16

Logarithms

INTRODUCTION

Questions based on this chapter are not so frequent in management entrance exams. In exams, most problems that have used the concept of logs have been of an applied nature. However, the aspirants should know the basic concepts of logarithms to ensure there are no surprises in the paper.

While studying this chapter the student should pay particular attention to the basic rules of logarithms as well as develop an understanding of the range of the values of logs.

THEORY

Let a be a positive real number, $a \neq 1$ and $a^x = m$. Then x is called the logarithm of m to the base a and is written as $\log_a m$, and conversely, if $\log_a m = x$, then $a^x = m$.

Note: Logarithm to a negative base is not defined.

Also, logarithm of a negative number is not defined. Hence, in the above logarithmic equation, $\log_a m = x$, and we can say that $m > 0$ and $a > 0$.

Thus $a^x = m \Rightarrow x = \log_a m$ and $\log_a m = x \Rightarrow a^x = m$

In short, $a^x = m \Rightarrow x = \log_a m$.

$x = \log_a m$ is called the logarithmic form and $a^x = m$ is called the exponential form of the equation connecting a, x and m.

Two Properties of Logarithms

1. **$\log_a 1 = 0$ for all $a > 0$, $a \neq 1$**
   That is, log 1 to any base is zero
   Let $\log_a 1 = x$. Then by definition, $a^x = 1$
   But $a^0 = 1 \therefore a^x = a^0 \Leftrightarrow x = 0$.
   Hence $\log_a 1 = 0$ for all $a > 0$, $a \neq 1$
2. **$\log_a a = 1$ for all $a > 0$, $a \neq 1$**
   That is, log of a number to the same base is 1
   Let $\log_a a = x$. Then by definition, $a^x = a$.
   But $a^1 = a \therefore a^x = a^1 \Rightarrow x = 1$.
   Hence $\log_a a = 1$ for all $a > 0$, $a \neq 1$.

Laws of Logarithms

First Law: $\log_a (mn) = \log_a m + \log_a n$
That is, log of product = sum of logs

Second Law: $\log_a (m/n) = \log_a m - \log_a n$
That is, log of quotient = difference of logs

Note: The first theorem converts a problem of multiplication into a problem of addition and the second theorem converts a problem of division into a problem of subtraction, which are far easier to perform than multiplication or division. That is why logarithms are so useful in all numerical calculations.

Third Law: $\log_a m^n = n \log_a m$

Generalisation

1. $\log (mnp) = \log m + \log n + \log p$
2. $\log (a_1 a_2 a_3 \ldots a_k) = \log a_1 + \log a_2 + \ldots + \log a_k$

Note: *Common logarithms*: We shall assume that the base $a = 10$ whenever it is not indicated. Therefore, we shall denote $\log_{10} m$ by $\log m$ only. The logarithm calculated to base 10 are called common logarithms.

The Characteristic and Mantissa of a Logarithm

The logarithm of a number consists of two parts: the *integral* part and the *decimal* part. The integral part is known as the *characteristic* and the decimal part is called the *mantissa*.

For example,

In log 3257 = 3.5128, the integral part is 3 and the decimal part is .5128; therefore, characteristic = 3 and mantissa = .5128.

It should be remembered that the mantissa is always written as positive.

Rule: To make the mantissa positive (in case the value of the logarithm of a number is negative), subtract 1 from the integral part and add 1 to the decimal part.

Thus, $-3.4328 = -(3 + .4328) = -3 - 0.4328$
$= (-3 - 1) + (1 - 0.4328)$
$= -4 + .5672.$

so the mantissa is = .5672.

Note: The characteristic may be positive or negative. When the characteristic is negative, it is represented by putting a bar on the number.

Thus instead of –4, we write $\bar{4}$.

Hence we may write –4 + .5672 as $\bar{4}.5672$.

Base Change Rule

Till now all rules and theorems you have studied in Logarithms have been related to operations on logs with the same basis. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different basis. The base change rule is used in such situations.

This rule states that

(i) $\log_a (b) = \log_c (b)/\log_c (a)$

It is one of the most important rules for solving logarithms.

