus is 30°. How far is the bus from the tower?

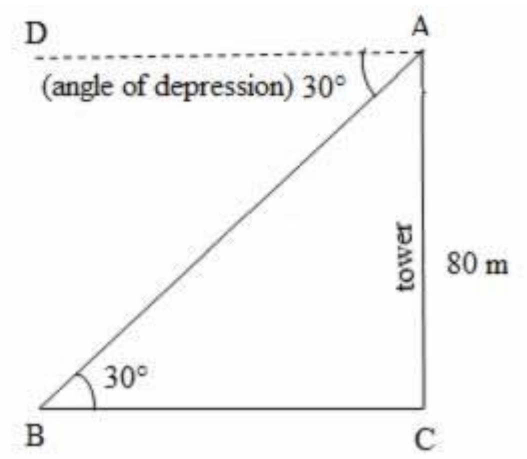
A. 40 m B. 138.4 m

C. 46.24 m D. 160 m

answer with explanation

Answer: Option B

Explanation:



Let AC be the tower and B be the position of the bus.
Then BC = the distance of the bus from the foot of the tower.

Given that height of the tower, AC = 80 m and the angle of depression, ∠DAB = 30°

∠ABC = ∠DAB = 30° (because DA || BC)

tan30°=ACBC=>tan30°=80BC=>BC = 80tan30°=80(1√3)=80×1.73=138.4 mtan30°=ACBC=>tan30°=80BC=>BC = 80tan30°=80(13)=80×1.73=138.4 m

i.e., Distance of the bus from the foot of the tower = 138.4 m

14. The angle of elevation of the top of a lighthouse 60 m high, from two points on the ground on its opposite sides are 45° and 60°. What is the distance between these two points?

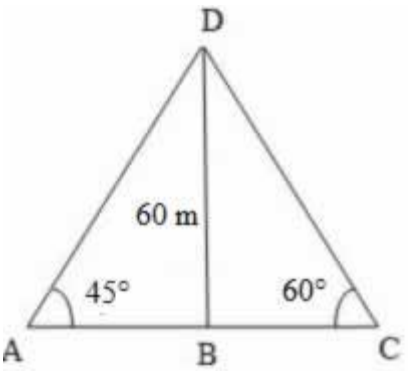
A. 45 m B. 30 m

C. 103.8 m D. 94.6 m

answer with explanation

Answer: Option D

Explanation:



Let BD be the lighthouse and A and C be the two points on ground.
Then, BD, the height of the lighthouse = 60 m

∠BAD = 45° , ∠BCD = 60°

tan45°=BDBA⇒1=60BA⇒BA=60 m⋯(1)tan45°=BDBA⇒1=60BA⇒BA=60 m⋯(1)

tan60°=BDBC⇒√3=60BC⇒BC=60√3=60×√3√3×√3=60√33=20√3=20×1.73=34.6 m⋯(2)tan60°=BDBC⇒3=60BC⇒BC=603=60×33×3=6033=203=20×1.73=34.6 m⋯(2)

Distance between the two points A and C
= AC = BA + BC
= 60 + 34.6 [∵ Substituted value of BA and BC from (1) and (2)]
= 94.6 m

15. From the top of a hill 100 m high, the angles of depression of the top and bottom of a pole are 30° and 60° respectively. What is the height of the pole?

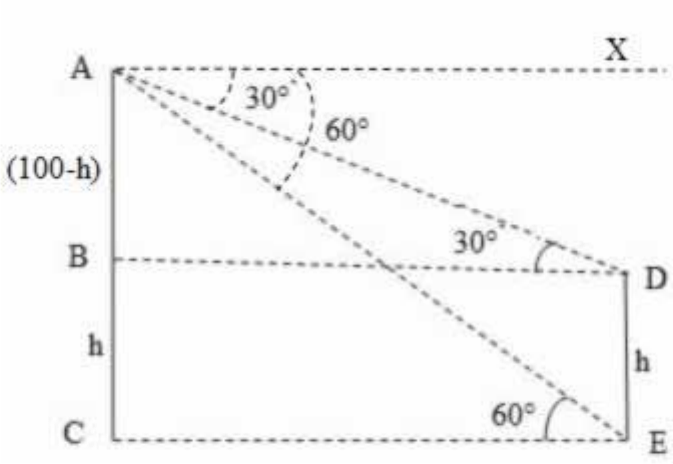
A. 52 m B. 50 m

C. 66.67 m D. 33.33 m

answer with explanation

Answer: Option C

Explanation:



