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Set Theory

Set Theory is an important concept of mathematics which is often asked in aptitude exams. There are two types of questions in this chapter:

(i) Numerical questions on set theory based on venn diagrams

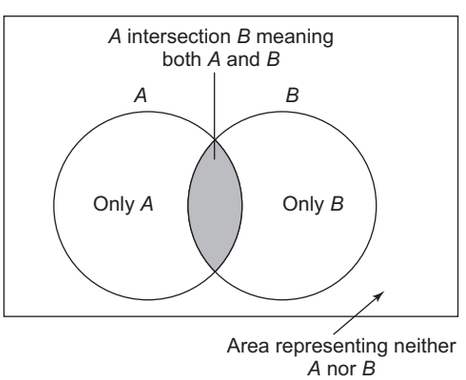
(ii) Logical questions based on set theory

Let us first take a look at some standard theoretical inputs related to set theory.

SET THEORY

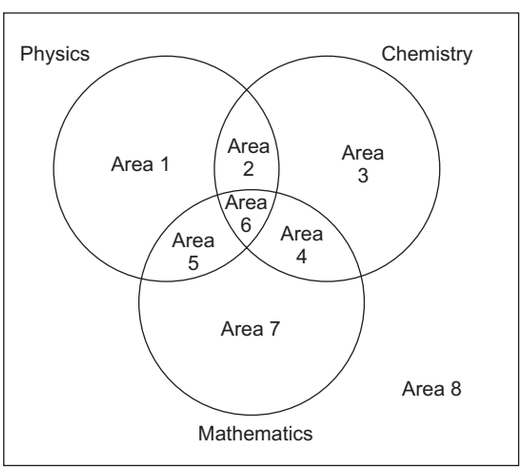
Look at the following diagrams:

Figure 1: Refers to the situation where there are two attributes *A* and *B*. (Let's say *A* refers to people who passed in Physics and *B* refers to people who passed in Chemistry.) Then the shaded area shows the people who passed both in Physics and Chemistry.



In mathematical terms, the situation is represented as:
Total number of people who passed at least 1 subject = $A + B - A \cap B$

Figure 2: Refers to the situation where there are three attributes being measured. In the figure below, we are talking about people who passed Physics, Chemistry and/or Mathematics.



In the above figure, the following explain the respective areas:

Area 1: People who passed in Physics only

Area 2: People who passed in Physics and Chemistry only (in other words—people who passed Physics and Chemistry but not Mathematics)

Area 3: People who passed Chemistry only

Area 4: People who passed Chemistry and Mathematics only (also, can be described as people who passed Chemistry and Mathematics but not Physics)

Area 5: People who passed Physics and Mathematics only (also, can be described as people who passed Physics and Mathematics but not Chemistry)

Area 6: People who passed Physics, Chemistry and Mathematics

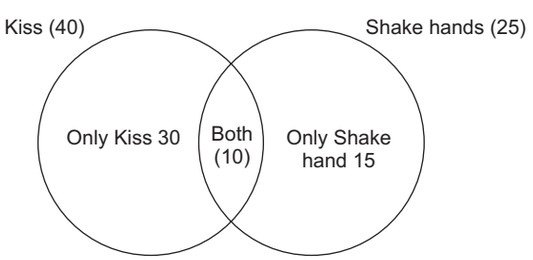
Area 7: People who passed Mathematics only
Area 8: People who passed in no subjects
Also take note of the following language which there is normally confusion about:
People passing Physics and Chemistry—Represented by the sum of areas 2 and 6
People passing Physics and Maths—Represented by the sum of areas 5 and 6
People passing Chemistry and Maths—Represented by the sum of areas 4 and 6
People passing Physics—Represented by the sum of the areas 1, 2, 5 and 6
In mathematical terms, this means:
Total number of people who passed at least 1 subject =
$P + C + M - P \cap C - P \cap M - C \cap M + P \cap C \cap M$
Let us consider the following questions and see how these figures work in terms of real time problem solving:

Illustration 1

At the birthday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. 10 persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

(a) 35 (b) 75
(c) 55 (d) 25

Solution:



From the figure, it is clear that the number of people at the party were 30 + 10 + 15 = 55.
We can of course solve this mathematically as below:
Let n(A) = No. of persons who kissed Sherry = 40
n(B) = No. of persons who shake hands with Sherry = 25
and $n(A \cap B)$ = No. of persons who shook hands with Sherry and kissed him both = 10
Then using the formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 40 + 25 - 10 = 55$

Illustration 2

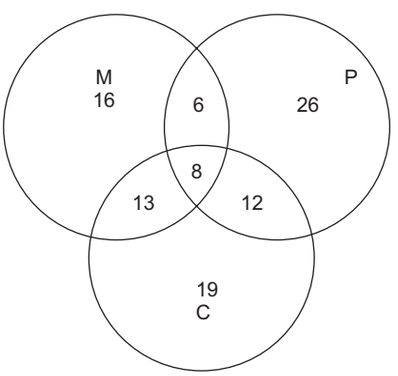
Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow:

In an examination 43% passed in Math, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Math and Physics and 21% passed in Math and Chemistry and 20% passed in Physics and Chemistry. Number of students who took the exam is 200.

Let Set P, Set C and Set M denotes the students who passed in Physics, Chemistry and Math respectively. Then

1. How many students passed in Math only?
   (a) 16 (b) 32
   (c) 48 (d) 80
2. Find the ratio of students passing in Math only to the students passing in Chemistry only?
   (a) 16:37 (b) 29:32
   (c) 16:19 (d) 31:49
3. What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?
   (a) 34/46 (b) 26/84
   (c) 49/32 (d) None of these
4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is?
   (a) 33 (b) 66
   (c) 39 (d) 78

Sol. Let P denote Physics, C denote Chemistry and M denote Maths.
% of students who passed in P and C only is given by
% of students who passed in P and C - % of students who passed all three = 20% – 8% = 12%
% of students who passed in P and M only is given by
% of students who passed in P and M - % of students who passed all three = 14% – 8% = 6%
% of students who passed in M and C only is:
% of students who passed in C and M - % of students who passed all three = 21% – 8% = 13%
So, % of students who passed in P only is given by:
Total no. passing in P – No. Passing in P & C only – No. Passing P & M only – No. Passing in all three→
52% – 12% – 6% – 8%- = 26%
% of students who passed in M only is:
Total no. passing in M – No. Passing in M & C only – No. Passing P & M only – No. Passing in all three→
43% – 13% – 6% – 8%- = 16%



% of students who passed in Chemistry only is
Total no. passing in C – No. Passing in P & C only – No. Passing C & M only – No. Passing in all three →
52% – 12% – 13% – 8%– = 19%

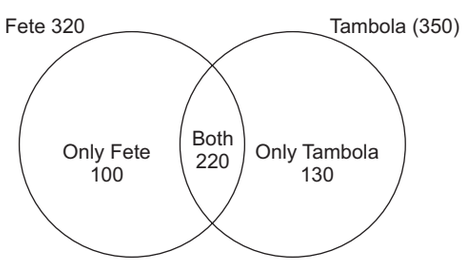
The answers are:

1. Only Math =16% = 32 people. Option (b) is correct.
2. Ratio of Only Math to Only Chemistry = 16:19. Option (c) is correct.
3. 26:84 is the required ratio. Option (b) is correct.
4. 39 % or 78 people. Option (d) is correct.

Illustration 3

In the Mindworkzz club all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?

(a) 410 (b) 550
(c) 440 (d) None of these

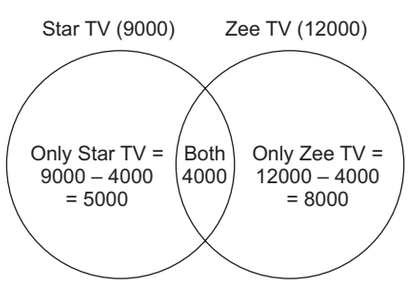


The total number of people = 100 + 220 + 130 = 450
Option (d) is correct.

