11

Geometry and Mensuration

The chapters on geometry and mensuration have their own share of questions in the CAT and other MBA entrance examinations. For doing well in questions based on this chapter, the student should familiarise himself/herself with the basic formulae and visualisations of the various shapes of solids and two-dimensional figures based on this chapter.

The following is a comprehensive collection of formulae based on two-dimensional and three-dimensional figures:

For the purpose of this chapter we have divided the theory in two parts:

- Part I consists of geometry and mensuration of two-dimensional figures
- Part II consists of mensuration of three-dimensional figures.

PART I: GEOMETRY

INTRODUCTION

Geometry and Mensuration are important areas in the CAT examination. In the Online CAT, the Quantitative Aptitude section has consisted of an average of 15–20% questions from these chapters. Besides, questions from these chapters appear prominently in all major aptitude based exams for MBAs, Bank POs, etc.

Hence, the student is advised to ensure that he/she studies this chapter completely and thoroughly. Skills to be developed while studying and practising this chapter will be based on the application of formula and visualisation of figures and solids.

The principal skill required for doing well in this chapter is the ability to apply the formulae and theorems.

The following is a comprehensive collection of formulae based on two-dimensional figures. The student is advised to remember the formulae in this chapter so that he is able to solve all the questions based on this chapter.

THEORY

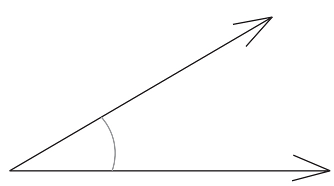
Basic Conversions

A. 1 m = 100 cm = 1000 mm 1 km = 1000 m = 5/8 miles 1 inch = 2.54 cm	B. 1 m = 39.37 inches 1 mile = 1760 yd = 5280 ft 1 nautical mile (knot) = 6080 ft
C. 100 kg = 1 quintal 10 quintal = 1 tonne = 1000 kg 1 kg = 2.2 pounds (approx.)	D. 1 litre = 1000 cc 1 acre = 100 sq m 1 hectare = 10000 sq m

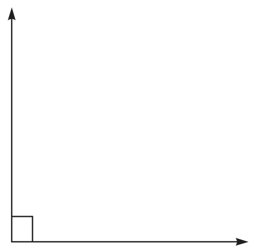
TYPES OF ANGLES

Basic Definitions

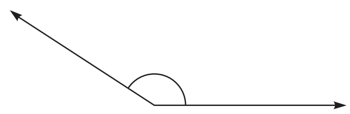
Acute angle: An angle whose measure is less than 90 degrees. The following is an acute angle.



Right angle: An angle whose measure is 90 degrees. The following is a right angle.

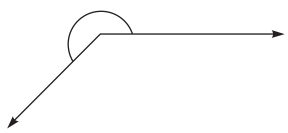


Obtuse angle: An angle whose measure is bigger than 90 degrees but less than 180 degrees. Thus, it is between 90 degrees and 180 degrees. The following is an obtuse angle.

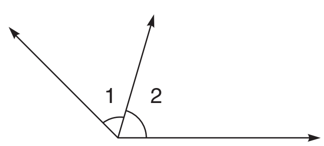


Straight angle: Is an angle whose measure is 180 degrees.

Reflex angle: An angle whose measure is more than 180 degrees but less than 360 degrees. The following is a reflex angle.

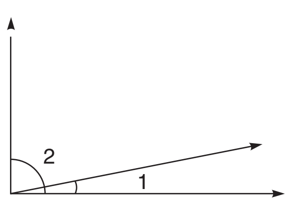


Adjacent angles: Angles with a common vertex and one common side. In the figure below, $\angle 1$ and $\angle 2$ are adjacent angles.

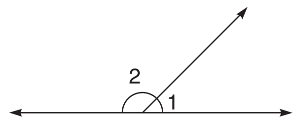


Complementary angles: Two angles whose measures add to 90 degrees $\angle 1$ and $\angle 2$ are complementary angles because together they form a right angle.

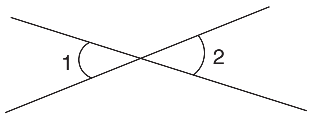
However, one thing that you should note is that, even though in the figure given here, the two angles are shown as adjacent, they need not be so to be called complementary. As long as two angles add up to 90 degrees, they would be called complementary (even if they are not adjacent to each other).



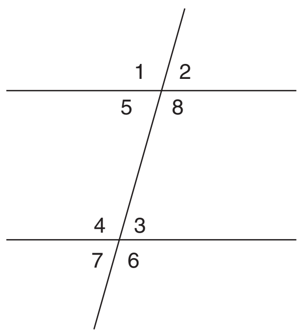
Supplementary angles: Two angles whose measures add up to 180 degrees. The following angles $\angle 1$ and $\angle 2$ are supplementary angles. However, supplementary angles do not need to be adjacent to be called supplementary (quite like complementary angles). The only condition for two angles to be called supplementary is if they are adding up to 180 degrees.



Vertical angles: Angles that have a common vertex and whose sides are formed by the same lines. The following($\angle 1$ and $\angle 2$) are vertical angles.



Angles formed when two parallel lines, are crossed by a transversal: When two parallel lines are crossed by a third line, (transversal), 8 angles are formed. Take a look at the following figure:



Angles 3,4,5,8 are interior angles.
Angles 1,2,6,7 are exterior angles.

Alternate interior angles: Pairs of interior angles on opposite sides of the transversal.

For instance, angle 3 and angle 5 are alternate interior angles. Angle 4 and angle 8 are also alternate interior angles. Both the angles in a pair of alternate interior angles are equal. Hence, in the figure we have: Angle 3 = Angle 5; Also Angle 4 = Angle 8.

Alternate exterior angles: Pairs of exterior angles on opposite sides of the transversal.

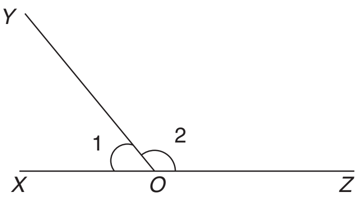
Angle 2 and angle 7 are alternate exterior angles. Angles 1 and 6 are also alternate exterior angles. Both the angles in a pair of alternate exterior angles are equal. Thus, in the figure Angle 2 = Angle 7 and Angle 1 = Angle 6.

Co-interior angles: When two lines are cut by a third line (transversal) co-interior angles are between the pair of lines on the same side of the transversal. If the lines that are being cut by the transversal are parallel to each other, the co-interior angles are supplementary (add up to 180 degrees). In the given figure, angles 3 and 8 are co-interior angles. Also, angles 4 and 5 are co-interior angles, since, the lines being cut are parallel in this case, $\angle 3 + \angle 8 = 180$. Also, $\angle 4 + \angle 5 = 180$.

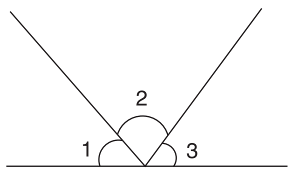
Corresponding angles: Are pairs of angles that are in similar positions when two parallel lines are intersected by a transversal.

Angle 3 and angle 2 are corresponding angles. Similarly, the pairs of angles, 1 and 4; 5 and 7; 6 and 8 are corresponding angles. Corresponding angles are equal. Thus, in the figure- $\angle 1 = \angle 4$; $\angle 5 = \angle 7$; $\angle 2 = \angle 3$ & $\angle 6 = \angle 8$.

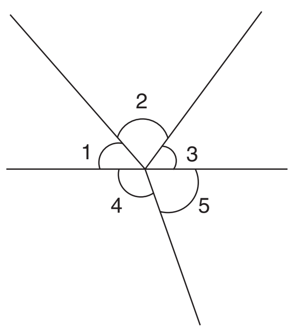
Linear pair: $\angle XOY$ and $\angle YOZ$ are linear pair angles. One side must be common (e.g. OY)and these two angles must be supplementary.



Angles on the side of a line: $\angle 1 + \angle 2 + \angle 3 = 180°$

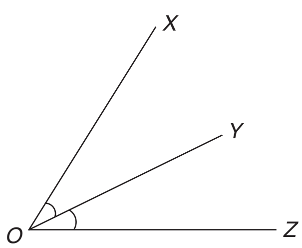


Angles around the point: $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$



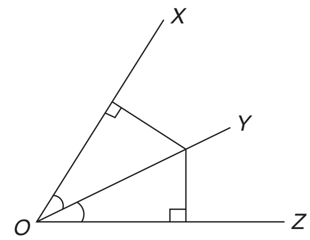
Angle Bisector: OY is the angle bisector for the $\angle$XOZ.

i.e., $\angle XOY = \angle ZOY = \frac{1}{2}\angle XOZ$



When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OY).

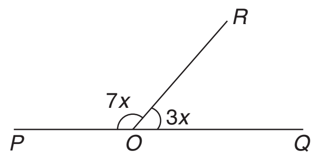
(Angle bisector is equidistant from the two sides of the angle.)



The disance between the lines OX & OY and the lines OY & OZ are equal to each other.

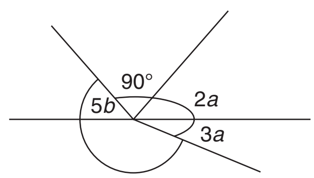
PRACTICE EXERCISE

1. What is the value of x in the given figure?



(a) 18° (b) 20°
(c) 28° (d) None of these

2. In the given figure, find the value of $(a + b)$



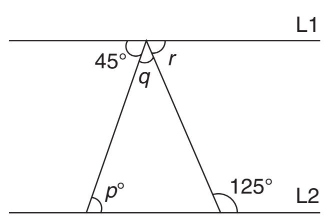
(a) 50° (b) 54°
(c) 60° (d) None of these

3. If $2a + 3$, $3a + 2$ are complementary, then a = ?
(a) 17° (b) 20°
(c) 23° (d) 26°

4. If $5x + 17°$ and $x + 13°$ are supplementary, then x = ?
(a) 20° (b) 25°
(c) 30° (d) None of these

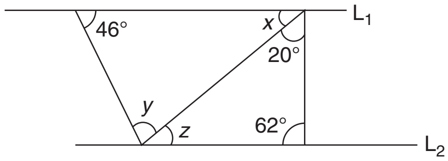
5. An angle is exactly half of its complementary angle, then find the angle.
(a) 30° (b) 40°
(c) 50° (d) 60°

6. In the following figure, lines L1 and L2 are parallel to each other. Find the value of q.



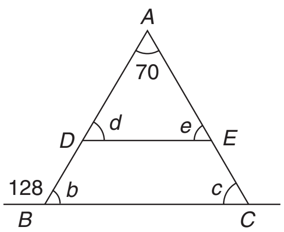
(a) 60° (b) 80°
(c) 90° (d) 85°

7. In the given figure if $L_1 \| L_2$ then values of x, y, z are:



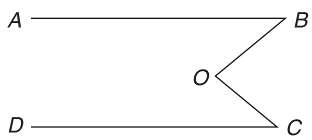
(a) 98°, 98°,36° (b) 98°, 36°, 98°
(c) 36°,98°,36° (d) None of these

8. In the given diagram if $BC \| ED$ and $\angle BAC = 70°$, then find the value of d and c.



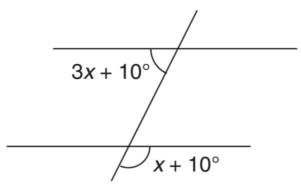
(a) 52°, 58° (b) 58°, 52°
(c) 44°, 36° (d) 36°, 44°

9. In the given diagram if $AB \| CD$ and $\angle ABO = 60°$ and $\angle BOC = 110°$, find $\angle OCD$



(a) 40° (b) 50°
(c) 60° (d) 70°

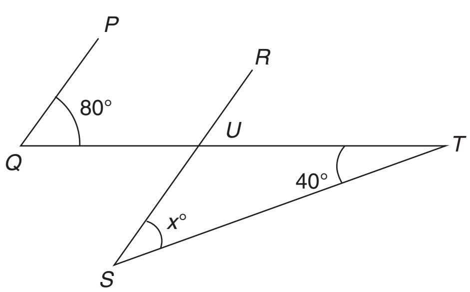
10. In the figure given, two parallel lines are intersected by a transversal. Then, find the value of x.



(a) 40° (b) 50°
(c) 55° (d) 65°

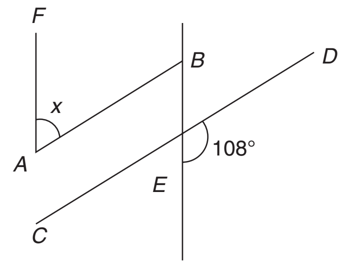
11. Maximum number of points of intersection of five lines on a plane is
(a) 6 (b) 8
(c) 10 (d) 12

12. If $PQ \| RS$ then find the value of x.



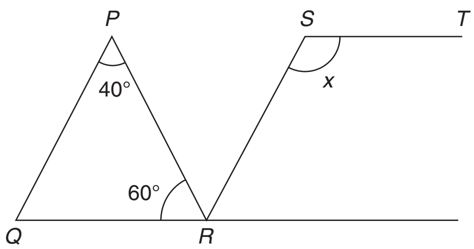
(a) 40° (b) 60°
(c) 70° (d) 80°

13. If $AB \| CD$ and $AF \| BE$ then the value of x is:



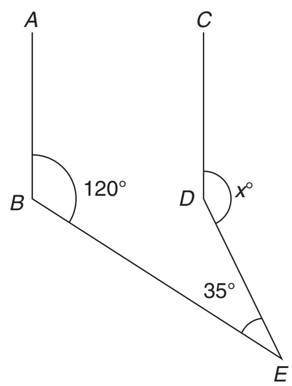
(a) 108° (b) 72°
(c) 88° (d) 82°

14. In the figure if $PQ \| SR$ and $ST \| QR$ then x = ?



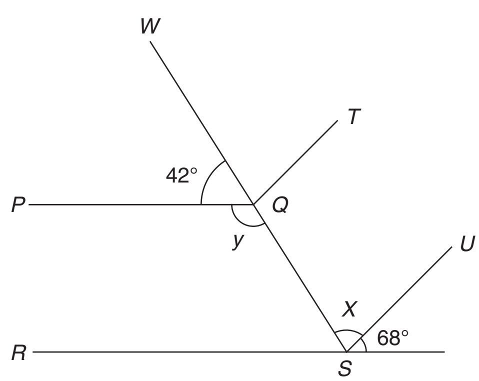
(a) 70° (b) 80°
(c) 90° (d) 100°

15. In the given figure, if $AB \| CD$ then the value of x =?



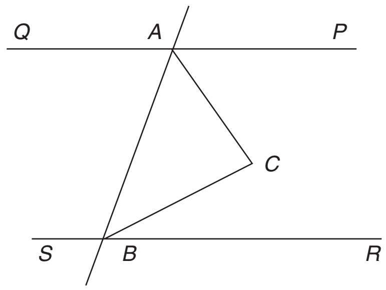
(a) 135° (b) 145°
(c) 155° (d) None of these