(ii) $\log_b (a) = \log_c (a)/\log_b (c)$

A corollary of this rule is

(iii) $\log_a (b) = 1/\log_b (a)$

(iv) log c to the base a^b is equal to $\dfrac{\log a^c}{b}$.

Results on Logarithmic Inequalities

(a) If $a > 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 > x_2$

(b) If $a < 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 < x_2$

Applied conclusions for logarithms

1. The characteristic of common logarithms of any positive number less than 1 is negative.
2. The characteristic of common logarithm of any number greater than 1 is positive.
3. If the logarithm to any base a gives the characteristic n, then we can say that the number of integers possible is given by $a^{n+1} - a^n$.

   Example: $\log_{10} x = 2.bcde\ldots$, then the number of integral values that x can take is given by: $10^{2+1} - 10^2 = 900$. This can be physically verified as follows. Log to the base 10 gives a characteristic of 2 for all 3 digit numbers with the lowest being 100 and the highest being 999. Hence, there are 900 integral values possible for x.
4. If $-n$ is the characteristic of $\log_{10} y$, then the number of zeros between the decimal and the first significant number after the decimal is $n - 1$.

   Thus if the log of a number has a characteristic of –3 then the first two decimal places after the decimal point will be zeros.

   Thus, the value will be $-3.00ab\ldots$

Space for Notes

WORKED-OUT PROBLEMS

Problem 16.1 Find the value of x in $3^{|3x-4|} = 9^{2x-2}$

(a) 8/7 (b) 7/8
(c) 7/4 (d) 16/7

Solution Take the log of both sides, then we get,

$$|3x - 4| \log 3 = (2x - 2) \log 9$$
$$= (2x - 2) \log 3^2$$
$$= (4x - 4) \log 3$$

Dividing both sides by log 3, we get

$$|3x - 4| = (4x - 4) \quad (1)$$

Now, $|3x - 4| = 3x - 4$ if $x > 4/3$

so if $x > 4/3$

$3x - 4 = 4x - 4$

or $3x = 4x$

or $3 = 4$

But this is not possible.

Let's take the case of $x < 4/3$

Then $|3x - 4| = 4 - 3x$

Therefore, $4 - 3x = 4x - 4$ from (1)

or $7x = 8$

or $x = 8/7$

Problem 16.2 Solve for x.

$$\log_{10}x - \log_{10}\sqrt{x} = 2 \log_x 10$$

Solution Now, $\log_{10}\sqrt{x} = \frac{1}{2} \times \log_{10}x$

Therefore, the equation becomes

$$\log_{10}x - \frac{1}{2}\log_{10}x = 2 \log_x 10$$

or $$\frac{1}{2}\log_{10}x = 2 \log_x 10 \quad (2)$$

Using base change rule ($\log_b a = 1/\log_a b$)

Therefore, equation (2) becomes

$$\frac{1}{2}\log_{10}x = 2/\log_{10}x$$

$\Rightarrow$ $(\log_{10}x)^2 = 4$

or $\log_{10}x = 2$

Therefore, $x = 100$

Problem 16.3 If $7^{x+1} - 7^{x-1} = 48$, find x.

Solution Take 7^{x-1} as the common term. The equation then reduces to

$7^{x-1}(7^2 - 1) = 48$

or $7^{x-1} = 1$

or $x - 1 = 0$ or $x = 1$

Problem 16.4 Calculate: $\log_2(2/3) + \log_4(9/4)$

$= \log_2(2/3) + (\log_2(9/4)/\log_2 4$

$= \log_2(2/3) + 1/2 \log_2(9/4)$

$= \log_2(2/3) + 1/2\,(2 \log_2 3/2)$

$= \log_2 2/3 + \log_2 3/2 = \log_2 1 = 0$

Problem 16.5 Find the value of the expression

$1/\log_3 2 + 2/\log_9 4 - 3/\log_{27} 8$

Passing to base 2

we get

$$\log_2 3 + 2\log_2 2^9 - 3\log_{2^3} 27$$
$$= \log_2 3 + \frac{4 \log_2 3}{2} - \frac{9 \log_2 3}{3}$$
$$= 3\log_2 3 - 3\log_2 3$$
$$= 0$$

Problem 16.6 Solve the inequality.