Illustration 4

There are 20000 people living in Defence Colony, Gurgaon. Out of them 9000 subscribe to Star TV Network and 12000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?

(a) 3000 (b) 2000
(c) 1000 (d) 4000



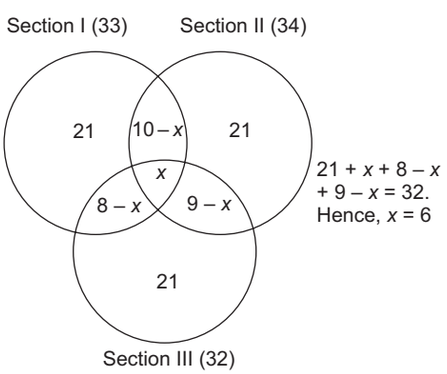
The required answer would be 20000 – 5000 – 4000 – 8000 = 3000.

Illustration 5

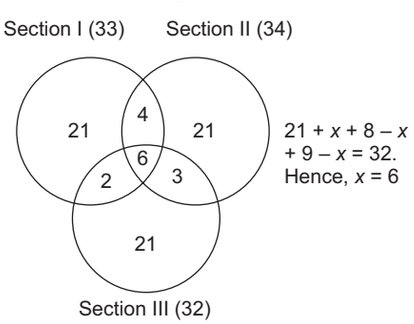
Directions for Questions 1 to 3: Refer to the data below and answer the questions that follow.

Last year, there were 3 sections in the Catalyst, a mock CAT paper. Out of them 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. 10 students cleared the cut-off in Section 1 and Section 2, 9 cleared the cut-off in Section 2 and Section 3, 8 cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

1. How many cleared all the three sections?
   (a) 3 (b) 6
   (c) 5 (d) 7
2. How many cleared only one of the three sections?
   (a) 21 (b) 63
   (c) 42 (d) 52
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cutoff in Section 1 alone is?
   (a) 78/21 (b) 3
   (c) 73/21 (d) None of these



Since, $x = 6$, the figure becomes:



The answers would be:

1. 6. Option (b) is correct.
2. 21+21+21=63. Option (b) is correct.
3. (21+21+21+6+4+3+2)/21 = 78/21. Option (a) is correct.

Illustration 6

In a locality having 1500 households, 1000 watch Zee TV, 300 watch NDTV and 750 watch Star Plus. Based on this information answer the questions that follow:

1. The minimum number of households watching Zee TV and Star Plus is:

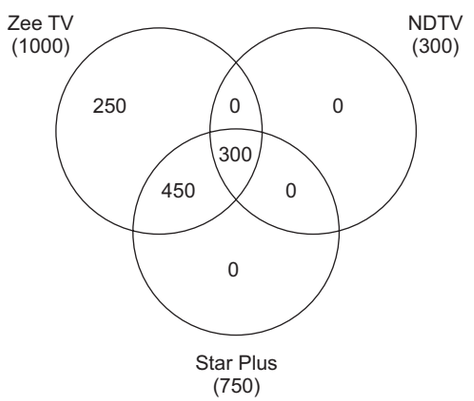
   Logic: If we try to consider each of the households watching Zee TV and Star Plus as independent of each other, we would get a total of 1000 + 750 = 1750 households. However, we have a total of only 1500 households in the locality and hence, there has to be a minimum interference of at least 250 households who would be watching both Zee TV and Star Plus. Hence, the answer to this question is 250.

2. The minimum number of households watching both Zee TV and NDTV is:

   In this case, the number of households watching Zee TV and NDTV can be separate from each other since there is no interference required between the households watching Zee TV and the households watching NDTV as their individual sum (1000 + 300) is smaller than the 1500 available households in the locality. Hence, the answer in this question is 0.