16. If $PQ \| RS$ and $QT \| SU$ then find the value of $x + y$



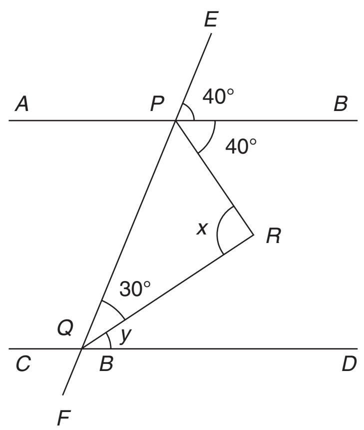
(a) 188° (b) 202°
(c) 208° (d) 212°

17. If $PQ \| RS$ and AC is angle bisector of $\angle PAB$, BC is angle bisector of $\angle RBA$. Then $\angle ACB$ = ?



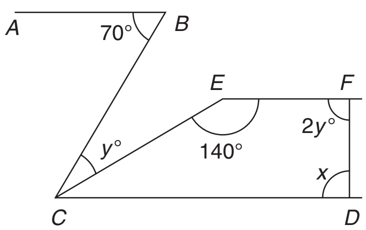
(a) 45° (b) 75°
(c) 90° (d) 110°

18. In the given figure if $AB \| CD$ then $x + y$ = ?



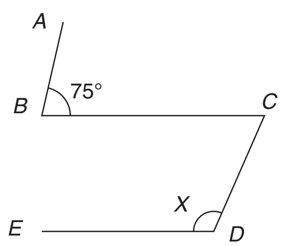
(a) 100° (b) 110°
(c) 60° (d) 125°

19. In the figure if $CD \| EF \| AB$ then, find the value of x.



(a) 70° (b) 90°
(c) 110° (d) 120°

20. If in the given figure, $AB \| CD$ and $BC \| DE$, then x =?



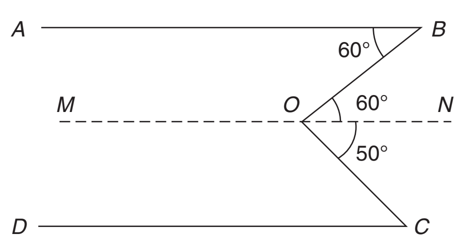
(a) 95° (b) 105°
(c) 115° (d) 125°

ANSWER KEY

1 (a)	2 (b)	3 (a)	4 (b)
5 (a)	6 (b)	7 (b)	8 (a)
9 (b)	10 (a)	11 (c)	12 (a)
13 (b)	14 (d)	15 (c)	16 (c)
17 (c)	18 (c)	19 (d)	20 (b)

Solutions

1. $7x + 3x = 180°$
   $10x = 180°$ or $x = 18°$.
   Option (a) is correct.
2. $90° + 2a + 3a + 5b = 360°$
   $5a + 5b = 270°$
   $a + b = 54°$
3. $2a + 3 + 3a + 2 = 90°$
   $5a + 5 = 90°$
   $$a = \frac{85°}{5} = 17°$$
4. $5x + 17° + x + 13° = 180°$
   $6x + 30° = 180°$
   $x = 25°$.
   Option b is correct.
5. We can solve this problem by checking the options. Option (a) = 30°, complementary angle of 30° is 60° and 30° is half of 60°.
   So option (a) is true. Alternately, we can also solve this using: $a + 2a = 90 \rightarrow a = 30°$.
6. $q + r = 180° - (45°) = 135°$
   $r + 125° = 180° \Rightarrow r = 55°$
   $q + 55° = 135°$
   $q = 80°$
7. $x + 20 + 62 = 180°$
   $x = 180° - 82° = 98°$
   $x = z$ [Alternate angles]
   $z = 98°$
   $x + y + 46° = 180°$
   $y = 180° - (46° + x) = 180° - (46° + 98°) = 36°$.
8. $\angle b = 180° - 128° = 52° = \angle d$ (Since they are corresponding angles).
   $\angle c = 180° - (70° + 52°) = 58°$
9. Draw line $MON \| AB \| CD$



$\angle ABO = \angle BON$ [Alternate angles]

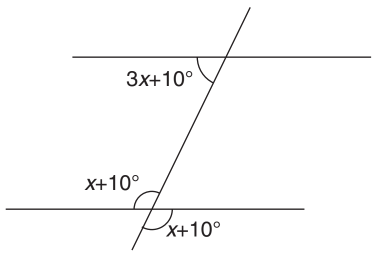
Hence, $\angle BON = 60^o$

$\angle NOC = 110^o - 60^o = 50^o$

Also, $\angle NOC = \angle OCD$ [Alternate angles]

$\angle OCD = 50^o$

10. $3x + 10^o + x + 10^o = 180^o$



$4x = 160^o$

$x = 40^o$

11. $^5C_2 = \dfrac{5!}{2! \times 3!} = 10$

Option (c) is correct.

12. $\angle PQU = \angle SUQ = 80^o$ [Alternate angles]

$\angle SUT = 180^o - 80^o = 100^o$

$\angle UST = x = 40^o$

13. If $AB \| CD$ then $\angle CEB + \angle ABE = 180^o$

$\angle CEB = 108^o$

$108^o + \angle ABE = 180^o$

$\angle ABE = 72^o$

If $AB \| BD$ then $\angle ABE = x$ [Alternate angles]

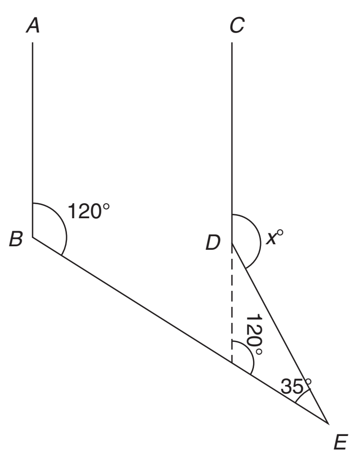
$x = 72^o$

14. $\angle QPR = \angle SRP = 40^o$ [Alternative angles]

$x = \angle SRQ = \angle SRP + \angle PRQ$ [Alternative angles]

$x = 60^o + 40^o = 100^o$

15. Extend CD to x



$\angle ABX = \angle CXE = 120^o$

$x = 120^o + 35^o$ [x is exterior angle of ΔDXE)

$x = 155^o$

16. $y = 180^o - 42 = 138^o$

$\angle PQW = \angle RSQ = 42^o$

$42^o + x + 68^o = 180^o$

$x = 70^o$

$x + y = 70^o + 138^o = 208^o$

17. $\angle ACB = 180^o - (\angle CAB + \angle CBA)$

$\angle PAB + \angle RBA = 180^o$

$\dfrac{\angle PAB}{2} + \dfrac{\angle RBA}{2} = 90°$

$\angle CAB + \angle CBA = 90^o$

$\angle ACB = 180^o - (90^o) = 90^o$

18. $\angle RPQ = 180^o - (40^o + 40^o) = 100^o$

$x = 180^o - (100^o + 30^o) = 50^o$

$30^o + y = 40^o$

($\angle BPQ$ and $\angle DQP$ are corresponding angles).

$y = 10^o$

$x + y = 50^o + 10^o = 60^o$

19. $\angle ABC = \angle BCD = 70^o$ [Alternate angles]

$\angle ECD = 180^o - 140^o = 40^o$

[As angles FEC and ECD are co-interior angles]

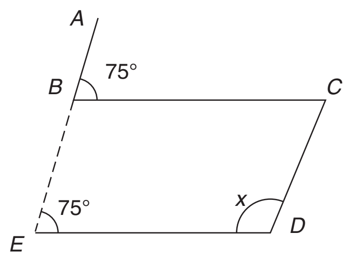
$40^o + y = 70^o$

$y = 30^o$

$EF \| CD$ then $2y + x = 180^o$

$x = 180^o - 2y = 180^o - 60^o = 120^o$

20. Extend AB to E



$BC \| DE$, so $\angle ABC = \angle AFD = 75^o$

$CD \| BE$, hence $x + 75^o = 180^o$

$x = 105^o$

POLYGONS

Polygons are plane figures formed by a closed series of rectilinear (straight) segments. The following are examples of polygons:

Triangle, Rectangle, Pentagon, Hexagon, Heptagon, Octagon, nonagon (9 sided), decagon, Undecagon or Hendecagon (11 sided), Dodecagon (12 sided), Triskaidecagon or Tridecagon (13 sided). Subsequent polygons are named as per the table below:

Number of sides	*Name of the Polygon*
14	Tetradecagon, Terakaidecagon
15	Pentadecagon, Pentakaidecagon
16	Hexadecagon, Hexakaidecagon

17	Heptadecagon, Heptakaidecagon
18	Octadecagon, Octakaidecagon
19	Enneadecagon, Enneakaidecagon
20	Icosagon
30	Triacontagon
40	Tetracontagon
50	Pentacontagon
60	Hexacontagon
70	Heptacontagon
80	Ontacontagon
90	Enneacontagon
100	Hectogon, Hecatontagon
1000	Chiliagon
10000	Myriagon

Polygons can broadly be divided into two types:

(a) *Regular polygons:* Polygons with all the sides and angles equal.

(b) *Irregular polygons:* Polygons in which all the sides or angles are not of the same measure.

Polygon can also be divided as *concave* or *convex* polygons.

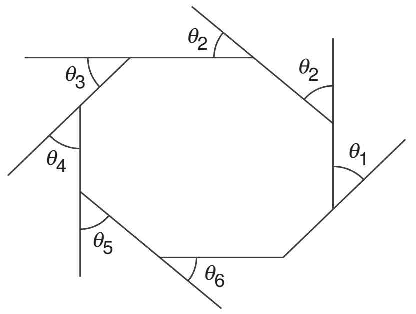
Convex polygons are the polygons in which all the diagonals lie inside the figure otherwise it's a concave polygon

Polygons can also be divided on the basis of the number of sides they have.

No. of sides	*Name of the polygon*	*Sum of all the angles*
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	500°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°

Properties

1. Sum of all the angles of a polygon with n sides $= (2n - 4)\pi/2$ or $(n - 2)\pi$ Radians $= (n - 2)\ 180°$ degrees
2. Sum of all exterior angles = 360°.
   i.e. In the figure below:
   $\theta_1 + \theta_2 + ... + \theta_6 = 360°$
   In general, $\theta_1 + \theta_2 + ... + \theta_n = 360°$
3. No. of sides = 360°/exterior angle.
   (Note: This property is true only for regular polygons)
4. Area $= (ns^2/4) \times \cot(180/n)$; where s = length of side, n = no. of sides.
   (Note: This property is true only for regular polygons)
5. Perimeter $= n \times s$.
   (Note: This property is true only for regular polygons)



PRACTICE EXERCISE

1. Each interior angle of a regular polygon is 140°. Then the number of sides is:
   (a) 6 (b) 8
   (c) 9 (d) 12
2. Each interior angle of a regular octagon is:
   (a) 90° (b) 115°
   (c) 125° (d) 135°
3. The sum of the interior angles of a polygon is 1440°. The number of sides of the polygon is:
   (a) 8 (b) 10
   (c) 12 (d) 14
4. Difference between interior and exterior angle of a polygon is 100°. Then the number of sides in the polygon is:
   (a) 8 (b) 9
   (c) 10 (d) 11
5. If the ratio of interior and exterior angles of a regular polygon is 2:1, then find the number of sides of the polygon.
   (a) 6 (b) 8
   (c) 10 (d) 12
6. The ratio of the measure of an angle of a regular octagon to the measure of its exterior angle is:
   (a) 2:1 (b) 1:3
   (c) 3:1 (d) 1:1
7. Ratio between, the number of sides of two regular polygons is 2:3 and the ratio between their interior angles is 3:4. The number of sides of these polygons respectively are:
   (a) 4,6 (b) 6,9
   (c) 8,12 (d) None of these
8. Number of diagonals of a 6-sided polygon is
   (a) 6 (b) 9
   (c) 12 (d) 15
9. Find the sum of all internal angles of a 5-point star.
   (a) 160° (b) 180°
   (c) 240° (d) 300°

10. If the length of each side of a hexagon is 6 cm, then the area of the hexagon is:
(a) 54 cm^2 (b) $54\sqrt{3}$ cm^2
(c) 68 cm^2 (d) None of these

ANSWER KEY

1. (c)	2. (d)	3. (b)	4. (b)
5. (a)	6. (c)	7. (a)	8. (b)
9. (b)	10. (b)		

Solutions

1. Exterior angle of given polygon = 180° – 140° = 40°
Number of sides = 360°/40° = 9. [Since the sum of all exterior angles of a polygon is 360°)
Option (c) is correct.
2. Total number of sides in octagon = 8
Each interior angle = $\frac{(8-2)\times 180°}{8} = \frac{6\times 180°}{8}$ = 135°
3. Let the number of sides be x.
Then according to the question
$(x – 2) \times 180° = 1440°$
$x – 2 = 8$
$x = 10$.
4. Let the internal angle be x and external angle be y, according to the question
$x + y = 180°$ (i)
$x – y = 100°$ (ii)
$x = 140°$, $y = 40°$
Number of sides = $\frac{360°}{40°}$ = 9
5. If interior angle '$2x$' and exterior angle be x
Then $2x + x = 180°$
$3x = 180°$
$x = 60°$
Number of sides = 360°/60° = 6
6. Interior angle of a regular octagon = 135°
Exterior angle of a regular octagon = 45°
Required ratio = $\frac{135°}{45°}$ = 3:1
7. We can solve this problem by checking the options.
Option (a) 4, 6
Interior angle of a 4 sided polygon = 90°
Interior angle of a 6-sided polygon = 120°
So the ratio of interior angles = 90°:120° = 3:4
Hence this option is correct.
8. Number of diagonals = ${}^6C_2 – 6$
$= \frac{6!}{2!4!} - 6 \Rightarrow 15 – 6 = 9$
9. Sum of the angles of an x-pointed star = $(x – 4) \times \pi$
So the required sum = $(5 – 4) \times \pi$ = 180°
10. Required area = $6\times\frac{\sqrt{3}}{4}\times 6^2$
$= 54\sqrt{3}$ cm^2

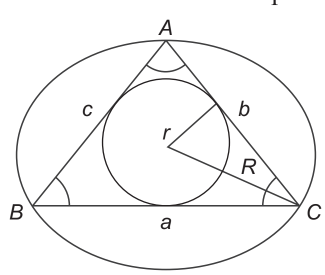
TRIANGLES (Δ)

A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.