Consider the diagram shown above. AC represents the hill and DE represents the pole

Given that AC = 100 m
∠XAD = ∠ADB = 30° (∵ AX || BD)
∠XAE = ∠AEC = 60° (∵ AX || CE)

Let DE = h

Then, BC = DE = h,
AB = (100-h) (∵ AC=100 and BC = h),
BD = CE

tan60°=ACCE=>√$\boxed{3}$=100CE=>CE = 100√$\boxed{3}$⋯(1)tan60°=ACCE=>3=100CE=>CE = 1003⋯(1)

tan30°=ABBD=>1√$\boxed{3}$=100−hBDtan30°=ABBD=>13=100−hBD
=>1√$\boxed{3}$=100−h(100√$\boxed{3}$)=>13=100−h(1003)(∵ BD = CE and substituted the value of CE from equation 1)
=>(100−h)=1√$\boxed{3}$×100√$\boxed{3}$=1003=33.33 =>h=100−33.33=66.67 m=>(100−h)=13×1003=1003=33.33 =>h=100−33.33=66.67 m

i.e., the height of the pole = 66.67 m

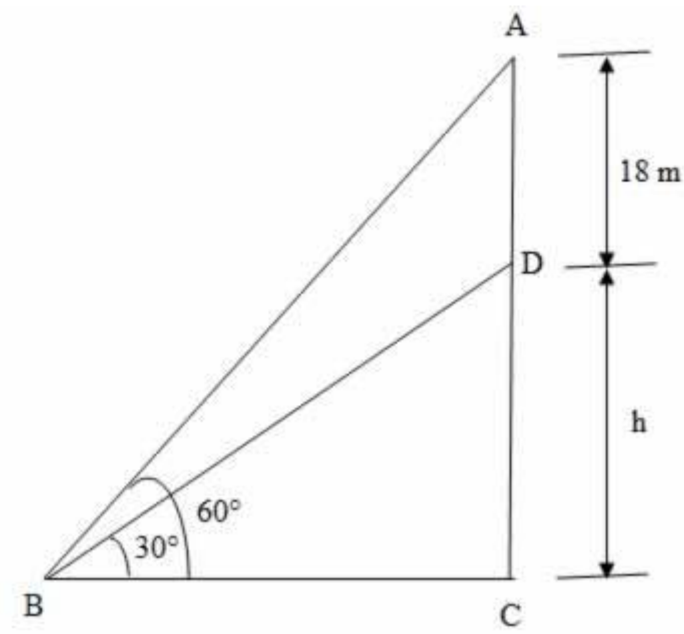
16. A vertical tower stands on ground and is surmounted by a vertical flagpole of height 18 m. At a point on the ground, the angle of elevation of the bottom and the top of the flagpole are 30° and 60° respectively. What is the height of the tower?

A. 9 m	B. 10.40 m
C. 15.57 m	D. 12 m

answer with explanation

Answer: Option A

Explanation:



Let DC be the vertical tower and AD be the vertical flagpole. Let B be the point of observation.

Given that AD = 18 m, ∠ABC = 60°, ∠DBC = 30°
Let DC be h.

tan30°=DCBC1√$\boxed{3}$=hBCh=BC√$\boxed{3}$⋯(1)tan30°=DCBC13=hBCh=BC3⋯(1)

tan60°=ACBC√$\boxed{3}$=18+hBC18+h=BC×√$\boxed{3}$⋯(2)tan60°=ACBC3=18+hBC18+h=BC×3⋯(2)

(1)(2)=>h18+h=(BC√$\boxed{3}$)(BC×√$\boxed{3}$)=13=>3h=18+h=>2h=18=>h=9 m(1)(2)=>h18+h=(BC3)(BC×3)=13=>3h=18+h=>2h=18=>h=9 m i.e., the height of the tower = 9 m

17. A balloon leaves the earth at a point A and rises vertically at uniform speed. At the end of 2 minutes, John finds the angular elevation of the balloon as 60°. If the point at which John is standing is 150 m away from point A, what is the speed of the balloon?