(a) $\log_2(x + 3) < 2$

$\Rightarrow$ $2^2 > x + 3$

$\Rightarrow$ $4 > x + 3$

$1 > x$

or $x < 1$

But log of negative number is not possible.

Therefore, $x + 3 \geq 0$

That is, $x \geq -3$

Therefore, $-3 \leq x < 1$

(b) $\log_2(x^2 - 5x + 5) > 0$

$= x^2 - 5x + 5 > 1$

$\rightarrow x^2 - 5x + 4 > 0$

$\rightarrow (x - 4)(x - 1) > 0$

Therefore, the value of x will lie outside 1 and 4.

That is, $x > 4$ or $x < 1$.

Space for Rough Work

LEVEL OF DIFFICULTY (I)

1. log 32700 = ?
   (a) log 3.27 + 4 (b) log 3.27 + 2
   (c) 2 log 327 (d) $100 \times \log 327$
2. log . 0867 = ?
   (a) log 8.67 + 2 (b) log 8.67 – 2
   (c) $\frac{\log 867}{1000}$ (d) –2 log 8.67
3. If $\log_{10}2 = .301$ find $\log_{10}125$.
   (a) 2.097 (b) 2.301
   (c) 2.10 (d) 2.087
4. $\log_{32} 8$ = ?
   (a) 2/5 (b) 5/3
   (c) 3/5 (d) 4/5

Find the value of x in equations 5–6.

5. $\log_{0.5}x = 25$
   (a) 2^{-25} (b) 2^{25}
   (c) 2^{-24} (d) 2^{24}
6. $\log_3 x = \frac{1}{2}$
   (a) 3 (b) $\sqrt{3}$
   (c) $\frac{3}{2}$ (d) $\frac{2}{3}$
7. $\log_{15}3375 \times \log_4 1024$ = ?
   (a) 16 (b) 18
   (c) 12 (d) 15
8. $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. Then a = ?
   (a) 4 (b) 2
   (c) 8 (d) 5
9. $\log_{625}\sqrt{5}$ = ?
   (a) 4 (b) 8
   (c) 1/8 (d) 1/4
10. If log x + log (x + 3) = 1 then the value(s) of x will be, the solution of the equation
    (a) $x + x + 3 = 1$ (b) $x + x + 3 = 10$
    (c) $x(x + 3) = 10$ (d) $x(x + 3) = 1$
11. If $\log_{10}a = b$, find the value of 10^{3b} in terms of a.
    (a) a^3 (b) $3a$
    (c) $a \times 1000$ (d) $a \times 100$
12. 3 log 5 + 2 log 4 – log 2 = ?
    (a) 4 (b) 3
    (c) 200 (d) 1000

Solve equations 13–25 for the value of x.

13. log (3x – 2) = 1
    (a) 3 (b) 2
    (c) 4 (d) 6
14. log (2x – 3) = 2
    (a) 103 (b) 51.5
    (c) 25.75 (d) 26
15. log (12 – x) = –1
    (a) 11.6 (b) 12.1
    (c) 11 (d) 11.9
16. $\log (x^2 - 6x + 6) = 0$
    (a) 5 (b) 1
    (c) Both (a) and (b) (d) 3 and 2
17. $\log 2^x = 3$
    (a) 9.87 (b) 3 log 2
    (c) 3/log 2 (d) 9.31
18. $3^x = 7$
    (a) $1/\log_7 3$ (b) $\log_7 3$
    (c) $1/\log_3 7$ (d) $\log_3 7$
19. $5^x = 10$
    (a) log 5 (b) log 10/log 2
    (c) log 2 (d) 1/log 5
20. Find x, if $0.01^x = 2$
    (a) log 2/2 (b) 2/log 2
    (c) –2/log 2 (d) – log 2/2
21. Find x if log x = log 7.2 – log 2.4
    (a) 1 (b) 2
    (c) 3 (d) 4
22. Find x if log x = log 1.5 + log 12
    (a) 12 (b) 8
    (c) 18 (d) 15
23. Find x if log x = 2 log 5 + 3 log 2
    (a) 50 (b) 100
    (c) 150 (d) 200
24. log (x – 13) + 3 log 2 = log (3x + 1)
    (a) 20 (b) 21
    (c) 22 (d) 24
25. log (2x – 2) – log (11.66 – x) = 1 + log 3
    (a) 452/32 (b) 350/32
    (c) 11 (d) 11.33