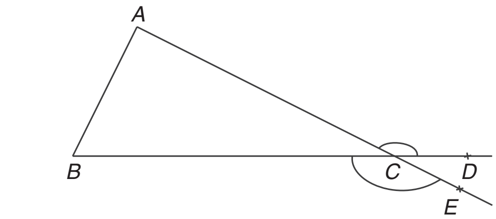
Types 1. *Acute angle triangle:* Triangles with all three angles acute (less than 90°).
2. *Obtuse angle triangle:* Triangles with one of the angles obtuse (more than 90°).
Note: We cannot have more than one obtuse angle in a triangle.
3. *Right angle triangle:* Triangle with one of the angles equal to 90°.
4. *Equilateral triangle:* Triangle with all sides equal. All the angles in such a triangle measure 60°.
5. *Isosceles triangle:* Triangle with two of its sides equal and consequently the angles opposite the equal sides are also equal.
6. *Scalene Triangle:* Triangle with none of the sides equal to any other side.

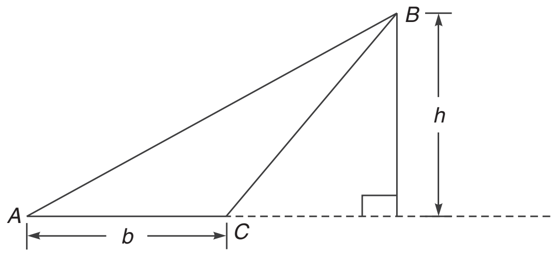
Properties (General)

- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: $a/\sin A = b/\sin B = c/\sin C = 2R$ (where R = circum radius.)
- The cosine rule: $a^2 = b^2 + c^2 – 2bc \cos A$
This is true for all sides and respective angles.



In case of a right triangle, the formula reduces to $a^2 = b^2 + c^2$
Since cos 90° = 0
- The exterior angle is equal to the sum of two interior angles not adjacent to it.
$\angle ACD = \angle BCE = \angle A + \angle B$

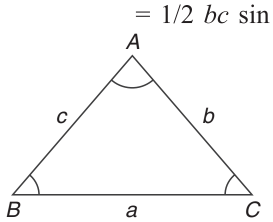




Area

1. Area = 1/2 base × height or 1/2 bh.
   Height = Perpendicular distance between the base and vertex opposite to it
2. Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)
   where $s = \dfrac{a+b+c}{2}$ (a, b and c being the length of the sides)
3. Area = rs (where r is in radius)
4. Area = 1/2 × product of two sides × sine of the included angle

$$= 1/2\ ac \sin B$$
$$= 1/2\ ab \sin C$$
$$= 1/2\ bc \sin A$$



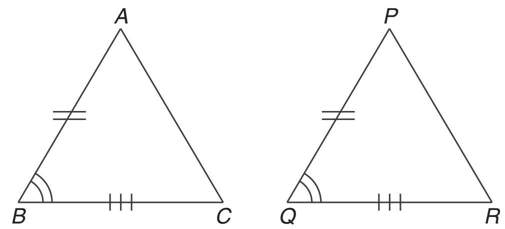
4. Area = $abc/4R$
   where R = circum radius

Congruency of Triangles Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is $(\cong)$.

Conditions for Congruency

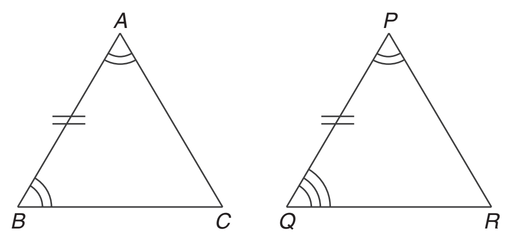
1. *SAS congruency*: If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent. (See figure below.)

   Here, $AB = PQ$
   $BC = QR$
   and $\angle B = \angle Q$
   So $\Delta ABC \cong \Delta PQR$



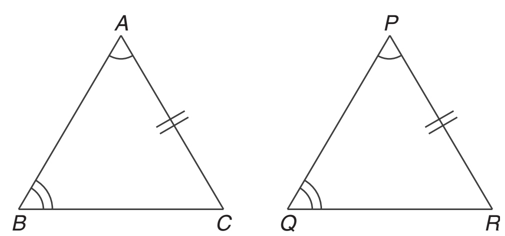
2. *ASA congruency*: If two angles and the included side of one triangle is equal to two angles and the included side of another, the triangles are congruent. (See figure below.)

   Here, $\angle A = \angle P$
   $\angle B = \angle Q$
   and $AB = PQ$
   So $\Delta ABC \cong \Delta PQR$



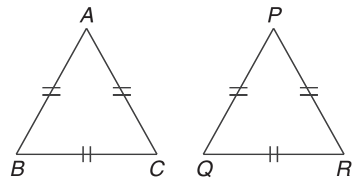
3. *AAS congruency*: If two angles and side opposite to one of the angles is equal to the corresponding angles and the side of another triangle, the triangles are congruent. In the figure below:

   $\angle A = \angle P$
   $\angle B = \angle Q$
   and $AC = PR$
   So $\Delta ABC \cong \Delta PQR$



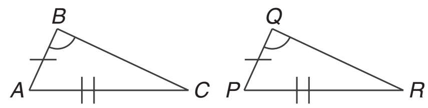
4. *SSS congruency*: If three sides of one triangle are equal to three sides of another triangle, the two triangles are congruent. In the figure below:

   $AB = PQ$
   $BC = QR$
   $AC = PR$
   $\therefore \Delta ABC \cong \Delta PQR$



5. *SSA congruency*: If two sides and the angle opposite the greater side of one triangle are equal to the two sides and the angle opposite to the greater side of another triangle, then the triangles are congruent. The congruency doesn't hold if the equal angles lie opposite the shorter side. In the figure below, if

$$AB = PQ$$
$$AC = PR$$
$$\angle B = \angle Q$$



Then the triangles are congruent.

i.e. $\Delta ABC \cong \Delta PQR$.

Similarity of triangles Similarity of triangles is a special case where if either of the conditions of similarity of polygons holds, the other will hold automatically.

Types of Similarity

1. *AAA similarity:* If in two triangles, corresponding angles are equal, that is, the two triangles are equiangular then the triangles are similar.

   Corollary (*AA similarity*)*:* If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar. The reason being, the third angle becomes equal automatically.
2. *SSS similarity:* If the corresponding sides of two triangles are proportional then they are similar.

   For ΔABC to be similar to ΔPQR, $AB/PQ = BC/QR = AC/PR$, must hold true.
3. *SAS similarity:* If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

   $$\Delta ABC \sim \Delta PQR$$

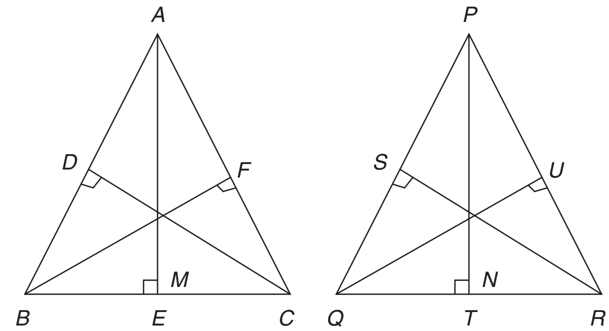
   If $AB/BC = PQ/QR$ and $\angle B = \angle Q$

Note: In similar triangles; the following identity holds:

Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of inradii = Ratio of angle bisectors

Properties of similar triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



1. Ratio of sides = Ratio of heights (altitudes)
   = Ratio of medians
   = Ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii
2. Ratio of areas = Ratio of square of corresponding sides.

   i.e., if $\Delta ABC \sim \Delta PQR$, then

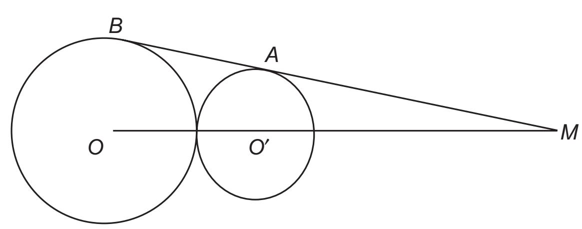
   $$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

While there are a lot of methods through which we see similarity of triangles, the one thing that all our Maths teachers forgot to tell us about similarity is the basic real life concept of similarity. i.e. **Two things are similar if they look similar!!**

If you have been to a toy shop lately, you would have come across models of cars or bikes which are made so that they look like the original—but are made in a different size from the original. Thus you might have seen a toy Maruti car which is built in a ratio of 1:25 of the original car. The result of this is that the toy car would look very much like the original car (of course if it is built well!!). Thus if you have ever seen a father and son looking exactly like each other, you have experienced similarity!!

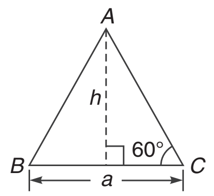
You should use this principle to identify similar triangles. In a figure two triangles would be similar simply if they look like one another.

Thus, in the figure below if you were to draw the radii OB and O′A the two triangles MOB and MO′A will be similar to each other. Simply because they look similar. Of course, the option of using the different rules of similarity of triangles still remains with you.



Equilateral Triangles (of side *a*):

1. $h = \frac{a\sqrt{3}}{2}$ $(\because \sin 60 = \sqrt{3}/2 = h/\text{side})$
2. Area = 1/2 (base) × (height) = $\frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

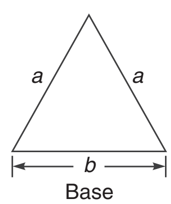


3. R (circum radius) = $\frac{2h}{3} = \frac{a}{\sqrt{3}}$.
4. r (in radius) = $\frac{h}{3} = \frac{a}{2\sqrt{3}}$.

Properties

1. The incentre and circumcentre lies at a point that divides the height in the ratio 2 : 1.
2. The circum radius is always twice the in radius. [$R = 2r$.]
3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
4. An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

Isosceles Triangle



Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

In an isosceles triangle, the angles opposite to the equal sides are equal.

Right-Angled Triangle

Pythagoras Theorem In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In the figure below, for triangle ABC, $a^2 = b^2 + c^2$

Area = 1/2 (product of perpendicular sides)

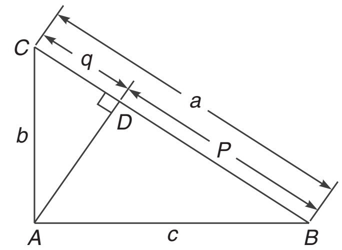
$R(\text{circumradius}) = \frac{\text{hypotenuse}}{2}$

Area = rs

(where r = in radius and $s = (a + b + c)/2$ where a, b and c are sides of the triangle)

$\Rightarrow \quad 1/2\ bc = r(a + b + c)/2$

$\Rightarrow \quad r = (bc)/(a + b + c)$



In the triangle ABC,

$$\Delta ABC \sim \Delta DBA \sim \Delta DAC$$

(*Note:* A lot of questions are based on this figure.)

Further, we find the following identities:

1. $\Delta ABC \sim \Delta DBA$

$\therefore \quad AB/BC = DB/BA$

$\Rightarrow \quad AB^2 = DB \times BC$

$\Rightarrow \quad c^2 = pa$

2. $\Delta ABC \sim \Delta DAC$

$AC/BC = DC/AC$

$\Rightarrow \quad AC^2 = DC \times BC$

$\Rightarrow \quad b^2 = qa$

3. $\Delta DBA \sim \Delta DAC$

$DA/DB = DC/DA$

$DA^2 = DB \times DC$

$\Rightarrow \quad AD^2 = pq$

Basic Pythagorean Triplets

→ 3, 4, 5 → 5, 12, 13 → 7, 24, 25 → 8, 15, 17 → 9, 40, 41 → 11, 60, 61 → 12, 35, 37 → 16, 63, 65 → 20, 21, 29 → 28, 45, 53. These triplets are very important since a lot of questions are based on them.

Any triplet formed by either multiplying or dividing one of the basic triplets by any positive real number will be another Pythagorean triplet.

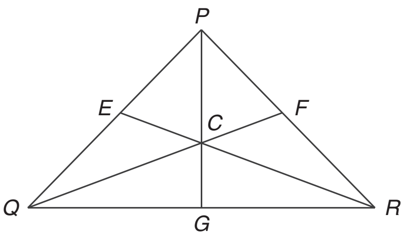
Thus, since 3, 4, 5 form a triplet so also will 6, 8 and 10 as also 3.3, 4.4 and 5.5.

Similarity of Right Triangles Two right triangles are similar if the hypotenuse and side of one is proportional to hypotenuse and side of another. (RHS–similarity–Right angle hypotenuse side).

Important Terms with Respect to a Triangle

1. Median A line joining the mid-point of a side of a triangle to the opposite vertex is called a median. In the figure the three medians are PG, QF and RE where G, E and F are mid-points of their respective sides.

- A median divides a triangle into two parts of equal area.
- The point where the three medians of a triangle meet is called the *centroid* of the triangle.
- The centroid of a triangle divides each median in the ratio 2 : 1.
  i.e. $PC:CG = 2:1 = QC:CF = RC:CE$



Important formula with respect to a median

$\rightarrow$ $2 \times (\text{median})^2 + 2 \times (1/2 \text{ the third side})^2$

= Sum of the squares of other two sides

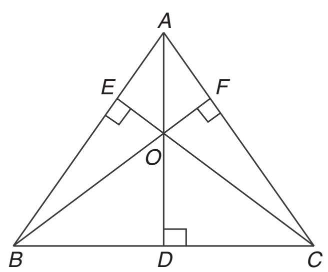
$$\Rightarrow \quad 2(PG)^2 + 2 \times \left(\frac{QR}{2}\right)^2$$

$= (PQ)^2 + (PR)^2$

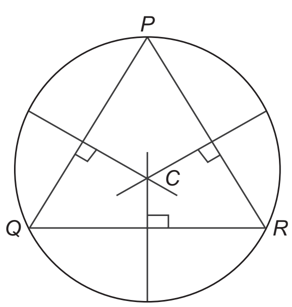
2. Altitude/Height A perpendicular drawn from any vertex to the opposite side is called the *altitude*. (In the figure, AD, BF and CE are the altitudes of the triangles).

- All the altitudes of a triangle meet at a point called the *orthocentre* of the triangle.
- The angle made by any side at the orthocentre and the vertical angle make a supplementary pair (i.e. they both add up to 180°). In the figure below:

$$\angle A + \angle BOC = 180° = \angle C + \angle AOB$$



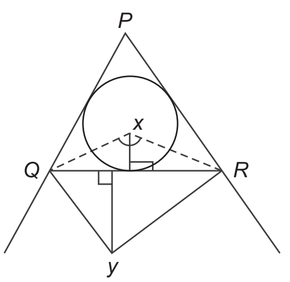
3. Perpendicular Bisectors A line that is a perpendicular to a side and bisects it is the perpendicular bisector of the side.