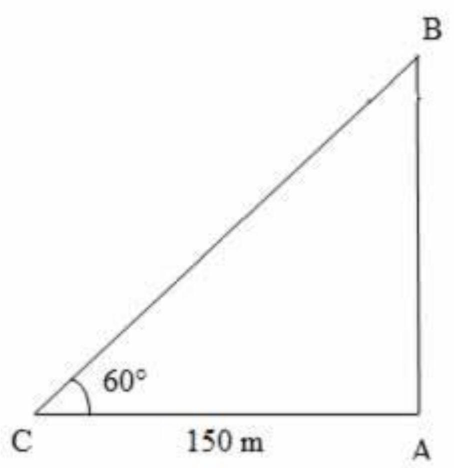
A. 0.63 meter/sec B. 2.16 meter/sec

C. 3.87 meter/sec D. 0.72 meter/sec

answer with explanation

Answer: Option B

Explanation:



Let C be the position of John. Let A be the position at which balloon leaves the earth and B be the position of the balloon after 2 minutes.

Given that CA = 150 m, ∠BCA = 60°

tan60°=BACA√3=BA150BA=150√3tan 60°=BACA3=BA150BA=1503

i.e, the distance travelled by the balloon =150√3=1503 meters

time taken = 2 min = 2 × 60 = 120 seconds

Speed =DistanceTime=DistanceTime =150√3120=1.25√3=1.25×1.73=2.16 meter/second=1503120=1.253=1.25×1.73=2.16 meter/second

18. The angles of depression and elevation of the top of a wall 11 m high from top and bottom of a tree are 60° and 30° respectively. What is the height of the tree?

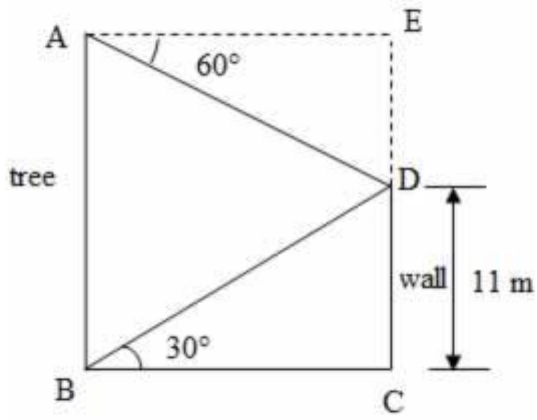
A. 22 m B. 44 m

C. 33 m D. None of these

answer with explanation

Answer: Option B

Explanation:



Let DC be the wall, AB be the tree.

Given that ∠DBC = 30°, ∠DAE = 60°, DC = 11 m

tan 30°=DCBC1√3=11BCBC = 11√3 mAE = BC =11√3 m⋯(1)tan 30°=DCBC13=11BCBC = 113 mAE = BC =113 m⋯(1)

tan60°=EDAEtan 60°=EDAE √3=ED11√33=ED113 [∵ Substituted value of AE from (1)]

ED =11√3×√3=11×3=33=113×3=11×3=33

Height of the tree
= AB = EC = (ED + DC)
= (33 + 11) = 44 m

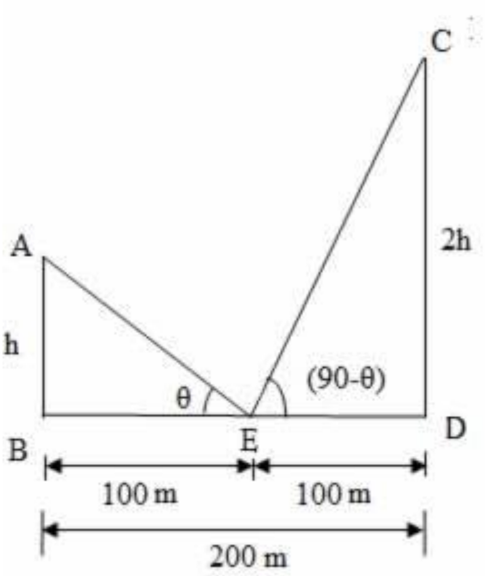
19. Two vertical poles are 200 m apart and the height of one is double that of the other. From the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary. Find the heights of the poles.

A. 141 m and 282 m	B. 70.5 m and 141 m
C. 65 m and 130 m	D. 130 m and 260 m

answer with explanation

Answer: Option B

Explanation:



Let AB and CD be the poles with heights h and 2h respectively.