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. Express log $\frac{\sqrt[3]{a^2}}{b^5\sqrt{c}}$ or $\frac{a^{2/3}}{b^5\sqrt{c}}$ in terms of log a, log b and log c.
   (a) $\frac{3}{2}$ log a + 5 log b – 2 log c
   (b) $\frac{2}{3}$ log a – 5 log b – $\frac{1}{2}$ log c
   (c) $\frac{2}{3}$ log a – 5 log b + $\frac{1}{2}$ log c
   (d) $\frac{3}{2}$ log a + 5 log b – $\frac{1}{2}$ log c
2. If log 3 = .4771, find log $(.81)^2 \times \log\left(\frac{27}{10}\right)^{\frac{2}{3}} \div \log 9$.
   (a) 2.689 (b) – 0.0552
   (c) 2.2402 (d) 2.702
3. If log 2 = .301, log 3 = .477, find the number of digits in $(108)^{10}$.
   (a) 21 (b) 27
   (c) 20 (d) 18
4. If log 2 = .301, find the number of digits in $(125)^{25}$.
   (a) 53 (b) 50
   (c) 25 (d) 63
5. Which of the following options represents the value of log $\sqrt{128}$ to the base .625?
   (a) $\frac{2+\log_8 2}{\log_8 5 - 1}$ (b) $\frac{\log_8 128}{2\log_8 0.625}$
   (c) $\frac{2+\log_8 2}{2(\log_8 5 - 1)}$ (d) Both (b) and (c)

6-8. Solve for x:

$\log\frac{75}{35} + 2\log\frac{7}{5} - \log\frac{105}{x} - \log\frac{13}{25} = 0.$

   (a) 90 (b) 65
   (c) 13 (d) 45

7. $2\log\frac{4}{3} - \log\frac{x}{10} + \log\frac{63}{160} = 0$
   (a) 7 (b) 14
   (c) 9 (d) 3
8. $\log\frac{12}{13} - \log\frac{7}{25} + \log\frac{91}{3} = x$
   (a) 0 (b) 1
   (c) 2 (d) 3

Questions 9 to 11: Which one of the following is true

9. (a) $\log_{17}275 = \log_{19}375$ (b) $\log_{17}275 < \log_{19}375$
   (c) $\log_{17}275 > \log_{19}375$ (d) Cannot be determined
10. (a) $\log_{11}1650 > \log_{13}1950$
    (b) $\log_{11}1650 < \log_{13}1950$
    (c) $\log_{11}1650 = \log_{13}1950$
    (d) None of these
11. (a) $\frac{\log_2 4096}{3} = \log_8 4096$
    (b) $\frac{\log_2 4096}{3} < \log_8 4096$
    (c) $\frac{\log_2 4096}{3} > \log_8 4096$
    (d) Cannot be determined
12. $\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log x,\ x = ?$
    (a) 2 (b) 3
    (c) 0 (d) None of these

If log 2 = 0.301 and log 3 = .4771 then find the number of digits in the following.

13. 60^{12}
    (a) 25 (b) 22
    (c) 23 (d) 24
14. 72^9
    (a) 17 (b) 20
    (c) 18 (d) 15
15. 27^{25}
    (a) 38 (b) 37
    (c) 36 (d) 35

Questions 16 to 18: Find the value of the logarithmic expression in the questions below.