- The point at which the perpendicular bisectors of the sides meet is called the *circumcentre* of the triangle
- The circumcentre is the centre of the circle that circumscribes the triangle. There can be only one such circle.
- Angle formed by any side at the circumcentre is two times the vertical angle opposite to the side. This is the property of the circle whereby angles formed by an arc at the centre are twice that of the angle formed by the same arc in the opposite arc. Here we can view this as:
  $\angle QCR = 2\ \angle QPR$ (when we consider arc QR and it's opposite arc QPR)

4. Incenter

- The lines bisecting the interior angles of a triangle are the angle bisectors of that triangle.
- The angle bisectors meet at a point called the *incentre* of the triangle.
- The incentre is equidistant from all the sides of the triangle.

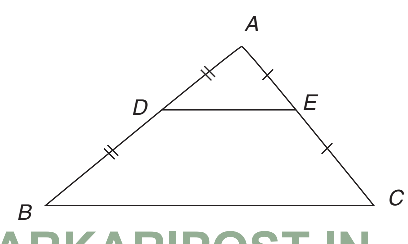


- From the incentre with a perpendicular drawn to any of the sides as the radius, a circle can be drawn touching all the three sides. This is called the *incircle* of the triangle. The radius of the incircle is known as *inradius*.
- The angle formed by any side at the incentre is always a right angle more than half the angle opposite to the side.

  This can be illustrated as $\angle QXR = 90 + \frac{1}{2}\angle P$
- If QY and RY are the angle bisectors of the exterior angles at Q and R, then:
  $\angle QYR = 90 - \frac{1}{2}\angle P$

Mid-Point Theorem

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half the third side.

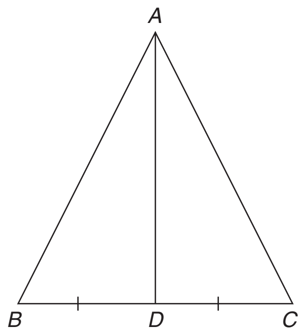


$$AD = BD \text{ and } AE = CE$$
$$DE \parallel BC$$

Apollonius' theorem

"The sum of the squares of any two sides of any triangle equals twice the square on half the third side plus twice the square of the median bisecting the third side"

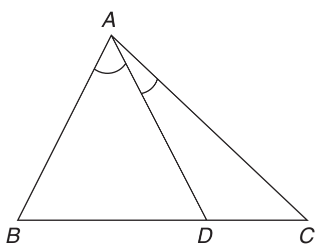
Specifically, in any triangle ABC, if AD is a median, then



$$BD = CD$$
AD is the median
$$AB^2 + AC^2 = 2(AD^2 + BD^2).$$

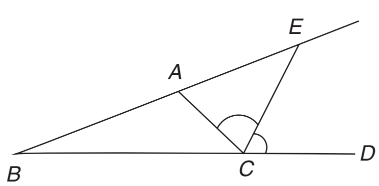
Angle bisector theorem

In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$ and $BD \times AC - CD \times AB = AD^2$



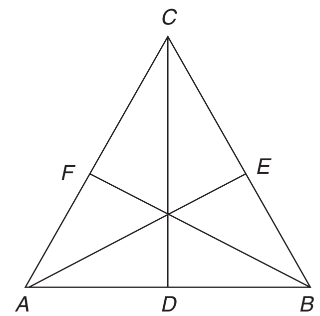
Exterior angle bisector theorem

In a triangle the angle bisector (represented by CE in the figure) of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e, $\frac{BE}{AE} \quad \frac{BC}{AC}$

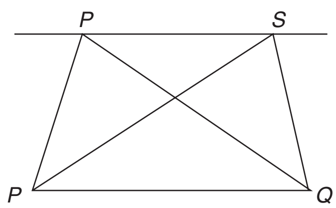


Few important results:

1. In a triangle AE, CD and BF are the medians then $3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$



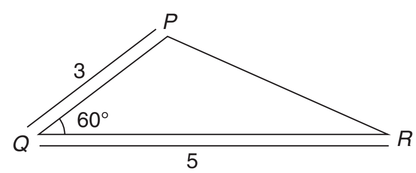
2. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.



i.e. Area (ΔPQR) = Area(ΔPQS)

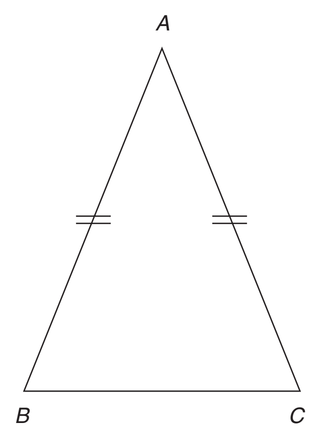
PRACTICE EXERCISE

1. Find the area of ΔPQR



(a) $\frac{13\sqrt{3}}{4}$ (b) $\frac{15\sqrt{3}}{4}$
(c) $5\sqrt{3}$ (d) None of these

2. In ΔABC, $AB = AC = 5$ cm, $BC = 4$ cm, then find the area of ΔABC



(a) $12\sqrt{3}$ cm^2 (b) $2\sqrt{21}$ cm^2
(c) $\sqrt{42}$ cm^2 (d) None of these

3. If we draw a Δ ABC inside a circle (A, B, C are on the circumference of a circle). Then area of the ΔABC is maximum when:
(a) $AB = BC \neq AC$
(b) $AB = BC = CA$
(c) $\angle BAC = 90^o$
(d) ΔABC is obtuse angle triangle

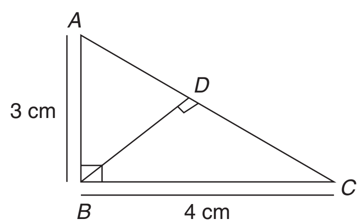
4. If height of an equilateral triangle is 10 cm, its area will be equal to:
(a) $100\sqrt{3}$ cm^2 (b) $\frac{100}{3}\sqrt{3}$ cm^2
(c) $\frac{100}{3}$ cm^2 (d) $\frac{200\sqrt{3}}{3}$ cm^2

5. Find the area of a triangle whose sides are 11, 60, 61
(a) 210 (b) 330
(c) 315 (d) 275

6. If AD, BE, CF are medians of a ΔABC and O is the centroid of ΔABC. If area of ΔAOF is 36 cm^2 then the area of ΔOFB + Area of ΔOEC = ?
(a) 36 cm^2 (b) 54 cm^2
(c) 72 cm^2 (d) None of these

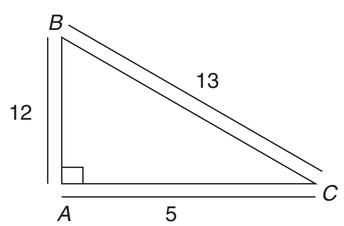
7. If three sides of a triangle are 5, 12, 13 then the circumradius of the triangle is:
(a) 6cm (b) 2.5cm
(c) 6.5cm (d) None of these

8. ΔABC, $\angle B = 90°$, $BD \perp AC$ then BD = ?



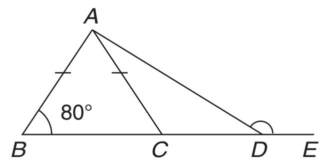
(a) 2.2 cm (b) 2.4 cm
(c) 2.6 cm (d) None of these

9. If $\angle A = 90°$ then in radius of ΔABC is:



(a) 2 cm (b) 4 cm
(c) 6 cm (d) 8 cm

10. In ΔABC $AB = AC$, $\angle B = 80°$, $\angle BAD = 90°$, $\angle ADE$ = ?



11. AD is the median of the triangle ABC and O is the centroid such that AO = 12 cm. The length of OD in cm is
(a) 4 (b) 5
(c) 6 (d) 8

12. If ΔABC is an isosceles triangle with $\angle C = 90°$ and AC = 7 cm, then AB is:
(a) 8.5 cm (b) 8.2 cm
(c) $7\sqrt{2}$ cm (d) 7.5 cm

13. In Δ ABC, $AB = AC$, $\angle BAC = 50°$, now CB is extended to D, then the external angle at $\angle DBA$ is:
(a) 90^o (b) 70^o
(c) 115^o (d) 80^o

14. The sides of a triangle are in the ratio 4:5:6. The triangle is:
(a) acute-angled
(b) right-angled
(c) obtuse-angled
(d) either acute-angled or right angled.

15. The sum of three altitudes of a triangle is
(a) equal to the sum of three sides
(b) less than the sum of sides
(c) $1/\sqrt{2}$ times of the sum of sides
(d) half the sum of sides

16. Two medians PS and RT of ΔPQR intersect at G at right angles. If PS = 9 cm and RT = 6 cm, then the length of RS in cm is
(a) 10 (b) 6
(c) 5 (d) 3

17. Two triangles ABC and PQR are similar to each other in which AB = 5 cm, PQ = 4 cm. Then the ratio of the areas of triangles ABC and PQR is
(a) 4:5 (b) 25:16
(c) 64:125 (d) 4:7

18. In ΔABC, the internal bisectors of $\angle ACB$ & $\angle ABC$ meet at X and $\angle BAC = 30°$. The measure of $\angle BXC$ is
(a) 95^o (b) 105^o
(c) 125^o (d) 130^o

19. The area of an equilateral triangle is $900\sqrt{3}$ sqm. Its perimeter is:
(a) 120 m (b) 150 m
(c) 180 m (d) 135 m

20. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid points of this triangle is:
(a) 6 (b) 3
(c) 3/2 (d) 3/4

ANSWER KEY

1. (b)	2. (b)	3. (b)	4. (b)
5. (b)	6. (c)	7. (c)	8. (b)
9. (a)	10. 170^0	11. (c)	12. (c)
13. (c)	14. (a)	15. (b)	16. (c)
17. (b)	18. (b)	19. (c)	20. (c)

Solutions

1. $\text{Area} = \frac{1}{2}\times 3\times 5\times \sin 60^o = \frac{15}{2}\times\frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}$

2. $\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{4}{4}\sqrt{4\times 25 - 16} = \sqrt{84} = 2\sqrt{21}\ \text{cm}^2$$

3. An equilateral triangle will have the maximum are compared to other triangles inside the same circle. So $AB = BC = CA$.

4. $h = 10$ cm

$$h = \frac{a\sqrt{3}}{2} \Rightarrow a = \frac{10\times 2}{\sqrt{3}} = \frac{20}{\sqrt{3}}\ \text{cm}$$

$$\text{Area} = \frac{1}{2}\times\frac{20}{\sqrt{3}}\times 10 = \frac{100}{\sqrt{3}}\ \text{cm}^2 \text{ or } \frac{100\sqrt{3}}{3}\ \text{cm}^2$$

5. 11, 60, 61 forms a Pythagoras triplet. Hence, the triangle is a right angled triangle.

$$\text{Area} = \frac{1}{2}\times 11\times 60 = 330$$

6. 'O' is the centroid of ΔABC
Then area of ΔAOF = area ΔOFB = area of ΔOEC
Area (ΔOFB) + Area(ΔOEC) = 36 + 36 = 72 cm^2

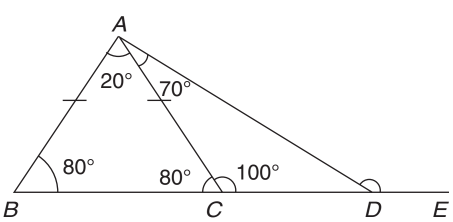
7. 5, 12, 13 forms a Pythagoras triplet.
Circumradius = 13/2 = 6.5 cm

8. $\frac{1}{BD^2} = \frac{1}{4^2} + \frac{1}{3^2} = \frac{25}{144}$

$$BD = \left(\frac{144}{25}\right)^{\frac{1}{2}} = \frac{12}{5} = 2.4\ \text{cm}$$

9. $\text{In radius} = \frac{12\times 5}{12+5+13} = \frac{60}{30} = 2\ \text{cm}$

10. $\angle B = \angle ACB = 80°$



$\angle BAC = 180° - (80° - 80°) = 20°$

$\angle ADE = \angle CAD + \angle ACD = 70° + 100° = 170°$

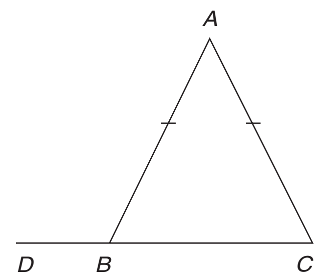
11. D, is the mid-point of side BC.
Centroid 'O' divides AD in the ratio 2:1

$$\therefore OD = \frac{12}{2} = 6\ \text{cm.}$$

12. $AC = BC = 7$ cm

$$\therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}\ \text{cm}$$

13.

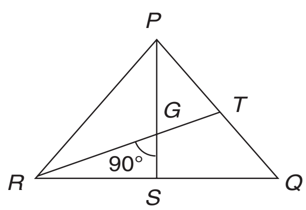


$\angle ABC = \angle ACB$
$\angle BAC = 50°$
$\therefore \angle ABC + \angle ACB = 130°$
$\angle ABC = 65°$
$\therefore \angle ABD = 180° - 65° = 115°$

14. Let the sides of the triangle be $3x$, $4x$ and $6x$ units.
Clearly, $(4x)^2 + (5x)^2 > (6x)^2$
$\therefore$ The triangle will be acute angled.

15. For a triangle PQR, let the altitudes be AP, BR and CQ respectively. Then:
$AP < PR$
$BR < RQ$
$CQ < PQ$
$\therefore AP + BR + CQ < PQ + QR + PR$

16. $PS = 9$ cm



$$\Rightarrow GS = \frac{1}{3}\times 9 = 3\ \text{cm}$$

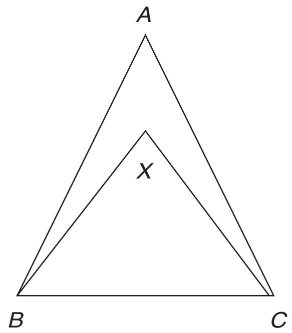
$RT = 6$ cm

$$\Rightarrow RG = \frac{2}{3}\times 6 = 4\ \text{cm}$$

$$\therefore RS = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5\ \text{cm}$$

17. $\frac{\text{area of }\Delta ABC}{\text{area of }\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{25}{16}$

18.



$\angle B + \angle C = 180° - 30° = 150°$

In ΔBXC,

$$\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BXC = 180°$$

$$\Rightarrow \angle BXC = 180° - \frac{1}{2}(\angle B + \angle C)$$

$$= 180° - \frac{150°}{2}$$

$= 180° - 75° = 105°$

19. Let the side of the equilateral triangle be X cm. Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (X)^2$$

$$\Rightarrow \frac{\sqrt{3}}{4} \times (X)^2 = 900\sqrt{3}$$

$$\Rightarrow (X)^2 = \frac{900\sqrt{3} \times 4}{\sqrt{3}}$$

$\therefore X = \sqrt{4 \times 900} = 60$ meters

$\therefore$ Perimeter $= 3 \times X = 3 \times 60 = 180$ meters

20. The area of the triangle formed by joining the mid-point of the triangle is 1/4th of the area of the original triangle.