Given that distance between the poles, BD = 200 m

Let E be the middle point of BD,
∠AEB = θ
∠CED = (90-θ) (∵ given that angular elevations are complementary)

Since E is the middle point of BD, we have BE = ED = 100 m

From the right △ ABE,
tanθ=ABBEtanθ=h100h = 100tanθ⋯(1)tan θ=ABBEtan θ=h100h = 100tan θ⋯(1)

From the right △ EDC,
tan(90−θ)=CDEDcotθ=2h100[∵tan(90−θ)=cotθ] 2h =100cotθ⋯(2)tan (90−θ)=CDEDcot θ=2h100[∵tan (90−θ)=cot θ] 2h =100cot θ⋯(2)

(1)×(2)=>2h2=1002(1)×(2)=>2h2=1002 [∵tanθ×cotθ=tanθ×1tanθ=1][∵tan θ×cot θ=tan θ×1tan θ=1]

=>√2h=100=>2h=100

=>h=100√2=100×√2√2×√2=50√2=50×1.41=70.5=>h=1002=100×22×2=502=50×1.41=70.5

=>2h=2×70.5=141=>2h=2×70.5=141

i.e., the height of the poles are 70.5 m and 141 m.

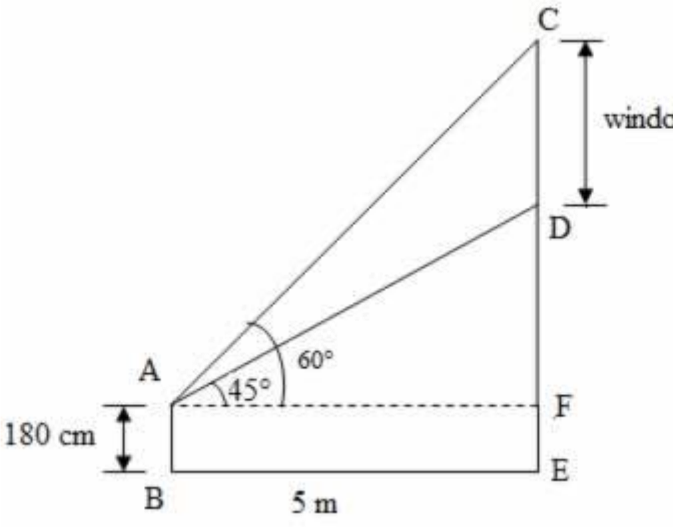
20. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and he is 5 m away from the wall, what is the length of the window?

A. 8.65 m	B. 2 m
C. 2.5 m	D. 3.65 m

answer with explanation

Answer: Option D

Explanation:



Let AB be the man and CD be the window

Given that the height of the man, AB = 180 cm, the distance between the man and the wall, BE = 5 m,
∠DAF = 45° , ∠CAF = 60°

From the diagram, AF = BE = 5 m

From the right △ AFD,
tan45°=DFAF1=DF5DF
= 5⋯(1)tan45°=DFAF1=DF5DF
= 5⋯(1)

From the right △ AFC,
tan60°=CFAF$\sqrt{\boxed{3}}$=CF5 CF=5$\sqrt{\boxed{3}}$⋯(2)tan6
0°=CFAF3=CF5 CF=53⋯(2)

Length of the window
= CD = (CF - DF)
=5$\sqrt{\boxed{3}}$−5=53−5[∵ Substituted the value of CF and DF from (1) and (2)]
=5($\sqrt{\boxed{3}}$−1)=5(1.73−1)=5×0.73=3.65 m

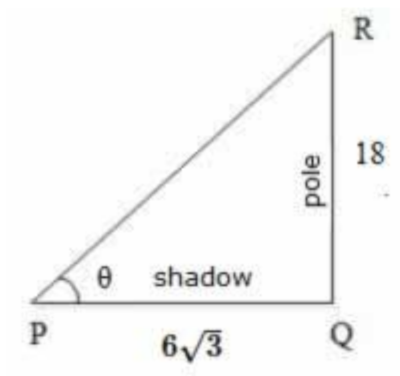
21.
Find the angle of elevation of the sun when the shadow of a pole of 18 m height is 6$\sqrt{\boxed{3}}$63m long?

A. 30° B. 60°
C. 45° D. None of these

answer with explanation

Answer: Option B

Explanation:



Let RQ be the pole and PQ be the shadow

Given that RQ = 18 m and
PQ =6$\sqrt{\boxed{3}}$=63 m

Let the angle of elevation, ∠RPQ = θ

From the right △ PQR,
tanθ=RQPQ=186$\sqrt{\boxed{3}}$=3$\sqrt{\boxed{3}}$=$\sqrt{\boxed{3}}$tanθ=RQ
PQ=1863=33=3

⇒θ=tan-1($\sqrt{\boxed{3}}$)=60°