16. $\frac{\log\sqrt{27} + \log 8 - \log\sqrt{1000}}{\log 1.2}$
    where, $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.4771213$
    (a) 1.77 (b) 1.37
    (c) 2.33 (d) 1.49
17. $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} =$
    (a) 1 (b) 2
    (c) 3 (d) 4
18. $\log a^n/b^n + \log b^n/c^n + \log c^n/a^n$
    (a) 1 (b) n
    (c) 0 (d) 2

19. $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$ then x = ?
(a) 50 (b) 100
(c) 150 (d) 200

20. $\left(\frac{21}{10}\right)^x = 2$. Then x = ?
(a) $\frac{\log 2}{\log 3 + \log 7 - 1}$ (b) $\frac{\log 2}{\log 3 + \log 7 + 1}$
(c) $\frac{\log 3}{\log 2 + \log 7 - 1}$ (d) $\frac{\log 2}{\log 3 - \log 7 + 1}$

21. $\log(x^3 + 5) = 3\log(x + 2)$ then x = ?
(a) $\frac{-2+\sqrt{2}}{2}$ (b) $\frac{-2-\sqrt{2}}{2}$
(c) Both (a) and (b) (d) None of these

22. $(a^4 - 2a^2b^2 + b^4)^{x-1} = (a - b)^{-2}(a + b)^{-2}$ then x = ?
(a) 1 (b) 0
(c) None of these (d) 2

23. If $\log_{10} 242 = a$, $\log_{10} 80 = b$ and $\log_{10} 45 = c$, express $\log_{10} 36$ in terms of a, b and c.
(a) $\frac{(c-1)(3c+b-4)}{2}$ (b) $\frac{(c-1)(3c+b-4)}{3}$
(c) $\frac{(c-1)(3c-b-4)}{2}$ (d) None of these

24. For the above problem, express $\log_{10} 66$ in terms of a, b and c.
(a) $\frac{(c-1)(3c+b-4)}{8}$ (b) $\frac{3(a+c)+(2b+-5)}{6}$
(c) $\frac{3(a+c)+(2b-5)}{6}$ (d) $\frac{3(c-1)(3c+b-4)}{6}$

25. $\log_2(9 - 2^x) = 10^{\log(3-x)}$. Solve for x.
(a) 0 (b) 3
(c) Both (a) and (b) (d) 0 and 6

26. If $\frac{\log_x}{b-c} = \frac{\log_y}{c-a} = \frac{\log_z}{a-b}$. Mark all the correct options. **IIFT 2006**
(a) $xyz = 1$ (b) $x^a y^b z^c = 1$
(c) $x^{b+c} y^{c+a} z^{a+b} = 1$ (d) All the options are correct.

27. What will be the value of x if it is given that:
$$\log_x\left[\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\ldots+\infty \text{ terms}\right]^2 = 2$$

28. $(\log_4 x^2)(x\log_{27}8)(\log_x 243)$ is equal to:
(a) $2x$ (b) $5x$
(c) $3x$ (d) 1

29. For how many real values of x will the equation $\log_3 \log_6(x^3 - 18x^2 + 108x) = \log_2 \log_4 16$ be satisfied?

30. If $n = 12\sqrt{3}$
$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\frac{1}{\log_6 n}+\frac{1}{\log_8 n}+\frac{1}{\log_9 n}+\frac{1}{\log_{18} n} = ?$$

31. $(\log_2 x)^2 + 2\log_2 x - 8 = 0$,Where x is a natural number. If $x^p = 64$, then what is the value of $x + p$.

Directions for 41 and 42: $A = \sum_{i=2}^{a}\log_3(i), B = \sum_{j=2}^{b}\log_3(j)$ & $C = \sum_{k=2}^{(a-b)}\log_3\log_3 k$, where $a \geq b$. If D = A – B – C. Then answer the following questions.