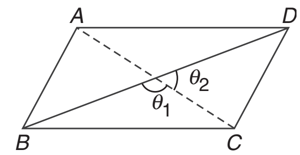
Area of the origional triangle $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2

$\therefore$ Required area $= \frac{1}{4} \times 6 = \frac{3}{2}$ cm^2

QUADRILATERALS

Area

(A) Area = 1/2 (product of diagonals) × (sine of the angle between, them)

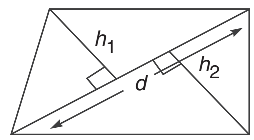


If θ_1 and θ_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines $\rightarrow \theta_1 + \theta_2 = 180°$

Then, the area of the quadrilateral $= \frac{1}{2}\ d_1 d_2 \sin \theta_1$

$= \frac{1}{2}\ d_1\ d_2 \sin \theta_2$.

(B) Area = 1/2 × diagonal × sum of the perpendiculars to it from opposite vertices $= \frac{d(h_1 + h_2)}{2}$.



(C) Area of a circumscribed quadrilateral

$$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$

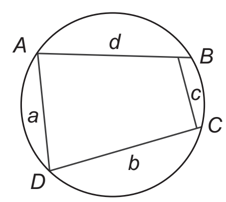
Where $S = \frac{a+b+c+d}{2}$

(where a, b, c and d are the lengths of the sides.)

Properties

1. In a convex quadrilateral inscribed in a circle, the product of the diagonals is equal to the sum of the products of the opposite sides. For example, in the figure below:

   $(a \times c) + (b \times d) = AC \times BD$



2. Sum of all the angles of a quadrilateral = 360°.

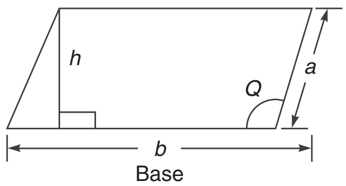
TYPES OF QUADRILATERALS

1. Parallelogram (|| gm)

A parallelogram is a quadrilateral with opposite sides parallel (as shown in the figure)

(A) Area = Base (b) × Height (h)

$= bh$



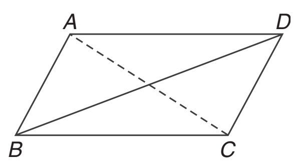
(B) Area = product of any two adjacent sides × sine of the included angle.
$= ab \sin Q$

(C) Perimeter = 2 $(a + b)$
where a and b are any two adjacent sides.

Properties

(a) Diagonals of a parallelogram bisect each other.
(b) Bisectors of the angles of a parallelogram form a rectangle.
(c) A parallelogram inscribed in a circle is a rectangle.
(d) A parallelogram circumscribed about a circle is a rhombus.
(e) The opposite angles in a parallelogram are equal.
(f) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure:

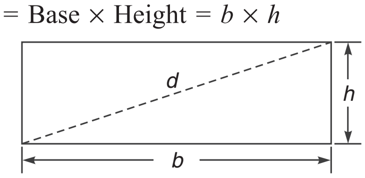
$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$$
$$= 2(AB^2 + BC^2)$$



2. Rectangles

A rectangle is a parallelogram with all angles 90°

(a) Area = Base × Height = $b \times h$



Note: Base and height are also referred to as the length and the breadth in a rectangle.

(b) Diagonal $(d) = \sqrt{b^2 + h^2}$ (by Pythagoras theorem)

Properties of a Rectangle

(a) Diagonals are equal and bisect each other.
(b) Bisectors of the angles of a rectangle (a parallelogram) form another rectangle.
(c) All rectangles are parallelograms but the reverse is not true.

3. Rhombus

A parallelogram having all the sides equal is a rhombus.

(a) Area = 1/2 × product of diagonals × sine of the angle between them.
$= 1/2 \times d_1 \times d_2 \sin 90°$ (Diagonals in a rhombus intersect at right angles)
$= 1/2 \times d_1\, d_2$ (since $\sin 90° = 1$)

(b) Area = product of adjacent sides × sine of the angle between them.

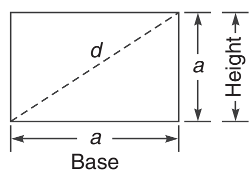
Properties

(a) Diagonals bisect each other at right angles.
(b) All rhombuses are parallelograms but the reverse is not true.
(c) A rhombus may or may not be a square but all squares are rhombuses.

4. Square

A square is a rectangle with adjacent sides equal or a rhombus with each angle 90°

(a) Area = base × height = a^2
(b) Area = 1/2 (diagonal)^2 = 1/2 d^2 (square is a rhombus too).
(c) Perimeter = $4a$ (a = side of the square)
(d) Diagonal = $a\sqrt{2}$
(e) In radius = $\dfrac{a}{2}$

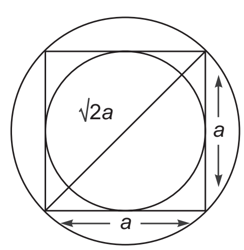


Properties

(a) Diagonals are equal and bisect each other at right angles.
(b) Side is the diameter of the inscribed circle.
(c) Diagonal is the diameter of the circumscribing circle.

$\Rightarrow$ Diameter = $a\sqrt{2}$.

Circumradius = $a/\sqrt{2}$



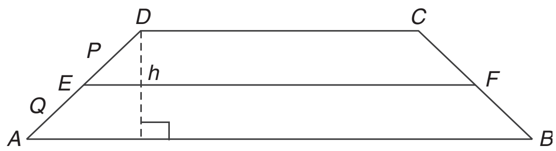
5. Trapezium

A trapezium is a quadrilateral with only two sides parallel to each other.

(a) Area = 1/2 × sum of parallel sides × height = 1/2 ($AB + DC$) × h—For the figure below.

(b) Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides)
For any line EF parallel to AB

$$EF = \frac{\{[P \times (AB)] + [Q \times (DC)]\}}{AD}$$

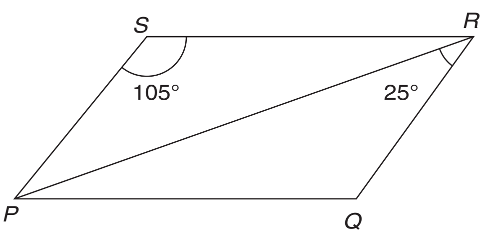


Properties

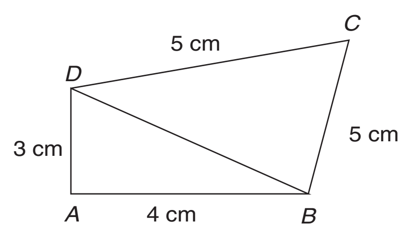
(a) If the non-parallel sides are equal then diagonals will be equal too.

PRACTICE EXERCISE

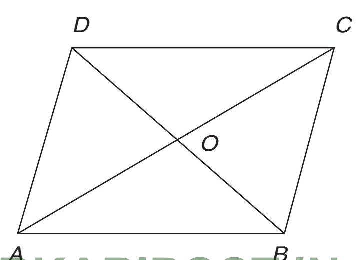
1. Find the smallest angle of a quadrilateral if the measure of it's interior angles are in the ratio of 1:2:3:4.
   (a) 18° (b) 36°
   (c) 54° (d) 72°
2. In a parallelogram PQRS if bisectors of P and Q meet at X, then the value of PXQ is
   (a) 45° (b) 90°
   (c) 75° (d) 60°
3. In a parallelogram $PQRS$, if S = 105° and PRQ = 25° then QPR = ?



   (a) 40° (b) 50°
   (c) 60° (d) 55°
4. If one diagonal of a rhombus is equal to it's side, then the diagonals of the rhombus are in the ratio.
   (a) $\sqrt{3}:1$ (b) 3 : 1
   (c) 2 : 1 (d) None of these
5. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is
   (a) 50 m (b) 100 m
   (c) 200 m (d) None of these
6. In a square $PQRS$, A is the mid point of PQ and B is the midpoint of QR, if area of ΔAQB is 100 m^2 then the area of the square $PQRS$ = ?
   (a) 400 m^2 (b) 250 m^2
   (c) 600 m^2 (d) 800 m^2
7. In the previous question length of diagonal PR = ?
   (a) 20 m (b) 30 m
   (c) 40 m (d) $20\sqrt{2}$ m
8. If a triangle with area x, rectangle with area y, parallelogram with area z were all constructed on the same base and all have the same altitude, then which of the following options is true?
   (a) $x = y = z$ (b) $x = y/2 = z$
   (c) $2x = y = z$ (d) $2x = 2y = z$
9. ▫$ABCD$ is a parallelogram, AC, BD are the diagonals & intersect at point O. X and Y are the centroids of ΔADC and ΔABC respectively. If BY = 6 cm, then OX = ?
   (a) 2 cm (b) 3 cm
   (c) 4 cm (d) 6 cm
10. If area of a rectangle with sides x and y is X and that of a parallelogram (which is strictly not a rectangle) with sides x and y is Y. Then:
    (a) $X = Y$ (b) $X \leq Y$
    (c) $X < Y$ (d) $X > Y$
11. In ▫$ABCD$, A = 90°, $BC = CD$ = 5 cm, AD = 3 cm, BA = 4 cm. Find the value of $\angle BCD$.

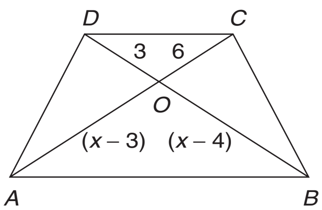


    (a) 45° (b) 60°
    (c) 75° (d) 85°
12. In the above question, what will be the area of ▫ABCD.
    (a) 16.83 cm^2 (b) 15.36 cm^2
    (c) 14.72 cm^2 (d) 13.76 cm^2
13. ▫$PQRS$ is a parallelogram. 'O' is a point within it, and area of parallelogram $PQRS$ is 50 cm^2. Find the sum of areas of ΔOPQ and ΔOSR (in cm^2):
    (a) 15 (b) 20
    (c) 25 (d) 30
14. $ABCD$ is a rhombus, such that AB = 5cm AC = 8 cm. Find the area of ▫$ABCD$



(a) 12 cm^2 (b) 18 cm^2
(c) 24 cm^2 (d) 36 cm^2

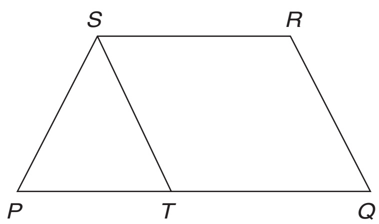
15. If $ABCD$ is a trapezium then find the value of x.



(a) 3 (b) 4
(c) 5 (d) 6

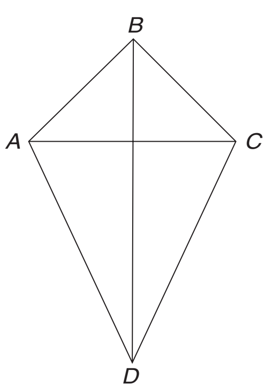
16. A square and a rhombus have the same base and the rhombus is inclined at 45° then what will be the ratio of area of the square to the area of the rhombus?
(a) 2:1 (b) $\sqrt{2}:1$
(c) $1:\sqrt{2}$ (d) $\sqrt{3}:1$

17. $PQRS$ is a quadrilateral and $PQ \parallel RS$. T is the mid-point of PQ. $ST \parallel RQ$. If area of the triangle ΔPST is 50 cm^2 then area of $\square PQRS$ is:



(a) 100 cm^2 (b) 125 cm^2
(c) 150 cm^2 (d) 175 cm^2

18. In $\square ABCD$, $AB = BC$, $AD = CD$. BD and AC are diagonals of $\square ABCD$. Such that BD = 10 cm, AC = 5 cm. Find area of $\square$ ABCD.



ANSWER KEY

1. (b)	2. (b)	3. (b)	4. (a)
5. (c)	6. (d)	7. (c)	8. (c)
9. (b)	10. (d)	11. (b)	12. (a)
13. (c)	14. (c)	15. (c)	16. (b)
17. (c)	18. 25 cm^2		

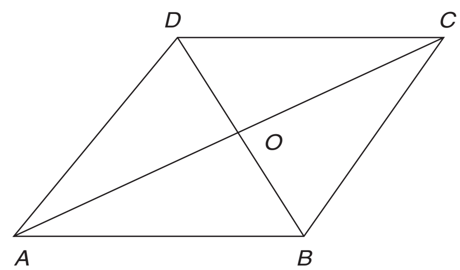
Solutions

1. Let the angles be x, $2x$, $3x$, $4x$ respectively
According to the question:
$x + 2x + 3x + 4x = 360^o$
$10x = 360^o$
$X = 36^o$
Smallest angle = 36^o

2. $P + Q = 180°$
$$\frac{\angle P}{2} + \frac{\angle Q}{2} = 90°$$
$$\angle PXQ = 180^o - \left[\frac{\angle P}{2} + \frac{\angle Q}{2}\right] = 180^o - 90^o = 90^o$$

3. $PQR = PSR = 105°$
$RPQ = 180° - (105° + 25°) = 50°$

4. Let $AB = BD = DC = a$, $AC = b$



In ΔCOD: $(CD)^2 = (OC)^2 + (OD)^2$
$$a^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$
$$\frac{3a^2}{4} = \frac{b^2}{4}$$
$$b = a\sqrt{3}$$
$$\frac{b}{a} = \frac{\sqrt{3}}{1}$$

5. If 'b' is the base and h_1, h_2 are altitudes of the triangle and parallelogram respectively.
Then according to the question:
$$\frac{1}{2} \times b \times h_1 = b \times h_2$$
$$h_1 = 2h_2$$
$$h_1 = 2 \times 100 = 200 \text{ m.}$$

6. Area of $\Delta AQB = \frac{1}{2}$ AQ BQ = 100 = 100
$$\frac{1}{2} \; \frac{PQ}{2} \; \frac{QR}{2} = 100$$
$$PQ \quad QR = 2 \quad 2 \quad 2 \quad 100 = 800 \text{ cm}^2.$$

7. $PQ.QR$ = 800 cm^2
$PQ = QR$ ($\square PQRS$ is a square)
$(PQ)^2 = 800$

$PQ = 20\sqrt{2}$ cm

Length of the diagonal = $PQ\sqrt{2} = 20\sqrt{2} \times \sqrt{2} =$ 40 m.

8. Area of triangle = $\frac{1}{2}$ *Area of Parallelogram*

$x = z/2$

Area of parallelogram = Area of rectangle.

$y = z$

$2x = y = z$

9. ΔABC & ΔADC are congruent to each other.

So $OD = OB$

$$\frac{OD}{3} = \frac{OB}{3}$$

$OX = OY$

$$OX = \frac{BY}{2} = \frac{6}{2} = 3 \text{ cm}$$

10. Area of rectangle $ABCD = X = xy$

Area of parallelogram $PQRS = Y = x.y\cos\theta$ (where θ is the angle between x and y and $\theta \neq 90°$)

As we know $\cos\theta < 1$ (For $\theta \neq 90°$)

$Y < xy$

or

$Y < X$.

11. ΔBAD is a right-angled triangle

$$BD = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm}$$

In ΔBCD all the sides are equal to each other, so ΔBCD is an equilateral triangle

$\therefore BCD = 60°$

12. Area of $\square ABCD$ = Area of ΔABC + Area of ΔBCD

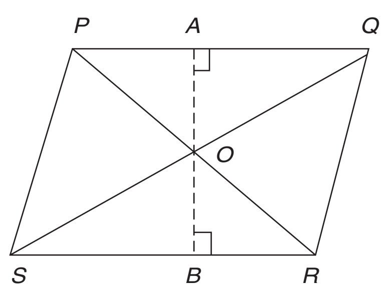
$$= \frac{1}{2} \times 3 \times 4 + \frac{\sqrt{3}}{4}(5)^2 \text{ cm}^2$$

$$= 6 + \frac{25\sqrt{3}}{4} \text{ cm}^2$$

$$= \left(6 + 6.25\sqrt{3}\right) \text{ cm}^2$$

$= 16.83$ cm^2

13. Draw $OA \perp PQ$ and $OB \perp SR$.



If $OA = x$, $OB = y$ and $PQ = SR = a$, $QR = PS = b$

Then area of $\Delta OPQ = \frac{1}{2} \times x \times a = \frac{ax}{2}$

Area of $\Delta OSR = \frac{1}{2} \times y \times a = \frac{ay}{2}$

Area of ΔOPQ + Area of $\Delta OSR = \frac{ax}{2} + \frac{ay}{2}$

$$= \frac{1}{2}a(x + y)$$

$x + y$ = Altitude of parallelogram $PQRS$

Area of $PQRS = a(x + y)$

Area of $(\Delta OPQ + \Delta OSR) = \frac{1}{2}$ Area of $\square PQRS$

$$= \frac{1}{2} \times 50 = 25 \text{ cm}^2$$

14. $OC = \frac{AC}{2} = \frac{8}{2} = 4$ cm

$\because \angle DOC = 90°$

$\therefore OD^2 + OC^2 = CD^2$

$OD^2 + 4^2 = 5^2$

$OD^2 = 9$

OD = 3 cm

$BD = 2 \times OD = 2 \times 3 = 6$ cm

Area of $ABCD = \frac{1}{2} \times 6 \times 8 = 24$ cm^2

15. $\frac{3}{x-4} = \frac{6}{x-3}$

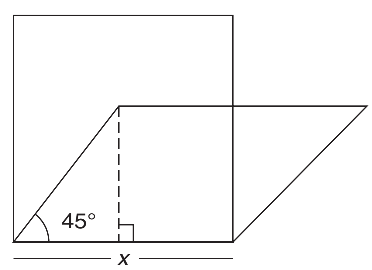
$3(x - 3) = 6(x - 4)$

$x - 3 = 2(x - 4)$

$x - 3 = 2x - 8$

$x = 5$

16. Let the length of base be 'x' units. Area of square = x^2



Area of rhombus = $x \times \sin 45°$

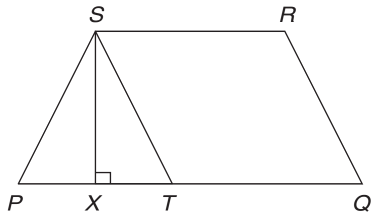
$$= x \times \frac{x}{\sqrt{2}} = \frac{x^2}{\sqrt{2}}$$

Required ratio $= x^2 : \frac{x^2}{\sqrt{2}}$

$$= 1 : \frac{1}{\sqrt{2}}$$

$$= \sqrt{2} : 1$$

17. T is the midpoint of PQ



$PT = TQ$

Draw $SX \perp PQ$, if $SX = h$ and $PT = TQ = a$

Area of $\Delta\, PST = \frac{1}{2} \times a \times h = \frac{ah}{2}$

Area of $\square\, PQRS$ = Area of ΔPST + Area of $\square\, STQR$

$= \frac{ah}{2} + ah$

$= \frac{3ah}{2}$

$= 3[50] = 150 \text{ cm}^2$

18. $\square\, ABCD$ has a kite like structure, so it's diagonals intersect each other perpendicularly

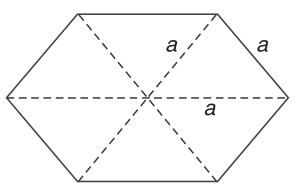
Area $= \frac{1}{2}$ (product of diagonals)

$= \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2$

REGULAR HEXAGON

(a) Area $= [(3\sqrt{3})/2]\ (\text{side})^2$

$= \frac{3\sqrt{3} \times a^2}{2}$



(b) A regular hexagon is actually a combination of 6 equilateral triangles all of side 'a'.
Hence, the area is also given by: 6 × Area of an equilateral triangle having the same side as the side of the hexagon $= 6 \times \frac{\sqrt{3}}{4} a^2$

(c) If you look at the figure closely it will not be difficult to realise that circumradius (R) = a; i.e the side of the hexagon is equal to the circumradius of the same.

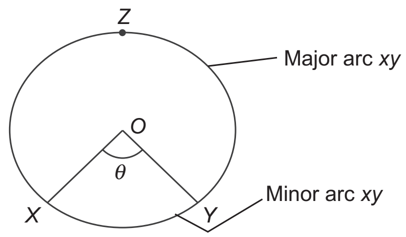
CIRCLES

(a) Area $= \pi r^2$

(b) Circumference $= 2\,\pi r = (r = \text{radius})$

(c) Area $= 1/2 \times$ circumference $\times\, r$
Arc: It is a part of the circumference of the circle. The bigger one is called the *major arc* and the smaller one the *minor arc*.

(d) Length (Arc XY) $= \frac{\theta}{360} \times 2\pi r$



(e) **Sector of a circle** is a part of the area of a circle between two radii.

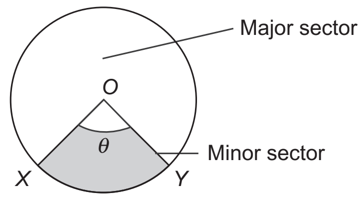
(f) Area of a sector $= \frac{\theta}{360} \times \pi r^2$

(where θ is the angle between two radii)

$= (1/2) r \times$ length (arc xy)

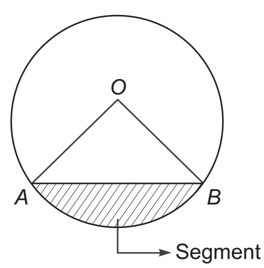
($\because\ \pi r\theta/180$ = length arc xy)

$= \frac{1}{2} \times r \times \frac{\pi r \theta}{360}$



(g) **Segment:** A sector minus the triangle formed by the two radii is called the segment of the circle.

(h) Area = Area of the sector – Area ΔOAB = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$



(i) Perimeter of segment = length of the arc + length of segment AB

$= \frac{\theta}{360} \times 2\pi r + 2r \sin\left(\frac{\theta}{2}\right)$

$= \frac{\pi r \theta}{180} + 2r \sin\left(\frac{\theta}{2}\right)$

(j) **Congruency:** Two circles can be congruent if and only if they have equal radii.

Properties

(a) The perpendicular from the centre of a circle to a chord bisects the chord. The converse is also true.

(b) The perpendicular bisectors of two chords of a circle intersect at its centre.

(c) There can be one and only one circle passing through three or more non-collinear points.

(d) If two circles intersect in two points then the line through the centres is the perpendicular bisector of the common chord.

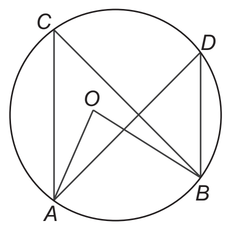
(e) If two chords of a circle are equal, then the centre of the circle lies on the perpendicular bisector of the two chords.

(f) Equal chords of a circle or congruent circles are equidistant from the centre.

(g) Equidistant chords from the centre of a circle are equal to each other in terms of their length.

(h) The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle. This can be clearly seen in the following figure:

With respect to the arc AB, $\angle AOB = 2\ \angle ACB$.

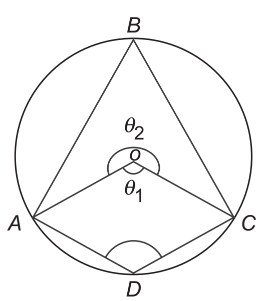


(i) Any two angles in the same segment are equal. Thus, $\angle ACB = \angle ADB$.

(j) The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

(k) Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.

In the figure below



$\angle ABC$ is acute and

$\angle ADC$ = obtuse

Also $\quad \theta_1 = 2\ \angle B$

And $\quad \theta_2 = 2\ \angle D$

$\therefore \quad \theta_1 + \theta_2 = 2(\angle B + \angle D)$

$= 360° = 2(\angle B + \angle D)$

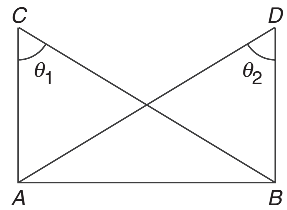
or $\angle B + \angle D = 180°$

or sum of opposite angles of a cyclic quadrilateral is 180°.

(l) If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic. Thus, in the following figure:

If, $\quad \theta_1 = \theta_2$

Then $ABCD$ are concyclic, that is, they lie on the same circle.



(m) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres.) The converse is also true.

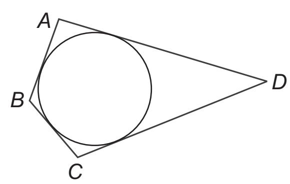
(n) If the sum of the opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Secant: A line that intersects a circle at two points.

Tangent: A line that touches a circle at exactly one point.

(o) If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two

$$AB + DC = AD + BC$$



(p) In two concentric circles, the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

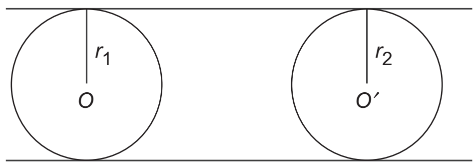
Tangents

- Length of direct common tangents is

$= \sqrt{(\text{Distance between their centres})^2 - (r_1 - r_2)^2}$

where r_1 and r_2 are the radii of the circles

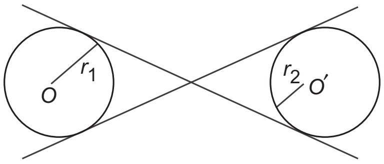
$= \sqrt{(OO')^2 - (r_1 - r_2)^2}$



- Length of transverse common tangents is

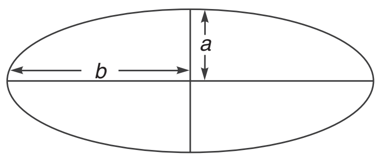
$$= \sqrt{(\text{distance between their centres})^2 - (r_1 + r_2)^2}$$

$$= \sqrt{(OO')^2 - (r_1 + r_2)^2}$$



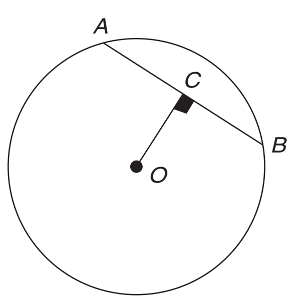
ELLIPSE

- Perimeter = $\pi (a + b)$
- Area = πab

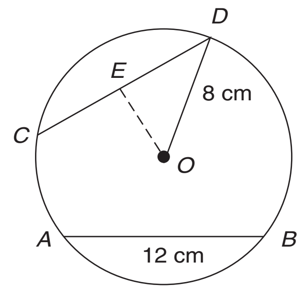


PRACTICE EXERCISE

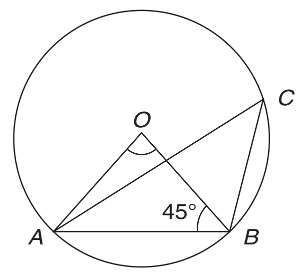
1. Find the area of a circle of radius 5cm.
   (a) 25π (b) 20π
   (c) 22π (d) None of these
2. Find the circumference of the circle in the previous question:
   (a) 10π (b) 5π
   (c) 7π (d) None of these
3. If O is the center of the circle and $OC \perp AB$ and $AC = x + 6$, $BC = 2x - 4$, then AB = ?