32. If $a = 10$ then for what value of b, D is minimum

33. For $a = 6$, D is maximum for b =

34. If 'p' and 'q' are integers and $\log_p(-q^2 + 6q - 8) + \log_q(-2p^2 + 20p - 48) = 0$ then $p \times q$ = ?

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (a)	2. (b)	3. (a)	4. (c)
5. (a)	6. (b)	7. (d)	8. (a)
9. (c)	10. (c)	11. (a)	12. (b)
13. (c)	14. (b)	15. (d)	16. (c)
17. (c)	18. (a)	19. (d)	20. (d)
21. (c)	22. (c)	23. (d)	24. (b)
25. (c)			

Level of Difficulty (II)

1. (b)	2. (b)	3. (a)	4. (a)
5. (d)	6. (c)	7. (a)	8. (c)
9. (b)	10. (a)	11. (a)	12. (d)
13. (b)	14. (a)	15. (c)	16. (d)
17. (b)	18. (c)	19. (b)	20. (a)
21. (c)	22. (b)	23. (d)	24. (c)
25. (a)	26. (d)	27. 25/48	28. (b)
29. 1	30. 4	31. 7	32. 10
33. 3	34. 15		

Solutions and Shortcuts

Level of Difficulty (I)

1. Log 32700 = log 3.27 + log 10000 = log 3.27 + 4
2. Log 0.0867 = log (8.67/100) = log 8.67 – log 100
   Log 8.67 – 2
3. $\log_{10} 125 = \log_{10}(1000/8) = \log 1000 - 3\log 2$
   $= 3 - 3 \times 0.301 = 2.097$
4. $\log_{32} 8 = \log 8/\log 32$ (By base change rule)
   $= 3 \log 2/5\log 2 = 3/5$.
5. $\log_{0.5} x = 25 \Rightarrow x = 0.5^{25} = (½)^{25} = 2^{-25}$
6. $x = 3^{1/2} = \sqrt{3}$.
7. $\log_{15}3375 \times \text{Log}_4 1024$
   $= 3 \log_{15} 15 \times 5 \log_4 4 = 3 \times 5 = 15$.
8. The given expression is:
   $\text{Log}_a (4 \times 16 \times 64 \times 256) = 10$
   i.e. $\log_a 4^{10} = 10$
   Thus, $a = 4$.
9. $1/2 \log_{625} 5 = [1/(2 \times 4)] \log_5 5 = 1/8$.
10. $\log x (x + 3) = 1 \Rightarrow 10^1 = x^2 + 3x$.
    or $x(x + 3) = 10$.
11. $\log_{10} a = b \Rightarrow 10^b = a \Rightarrow$ By definition of logs.
    Thus $10^{3b} = (10^b)^3 = a^3$.
12. 3 log 5 + 2 log 4 – log 2
    = log 125 + log 16 – log 2
    = log (125 × 16)/2 = log 1000 = 3.
13. $10^1 = 3x - 2 \Rightarrow x = 4$.
14. $10^2 = 2x - 3 \Rightarrow x = 51.5$
15. $1/10 = 12 - x \Rightarrow x = 11.9$
16. $x^2 - 6x + 6 = 10° \Rightarrow x^2 - 6x + 6 = 1$
    $\Rightarrow x^2 - 6x + 5 = 0$
    Solving gives us $x = 5$ and 1.
17. $x \log 2 = 3$
    $\log 2 = 3/x$.
    Therefore, $x = 3/\log 2$
18. $3^x = 7 \Rightarrow \log_3 7 = x$
    Hence $x = 1/\log_7 3$
19. $x = \log_5 10 = 1/\log_{10} 5 = 1/\log 5$.
20. $x = \log_{0.01} 2 = -\log 2/2$.
21. $\log x = \log (7.2/2.4) = \log 3 \Rightarrow x = 3$
22. $\log x = \log 18 \Rightarrow x = 18$
23. $\log x = \log 25 + \log 8 = \log (25 \times 8) = \log 200$.
24. $\log (x - 13) + \log 8 = \log [3x + 1]$
    $\Rightarrow \log (8x - 104) = \log (3x + 1)$
    $\Rightarrow 8x - 104 = 3x + 1$
    $5x = 105 \Rightarrow x = 21$
25. $\log (2x - 2)/(11.66 - x) = \log 30$
    $\Rightarrow (2x - 2)/(11.66 - x) = 30$
    $2x - 2 = 350 - 30x$
    Hence, $32x = 352 \Rightarrow x = 11$.