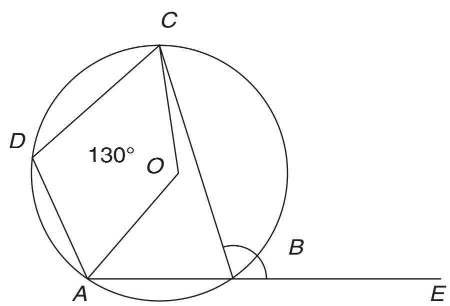
   (a) 22 (b) 31
   (c) 32 (d) 26
4. If $\overline{AB} = \overline{CD}$ and AB = 12 cm. 'O' is the center of the circle, OD = 8 cm, $OE \perp CD$, Then length of OE is



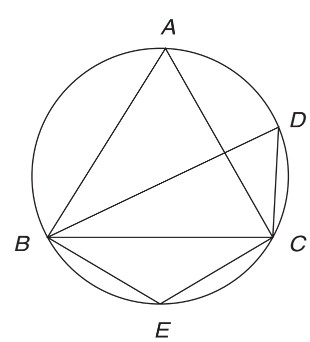
   (a) 2 cm (b) $2\sqrt{7}$ cm
   (c) $2\sqrt{11}$ cm (d) None of these
5. In the given figure, O is the centre of the circle. ABO = 45°. Find the value of ACB:



   (a) 60º (b) 75º
   (c) 90º (d) None of these
6. In the given figure, AOC = 130°, where O is the center. Find $\angle CBE$:

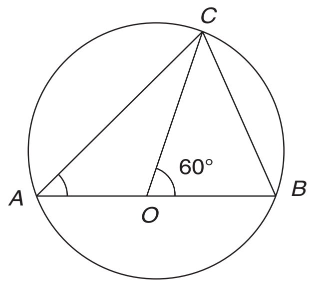


   (a) 100º (b) 70º
   (c) 115º (d) 130º
7. In the given figure, ΔABC is an equilateral triangle. Find $\angle BEC$:



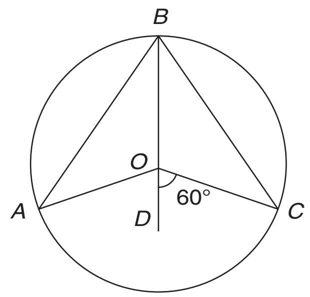
   (a) 120º (b) 60º
   (c) 80º (d) None of the above

8. In the given figure, $COB = 60°$, AB is the diameter of the circle. Find $\angle ACO$:



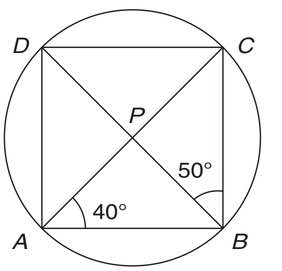
(a) 20° (b) 30°
(c) 35° (d) 40°

9. O is the center of the circle, line segment BOD is the angle bisector of AOC, $COD = 60°$. Find ABC:



(a) 30° (b) 40°
(c) 50° (d) 60°

10. In the given figure, $ABCD$ is a cyclic quadrilateral and the diagonals bisect each other at P. If CBD = 50° and CAB = 40°, then BCD is:



(a) 60° (b) 75°
(c) 90° (d) 105°

11. Two equal circles of radius 6 cm intersect each other such that each passes through the centre of the other. The length of the common chord is:
(a) $2\sqrt{3}$ cm (b) $6\sqrt{3}$ cm
(c) $2\sqrt{2}$ cm (d) 8 cm

12. The length of the chord of a circle is 6 cm and perpendicular distance between centre and the chord is 4 cm. Then the diameter of the circle is equal to:
(a) 12 cm (b) 10 cm
(c) 16 cm (d) 8 cm

13. The distance between two parallel chords of length 6 cm each in a circle of diameter 10 cm is
(a) 8 cm (b) 7 cm
(c) 6 cm (d) 5.5 cm

14. The length of the common chord of two intersecting circles is 24. If the diameters of the circles are 30 cm and 26 cm, then the distance between the centers of the circles (in cm) is
(a) 13 (b) 14
(c) 15 (d) 16

15. If two equal circles whose centers are O and O', intersect each other at the points A and B. OO' = 6 cm and AB = 8 cm, then the radius of the circles is
(a) 5 cm (b) 8 cm
(c) 12 cm (d) 14 cm

16. Chords BA and DC of a circle intersect externally at P. If AB = 7 cm, CD = 5 cm and PC = 1 cm, then the length of PB is
(a) 11 cm (b) 10 cm
(c) 9 cm (d) 8 cm

17. Two circles touch each other internally. Their radii are 3 cm and 4 cm. The biggest chord of the greater circle which is outside the inner circle is of length:
(a) $2\sqrt{3}$ cm (b) $3\sqrt{2}$ cm
(c) $4\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

18. If the radii of two circles be 8 cm and 4 cm and the length of the transverse common tangent be 13 cm, then the distance between the two centers is
(a) $\sqrt{313}$ cm (b) $\sqrt{125}$ cm
(c) $5\sqrt{2}$ cm (d) $\sqrt{135}$ cm

18. The distance between the centers of two equal circles, each of radius 6 cm, is 13 cm. the length of a transverse common tangent is
(a) 8 cm (b) 10 cm
(c) 5 cm (d) 6 cm

19. The radii of two circles are 9 cm and 4 cm, the distance between their centres is 13 cm. Then the length of the direct transverse common tangent is
(a) 12 cm (c) $12\sqrt{2}$ cm
(c) 5 cm (d) 15 cm

20. The radii of two circles are 9cm and 4cm, the distance between their centres is 13cm. Then the length of the direct common tangent is
(a) 12 cm (b) $12\sqrt{2}$ cm
(c) 5 cm (d) 15 cm

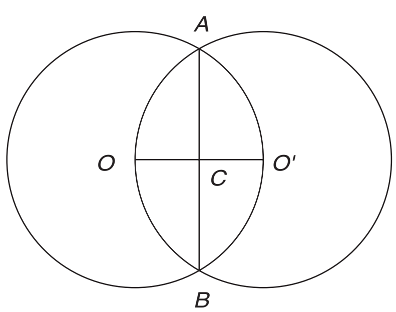
ANSWER KEY

1. (a)	2. (a)	3. (c)	4. (b)
5. (d)	6. (c)	7. (a)	8. (b)
9. (d)	10. (c)	11. (b)	12. (b)
13. (a)	14. (b)	15. (a)	16. (b)
17. (c)	18. (a)	19. (c)	20. (a)

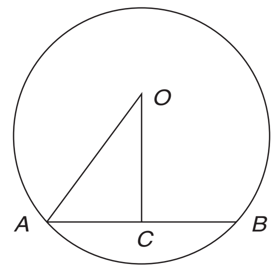
Solutions

1. Area = $\pi r^2 = \pi \times 5^2 = 25\pi$
2. Circumference = $2\pi \times 5 = 10\pi$
3. As $OC \perp AB$
   $AC = BC$
   $x + 6 = 2x - 4$
   $x = 10$
   $AB = x + 6 + 2x - 4 = 3x + 2 = 30 + 2 = 32$
4. If $\overline{AB} = \overline{CD}$
   Then $AB = CD = 12$ cm
   If $CD = 12$, then $CE = DE = 6$cm
   $OE = \sqrt{8^2 - 6^2} = \sqrt{28} = 2\sqrt{7}$ cm
5. $AO = BO$
   $ABO = BAO = 45°$
   $AOB = 180° - (45° + 45°) = 90°$
   $\angle ACB = \dfrac{\angle AOB}{2} = \dfrac{90°}{2} = 45°$

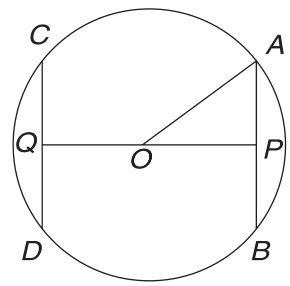
   $\angle ABC = \dfrac{130°}{2} = 65° \quad \angle CBE = 180° - 65° = 115°$
6.
7. $BAC = 60°$
   $\angle BEC = 180° - \angle BAC = 180° - 60° = 120°$
8. $\angle COB = 60°$
   $\angle AOC = 180° - 60° = 120°$
   $\angle CAO = \dfrac{60°}{2} = 30°$
   $\angle ACO = 180° - (120° + 30°) = 180° - 150° = 30°$
9. $\angle COD = 60°$
   $\angle AOC = 2 \times 60° = 120°$
   $\angle ABC = \dfrac{120°}{2} = 60°$
10. $\angle CDB = \angle CAB = 40°$
    In $\Delta BDC = \angle BCD + 50° + 40° = 180°$
    $\angle BCD = 90°$
11. $OO' = 6$ cm
    $OC = 3$ cm
    $OA = 6$ cm
    $\therefore AC = \sqrt{6^2 - 3^2} = \sqrt{36-9} = \sqrt{27} = 3\sqrt{3}$ cm
    $\therefore AB = 6\sqrt{3}$ cm



12. $AC = CB = 3$ cm
    $OC = 4$ cm
    $\therefore OA = \sqrt{OC^2 + CA^2}$
    $= \sqrt{4^2 + 3^2}$
    $= \sqrt{16+9} = \sqrt{25} = 5$ cm
    Diameter = 10 cm

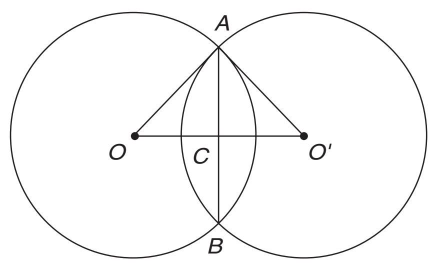


13.



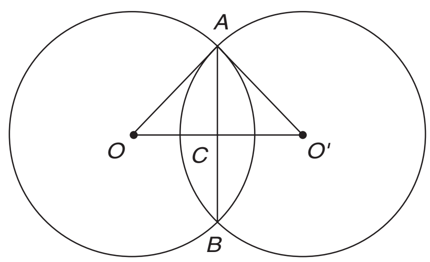
$AB = CD$
$OP = OQ$
From ΔOAP
$OP = \sqrt{OA^2 - AP^2} = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$ cm
$\therefore QP = 2 \times OP = 8$ cm

14.
    $AC = CB = 12$ cm



$OC = \sqrt{15^2 - 12^2} = \sqrt{225-144} = \sqrt{81} = 9$ cm
$O'C = \sqrt{13^2 - 12^2} = \sqrt{169-144} = \sqrt{25} = 5$ cm
$\therefore OO' = 9 + 5 = 14$ cm

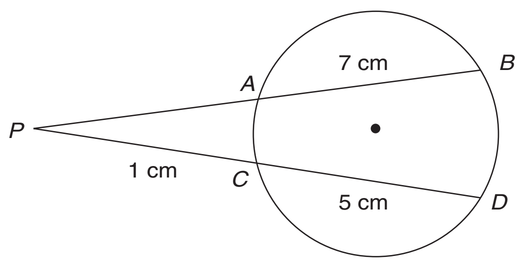
15.



AB = 8 cm
$AC = BC$ = 4 cm
$OC = CO'$ = 3 cm

$\therefore OA = \sqrt{OC^2 + CA^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ cm

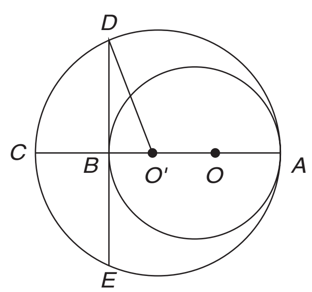
16.



AB = 7 cm, CD = 5 cm
PC = 1 cm, $PA = x$ cm
$PA \times PB = PC \times PD$
$\Rightarrow x(x+7) = 6 \times 5$

By solving we get x = 3 cm
PB = 3 + 7 = 10 cm

17.



$O'A$ = 4 cm
AB = 6 cm
$O'B = AB - O'A = 6 - 4 = 2$ cm
$BD = \sqrt{4^2 - 2^2} = 2\sqrt{3}$ cm
$DE = 4\sqrt{3}$ cm

18. Let the distance between the centers be x cm.

$\Rightarrow 13 = \sqrt{x^2 - (8+4)^2}$

$\Rightarrow 169 = x^2 - 144$

$\Rightarrow x^2 = 169 + 144 = 313$

$\Rightarrow x = \sqrt{313}$ cm

19. Transverse common tangent

$= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$

$= \sqrt{13^2 - 12^2} = \sqrt{25} = 5$ cm

20. Direct common tangent

$= \sqrt{(13)^2 - (9-4)^2} = \sqrt{169 - 25} = \sqrt{144} = 12$ cm

STAR

Sum of angles of a star = $(2n - 8) \times \pi/2 = (n - 4)\pi$

PART II: MENSURATION

The following formulae hold true in the area of mensuration:

1. Cuboid

A cuboid is a three dimensional box. It is defined by the virtue of it's length l, breadth b and height h. It can be visualised as a room which has its length, breadth and height different from each other.

1. Total surface area of a cuboid = $2(lb + bh + lh)$
2. Volume of the cuboid = lbh

2. Cube of side s

A cube is a cuboid which has all its edges equal i.e. length = breadth = height = s.

1. Total surface area of a cube = $6s^2$.
2. Volume of the cube = s^3.

3. Prism

A prism is a solid which can have any polygon at both its ends. It's dimensions are defined by the dimensions of the polygon at it's ends and its height.

1. Lateral surface area of a right prism = Perimeter of base × height
2. Volume of a right prism = area of base × height
3. Whole surface of a right prism = Lateral surface of the prism + the area of the two plane ends.

4. Cylinder

A cylinder is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base (r) and the height h. A gas cylinder is a close approximation of a cylinder.

1. Curved surface of a right cylinder = $2\pi rh$ where r is the radius of the base and h the height.
2. Whole surface of a right circular cylinder = $2\pi rh + 2\pi r^2$

3. Volume of a right circular cylinder = $\pi r^2 h$

5. Pyramid

A pyramid is a solid which can have any polygon as its base and its edges converge to a single apex. Its dimensions are defined by the dimensions of the polygon at its base and the length of its lateral edges which lead to the apex. The Egyptian pyramids are examples of pyramids.

1. Slant surface of a pyramid = 1/2 × Perimeter of the base × slant height
2. Whole surface of a pyramid = Slant surface + area of the base
3. Volume of a pyramid = $\frac{\text{area of the base} \times}{3}$ height

6. Cone

A cone is a solid which has a circle at its base and a slanting lateral surface that converges at the apex. Its dimensions are defined by the radius of the base (r), the height (h) and the slant height (l). A structure similar to a cone is used in ice cream cones.

1. Curved surface of a cone = πrl where l is the slant height
2. Whole surface of a cone = $\pi rl + \pi r^2$
3. Volume of a cone = $\frac{\pi r^2 h}{3}$

7. Sphere

A sphere is a solid in the form of a ball with radius r.

1. Surface Area of a sphere = $4\pi r^2$
2. Volume of a sphere = $\frac{4}{3}\pi r^3$

8. Frustum of a pyramid

When a pyramid is cut the left over part is called the frustum of the pyramid.

1. Slant surface of the frustum of a pyramid = 1/2 * sum of perimeters of end * slant height.
2. Volume of the frustum of a pyramid = $\frac{k}{3}[E_1 + (E_1 \cdot E_2)^{1/2} + E_2]$ where k is the thickness and E_1, E_2 the areas of the ends.

9. Frustum of a cone

When a cone is cut the left over part is called the frustum of the cone.

1. Slant surface of the frustum of a cone = $\pi(r_1 + r_2)l$ where l is the slant height.
2. Volume of the frustum of a cone = $\frac{\pi}{3}\ k(r_1^2 + r_1 r_2 + r_2^2)$

Space for Rough Work

WORKED-OUT PROBLEMS

Problem 11.1 A right triangle with hypotenuse 10 inches and other two sides of variable length is rotated about its longest side thus giving rise to a solid. Find the maximum possible volume of such a solid.

(a) $(250/3)\pi \text{in}^3$ (b) $(160/3)\pi \text{in}^3$
(c) $325/3\pi \text{in}^3$ (d) None of these

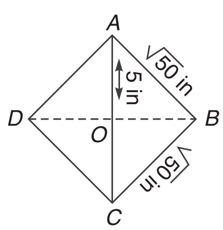
Solution Most of the questions like this that are asked in the CAT will not have figures accompanying them. Drawing a figure takes time, so it is always better to strengthen our imagination. The beginners can start off by trying to imagine the figure first and trying to solve the problem. They can draw the figure only when they don't arrive at the right answer and then find out where exactly they went wrong. The key is to spend as much time with the problem as possible trying to understand it fully and analysing the different aspects of the same without investing too much time on it.