Level of Difficulty (II)

1. $2/3 \log a - 5 \log b - 1/2 \log c$.
2. 2 Log (81/100) × 2/3 log (27/10) ÷ log 9
   $= 2 [\log 3^4 - \log 100] \times 2/3 [(\log 3^3 - \log 10)] \div 2 \log 3$
   $= 2 [\log 3^4 - \log 100] \times 2/3 [(3\log 3 - 1)] \div 2 \log 3$
   Substitute log 3 = 0.4771 ⇒ – 0.0552.
3. Let the number be y.
   $y = 108^{10}$
   $\Rightarrow \log y = 10 \log 108$
   $\text{Log } y = 10 \log (27 \times 4)$
   $\text{Log } y = 10 [3\log 3 + 2 \log 2]$
   $\text{Log } y = 10 [1.43 + 0.602]$
   Hence $\log y = 10[2.03] = 20.3$
   Thus, y has 21 digits.
4. $\log y = 25 \log 125$
   $= 25 [\log 1000 - 3 \log 2] = 25 \times (2.097)$
   = 52 +
   Hence 53 digits.
5. $0.5 \log_{0.625} 128$
   $= 0.5 [\log_8 128/\log_8 0.625]$
   $= 1/2 [\log_8 128/\log_8 0.625]$
   $$\frac{\text{Log}_8 128}{2(\log_8 5 - \log_8 8)} = \frac{\text{Log}_8 128}{2[\log_8 5 - 1]} = \frac{2 + \log_8 2}{2(\log_8 5 - 1)}$$
6. $(75/35) \times (49/25) \times (x/105) \times (25/13) = 1$
   $\Rightarrow x = 13$
7. $(16/9) \times (10/x) \times (63/160) = 1$
   $\Rightarrow x = 7$
8. Solve in similar fashion.

9. $\text{Log}_{17} 275 < \log_{19} 375$
Because the value of $\text{Log}_{17}275$ is less than 2 while $\log_{19}375$ is greater then 2.
10. $\log_{11}1650 > 3$
$\text{Log}_{13}1950 < 3$
Hence, $\log_{11}1650 > \log_{13}1950$
11. $\dfrac{\log_2 4096}{3} = \log_8 4096$
12. $x = (16/15) \times (25^5/24^5) \times (81^3/80^3)$
None of these is correct.

13 – 15.
Solve similarly as 3 and 4.

18. $\log (a^n b^n c^n / a^n b^n c^n) = \log 1 = 0$
19. $(1/2) \log x = 2 \log_x 10$
$\Rightarrow \quad \log x = 4 \log_x 10$
$\Rightarrow \quad \log x = 4/\log_{10} x \quad \Rightarrow \quad (\log x)^2 = 4$
So $\log x = 2$ and $x = 100$.
20. $x = \log_{(21/10)} 2$
$$= \frac{\text{Log}\, 2}{\text{Log}\, 21 - \log 10} = \frac{\text{Log}\, 2}{[\text{Log}\, 3 + \log 7 - 1]}$$
21. $6x^2 + 12x + 3 = 0$ or $2x^2 + 4x + 1 = 0$
Solving we get both the options (a) and (b) as correct. Hence, option (c) is the correct answer.
25. For $x = 0$, we have LHS
$\text{Log}_2 8 = 3$.
RHS: $10^{\log 3} = 3$.
We do not get LHS = RHS for either $x = 3$ or $x = 6$.
Thus, option (a) is correct.
26. $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b} = k$

$\Rightarrow x = 10^{k(b-c)}, y = 10^{k(c-a)}, z = 10^{k(a-b)}$
$\therefore xyz = 10^{k(b-c+c-a+a-b)} = 10^0 = 1$
Therefore option (a) is correct.
$x^a y^b z^c = 10^{k[a(b-c) + b(c-a) + c(a-b)]}$
$= 10^{k(ab - ac + bc - ab + ca - bc)}$
$= 10^{K.0} = 1$
Therefore option (b) is correct.
$x^{b+c} y^{c+a} z^{a+b} = 10^{k[(b+c)(b-c) + (c+a)(c-a) + (a+b)(a-b)]}$
$= 10^{k.0} = 1$
Therefore option (c) is also correct.
Since all the first three options are correct, we choose option (d) as the correct answer.
27. $\log_x \left[\dfrac{1}{5} + \dfrac{1}{12} + \dfrac{1}{21} + \dfrac{1}{32} + \dfrac{1}{45} + \ldots + \infty \text{ terms}\right]^2$
$$= 2\log_x \left(\frac{1}{5} + \frac{1}{12} + \frac{1}{21} + \frac{1}{32} + \frac{1}{45} + \ldots + \infty \text{ terms}\right)$$
Let $\dfrac{1}{5} + \dfrac{1}{12} + \dfrac{1}{21} + \dfrac{1}{32} + \dfrac{1}{45} + \ldots + \infty \text{ terms} = P$
$$P = \frac{1}{4}\left[\frac{4}{1\times 5} + \frac{4}{2\times 6} + \frac{4}{3\times 7} + \frac{4}{4\times 8} + \frac{4}{5\times 9} + \ldots \infty\right]$$
$$4P = \left[1 - \frac{1}{5} + \frac{1}{2} - \frac{1}{6} + \frac{1}{3} - \frac{1}{7} + \frac{1}{4} - \frac{1}{8} + \frac{1}{5} - \frac{1}{9}\right]$$
$$4P = \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right]$$
$$4P = \frac{25}{12}$$
$$P = \frac{25}{48}$$
$$\log_x \frac{25}{48} = 1 \text{ or } x = 25/48$$
28. $\log_4 x^2 . x\log_{27} 8 . \log_x 243 = \dfrac{2\log x}{\log 4} . \dfrac{x\log 8}{\log 27} . \dfrac{\log 243}{\log x}$
$$= \frac{\log x}{\log 2} . \frac{3x\log 2}{3\log 3} . \frac{5\log 3}{\log x} = 5x$$
29. $\log_2(\log_4 16) = \log_2 \log_4 4^2 = \log_2 2 = 1$
$\log_3 \log_6 (x^3 - 18x^2 + 108x) = 1$
$\log_6(x^3 - 18x^2 + 108x) = 3$
$x^3 - 18x^2 + 108x = 6^3$
$x^3 - 18x^2 + 108x - 216 = 0$
$(x - 6)^3 = 0$
$x = 6$ is the only value for which the above equation is true.
30. $n = 12\sqrt{3} = 2^2 \times 3^{1.5}$
$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \frac{1}{\log_6 n} + \frac{1}{\log_8 n} + \frac{1}{\log_9 n} + \frac{1}{\log_{18} n}$$
$= \log_n 2 + \log_n 3 + \log_n 4 + \log_n 6 + \log_n 8 + \log_n 9 + \log_n 18$
$= \log_n (2 \times 3 \times 4 \times 6 \times 8 \times 9 \times 18)$
$= \log_n (2^8 \times 3^6)$
$= \log_n (2^2 \times 3^{1.5})^4$
$= 4 \log_n (2^2 \times 3^{1.5})$
$= 4\log_{2^2 \times 3^{1.5}} \left(2^2 \times 3^{1.5}\right) = 4$
31. $(\log_2 x)^2 + 2\log_2 x - 8 = 0$
$(\log_2 x)^2 + 4\log_2 x - 2\log_2 x - 8 = 0$
$\log_2 x\, [\log_2 x + 4] - 2\, [\log_2 x + 4] = 0$
$[\log_2 x - 2]\, [\log_2 x + 4] = 0$
Since, x is a natural number hence $[\log_2 x + 4]$ cannot be zero. Hence, $\log_2 x - 2 = 0$
$\log_2 x = 2$
$x = 2^2 = 4$
We are given that: $x^p = 64$. Since x is 4, this means