Let's now look into this problem. The key here lies in how quickly you are able to visualise the figure and are able to see that

(i) the triangle has to be an isosceles triangle,
(ii) the solid thus formed is actually a combination of two cones,
(iii) the radius of the base has to be the altitude of the triangle to the hypotenuse.

After you have visualised this comes the calculation aspect of the problem. This is one aspect where you can score over others.

In this question the figure would be somewhat like this (as shown alongside) with triangles *ABC* and *ADC* representing the cones and *AC* being the hypotenuse around which the triangle *ABC* revolves. Now that the area has to be maximum with *AC* as the hypotenuse, we must realise that *ADB* has to be an isosceles triangle, which automatically makes *BCD* an isosceles triangle too. The next step is to calculate the radius of the base, which is essentially the height of the triangle *ABC*. To find that, we have to first find *AB*. We know



$$AC^2 = AB^2 + BC^2$$

For triangle to be one with the greatest possible area *AB* must be equal to *BC* that is, $AB = BC = \sqrt{50}$, since $AO = 1/2AC = 5$ inches.

Now take the right angle triangle *ABO*, *BO* being the altitude of triangle *AOC*. By Pythagoras theorem, $AB^2 = AO^2 + BO^2$, so $BO^2 = 25$ inches

The next step is to find the volume of the cone *ABD* and multiply it with two to get the volume of the whole solid.

Volume of the cone $ADB = \dfrac{\pi}{3} \times BO^2 \times AO = \dfrac{125\ \pi}{3}$

Therefore volume of the solid $ABCD = 2 \times \dfrac{250\ \pi}{3} = \dfrac{250\ \pi}{3}$

Problem 11.2 A right circular cylinder is to be made out of a metal sheet such that the sum of its height and radius does not exceed 9 cm can have a maximum volume of.

(a) 54 p cm^2 (b) 108 p cm^2
(c) 81 p cm^2 (d) None of these

Solution Solving this question requires the knowledge of ratio and proportion also. To solve this question, one must know that for $a^2b^3c^4$ to have the maximum value when $(a + b + c)$ is constant, *a*, *b* and *c* must be in the ratio 1 : 2 : 3.

Now lets look at this problem.

Volume of a cylinder = $\pi r^2 h$.

If you analyse this formula closely, you will find that *r* and *h* are the only variable term. So for volume of the cylinder to be maximum, r^2h has to be maximum under the condition that $r + h = 9$. By the information given above, this is possible only when $r : h = 2 : 1$, that is, $r = 6$, $h = 3$. So,

$$\text{Volume of the Cylinder} = \pi r^2 h = \pi \times 6^2 \times 3 = \mathbf{108\pi}$$

Problem 11.3 There are five concentric squares. If the area of the circle inside the smallest square is 77 square units and the distance between the corresponding corners of consecutive squares is 1.5 units, find the difference in the areas of the outermost and innermost squares.

Solution Here again the ability to visualise the diagram would be the key. Once you gain expertise in this aspect, you will be able to see that you will be able to see that the diameter of the circle is equal to the side of the innermost square that is

$$\pi r^2 = 77$$

or $$r = 3.5\sqrt{2}$$

or $$2r = 7\sqrt{2}$$

Then the diagonal of the square is 14 sq units.

Which means the diagonal of the fifth square would be 14 + 12 units = 26.

Which means the side of the fifth square would be $13\sqrt{2}$.

Therefore, the area of the fifth square = 338 sq units

Area of the first square = 98 sq units

Hence, the difference would be 240 sq. units.

Problem 11.4 A spherical pear of radius 4 cm is to be divided into eight equal parts by cutting it in halves along the same axis. Find the surface area of each of the final piece.

(a) 20 π (b) 25 π
(c) 24 π (d) 19 π

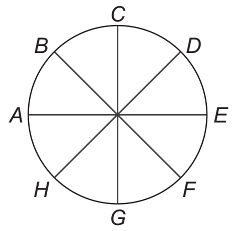


Fig. (a)

Solution The pear after being cut will have eight parts each of same volume and surface area. The figure will be somewhat like the above Figure (a) if seen from the top before cutting. After cutting it look something like the Figure (b).

Now the surface area of each piece = Area *ACBD* + 2 (Area *CODB*).

The darkened surface is nothing but the arc *AB* from side glance which means its surface area is one eighth the area of the sphere, that is, $1/8 \times 4\pi r^2 = (1/2)\pi r^2$.

Now *CODB* can be seen as a semicircle with radius 4 cm.

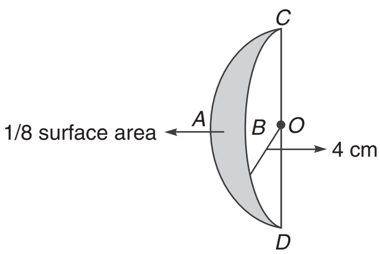


Fig. (b)

Therefore, 2 (Area *CODB*) = $2[(1/2)]\ \pi r^2 = \pi r^2$

$\Rightarrow$ surface area of each piece $= (1/2)\ \pi r^2 + \pi r^2$

$= (3/2)\ \pi r^2$

$= 24\pi$

Problem 11.5 A solid metal sphere is melted and smaller spheres of equal radii are formed. 10% of the volume of the sphere is lost in the process. The smaller spheres have a radius, that is 1/9th the larger sphere. If 10 litres of paint were needed to paint the larger sphere, how many litres are needed to paint all the smaller spheres?

(a) 90 (b) 81
(c) 900 (d) 810

Solution Questions like this require, along with your knowledge of formulae, your ability to form equations. Stepwise, it will be something like this

Step 1: Assume values.

Step 2: Find out volume left.

Step 3: Find out the number of small spheres possible.

Step 4: Find out the total surface area of each small spheres as a ratio of the original sphere.

Step 5: Multiply it by 10.

Step 1: Let radius of the larger sphere be *R* and that of smaller ones be *r*.

Then, volume $= \dfrac{4}{3}\pi R^3$ and $\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi (R/9)^3$ respectively for the larger and smaller spheres.

Step 2: Volume lost due to melting $= \dfrac{4}{3}\pi R^3 \times \dfrac{10}{100} = \dfrac{4\pi R^3}{30}$

$$\text{Volume left} = \frac{4}{3}\pi R^3 \times \frac{90}{100} = \frac{4\pi R^3 \times 0.9}{3}$$

Step 3: Number of small spheres possible = Volume left/ Volume of the smaller sphere

$$= \frac{\frac{4}{3}\pi R^3 \times 0.9}{\frac{4}{3}\pi \times (R/9)^3} = 9^3 \times 0.9$$

Step 4: Surface area of larger sphere = $4\pi R^2$

Surface area of smaller sphere $= 4\pi r^2 = 4\pi\ (R/9)^2 = \dfrac{4\pi R^2}{81}$

Surface area of all smaller spheres = Number of small spheres × Surface area of smaller sphere

$= (9^3 \times .9) \times (4\pi R^2)/81$

$= 8.1 \times (4\pi R^2)$

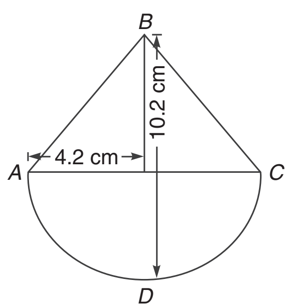
Therefore, ratio of the surface area is $\dfrac{[8.1 \times (4\pi R^2)]}{4\pi R^2} = 8.1$

Step 5:

8.1 × number of litres = 8.1 × 10 = 81

Problem 11.6 A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^2



Solution Volume of the cone is given by $-1/3 \times \pi r^2 h$

Here, $r = 4.2$ cm; $h = 10.2 - r = 6$ cm

Therefore the volume of the cone $= 1/3\ \pi \times (4.2)^2 \times 6$ cm

$= 110.88\ cm^3$

Volume of the hemisphere $= \dfrac{1}{2} \times \dfrac{4}{3}\ \pi r^3 = 155.23$

Total volume = 110.88 + 155.232 = 266.112

Problem 11.7 A vessel is in the form of an inverted cone. Its depth is 8 cm and the diameter of its top, which is open, is 10 cm. It is filled with water up to the brim. When bullets, each of which is a sphere of radius 0.5 cm, are dropped into the vessel 1/4 of the water flows out. Find the number of bullets dropped in the vessel.

(a) 50 (b) 100 (c) 150 (d) 200

Solution In these type of questions it is just your calculation skills that is being tested. You just need to take care that while trying to be fast you don't end up making mistakes like taking the diameter to be the radius and so forth. The best way to avoid such mistakes is to proceed systematically. For example, in this problem we can proceed thus:

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{200}{3}\pi$ cm^3

volme of all the lead shots = Volume of water that spilled out = $\frac{50}{3}\pi$ cm^3

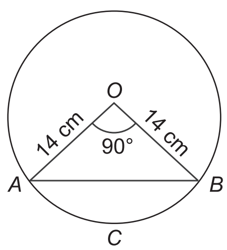
Volume of each lead shot = $\frac{4}{3}\pi r^3 = \frac{\pi}{6}$ cm^3

Number of lead shots = (Volume of water that spilled out)/(Volume of each lead shot)

$$= \frac{\frac{50}{3}\pi}{\frac{\pi}{6}} = \frac{50}{3} \times 6 = 100$$

Problem 11.8 *AB* is a chord of a circle of radius 14 cm. The chord subtends a right angle at the centre of the circle. Find the area of the minor segment.

(a) 98 sq cm (b) 56 sq cm
(c) 112 sq cm (d) None of these



Solution Area of the sector *ACBO* = $\frac{90\pi \times 14^2}{360}$

= 154 sq cm

Area of the triangle *AOB* = $\frac{14 \times 14}{2}$

= 98 sq cm

Area of the segment *ACB* = Area sector *ACBO* – Area of the triangle *AOB* = **56 sq cm**

Problem 11.9 A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. Find the diameter of the base of the cone.

(a) 158.76 cm (b) 79.38 cm
(c) 39.64 cm (d) None of these

Solution In questions like this, do not go for complete calculations. As far as possible, try to cancel out values in the resulting equations.

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.3)^3$

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{\pi}{3} r^2(25.2)$

Now, volume of the cone = volume of the sphere

Therefore, r (radius of the cone) = 39.69 cm

Hence the diameter = **79.38 cm**

Problem 11.10 A chord *AB* of a circle of radius 5.25 cm makes an angle of 60° at the centre of the circle. Find the area of the major and minor segments. (Take π = 3.14)

(a) 168 cm^2 (b) 42 cm^2
(c) 84 cm^2 (d) None of these

Solution The moment you finish reading this question, it should occur to you that this has to be an equilateral triangle. Once you realise this, the question is reduced to just calculations.

Area of the minor sector = $\frac{60}{360} \times \pi \times 5.25^2$

= 14.4375 cm^2

Area of the triangle = $\frac{\sqrt{3}}{4} \times 5.25^2$ = 11.93 cm^2

Area of the minor segment = Area of the minor sector – Area of the triangle = 2.5 cm^2

Area of the major segment = Area of the circle – Area of the minor segment.

= 86.54 cm^2 – 2.5 cm^2 = **84 cm^2**

Problem 11.11 A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

(a) 1 : 2 (b) 2 : 1
(c) 3 : 1 (d) None of these

Solution Questions of this type should be solved without the use of pen and paper. A good authority over formulae will make things easier.

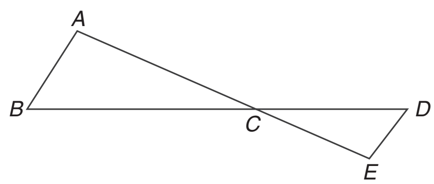
Volume of the cone = $\frac{\pi r^2 h}{3}$ = Volume of a hemisphere = $\frac{2}{3}\pi r^3$.

Height of a hemisphere = Radius of its base
So the question is effectively asking us to find out h/r
By the formula above we can easily see that h/r = **2/1**

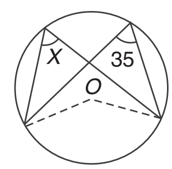
GEOMETRY

LEVEL OF DIFFICULTY (I)

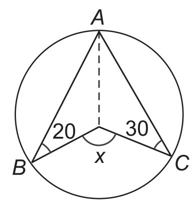
1. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts the shadow 50 m long on the ground. Find the height of the tower.
   (a) 100 m (b) 120 m
   (c) 25 m (d) 200 m
2. In the figure, ΔABC is similar to ΔEDC.



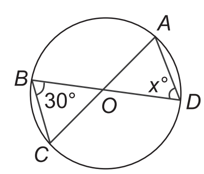
   If we have $AB = 4$ cm,
   $ED = 3$ cm, $CE = 4.2$ and
   $CD = 4.8$ cm, find the value of CA and CB
   (a) 6 cm, 6.4 cm (b) 4.8 cm, 6.4 cm
   (c) 5.4 cm, 6.4 cm (d) 5.6 cm, 6.4 cm
3. The area of similar triangles, ABC and DEF are 144 cm^2 and 81 cm^2 respectively. If the longest side of larger ΔABC be 36 cm, then the longest side of smaller ΔDEF is
   (a) 20 cm (b) 26 cm
   (c) 27 cm (d) 30 cm
4. Two isosceles Δs have equal angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.
   (a) 4/5 (b) 5/4
   (c) 3/2 (d) 5/7
5. The areas of two similar Δs are respectively 9 cm^2 and 16 cm^2. Find the ratio of their corresponding sides.
   (a) 3 : 4 (b) 4 : 3
   (c) 2 : 3 (d) 4 : 5
6. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, find the distance between their tops.
   (a) 12 m (b) 14 m
   (c) 13 m (d) 11 m
7. The radius of a circle is 9 cm and length of one of its chords is 14 cm. Find the distance of the chord from the centre.
   (a) 5.66 cm (b) 6.3 cm
   (c) 4 cm (d) 7 cm
8. Find the length of a chord that is at a distance of 12 cm from the centre of a circle of radius 13 cm.
   (a) 9 cm (b) 8 cm
   (c) 12 cm (d) 10 cm
9. If O is the centre of circle, find $\angle x$



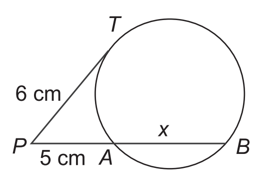
   (a) 35° (b) 30°
   (c) 39° (d) 40°
10. Find the value of $\angle x$ in the given figure.



   (a) 120° (b) 130°
   (c) 110° (d) 100°
11. Find the value of x in the figure, if it is given that AC and BD are diameters of the circle.



   (a) 60° (b) 45°
   (c) 15° (d) 30°
12. Find the value of x in the given figure.



   (a) 2.2 cm (b) 1.6 cm
   (c) 3 cm (d) 2.6 cm