3. The maximum number of households who watch neither of the the three channels is:

   For this to occur the following situation would give us the required solution:



   As you can clearly see from the figure, all the requirements of each category of viewers is fulfilled by the given allocation of 1000 households. In this situation, the maximum number of households who do not watch any of the three channels is visible as 1500 –1000 = 500.

Illustration 7

1. In a school, 90% of the students faced problems in Mathematics, 80% of the students faced problems in Computers, 75% of the students faced problems in Sciences, and 70% of the students faced problems in Social Sciences. Find the minimum percent of the students who faced problems in all four subjects.

   Solution: In order to think about the minimum number of students who faced problems in all four subjects you would need to think of keeping the students who did not face a problem in any of the subjects separate from each other. We know that 30% of the students did not face problems in Social Sciences, 25% of the students did not face problems in Sciences, 20% students did not face problems in computers and 10% students did not face problems in Mathematics. If each of these were separate from each other, we would have 30+25+20+10=85% people who did not face a problem in one of the four subjects. Naturally, the remaining 15% would be students who faced problems in all four subjects. This represents the minimum percentage of students who faced problems in all the four subjects.

2. For the above question, find the maximum possible percentage of students who could have problems in all 4 subjects.

   In order to solve this, you need to consider the fact that 100 (%) people are counted 315 (%) times, which means that there is an extra count of 215 (%). When you put a student into the 'has problems in each of the four subjects' he is one student counted four times — an extra count of 3. Since, 215/3= 71 (quotient) we realise that if we have 71 students who have problems in all four subjects — we will have an extra count of 213 students. The remaining extra count of 2 more can be matched by putting 1 student in 'has problems in 3 subjects' or by putting 2 students in 'has problems in 2 subjects'. Thus, from the extra count angle, we have a limit of 71% students in the 'have problems in all four categories.'

   However, in this problem there is a constraint from another angle — i.e. there are only 70% students who have a problem in Social Sciences — and hence it is not possible for 71% students to have problems in all the four subjects. Hence, the maximum possible percentage of people who have a problem in all four subjects would be 70%.

3. In the above question if it is known that 10% of the students faced none of the above mentioned four problems, what would have been the minimum number of students who would have a problem in all four subjects?

   If there are 10% students who face none of the four problems, we realise that these 10% would be common to students who face no problems in Mathematics, students who face no problems in Sciences, students who face no problems in Computers and students who face no problems in Social Sciences.

   Now, we also know that overall there are 10% students who did not face a problem in Mathematics; 20%

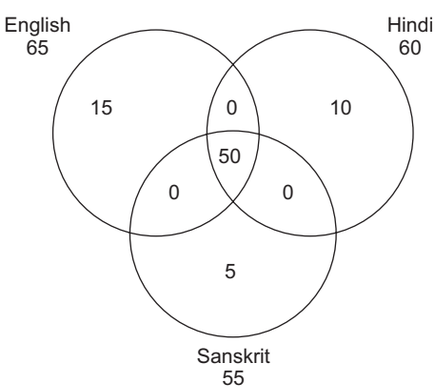
students who did not face a problem in computers; 25% students who did not face a problem in Sciences and 30% students who did not face a problem in Social Sciences. The 10% students who did not face a problem in any of the subjects would be common to each of these 4 counts. Out of the remaining 90% students, if we want to identify the minimum number of students who had a problem in all four subjects we will take the same approach as we took in the first question of this set — i.e. we try to keep the students having problems in the individual subjects separate from each other. This would result in: 0% additional students having no problem in Mathematics; 10% additional students having no problem in Computers; 15% additional students having no problem in Sciences and 20% additional students having no problem in Social Sciences. Thus, we would get a total of 45% (0+10+15+20=45) students who would have no problem in one of the four subjects. Thus, the minimum percentage of students who had a problem in all four subjects would be 90 – 45 = 45%.

Illustration 8

In a class of 80 students, each of them studies at least one language — English, Hindi and Sanskrit. It was found that 65 studied English, 60 studied Hindi and 55 studied Sanskrit.

1. Find the maximum number of people who study all three languages.

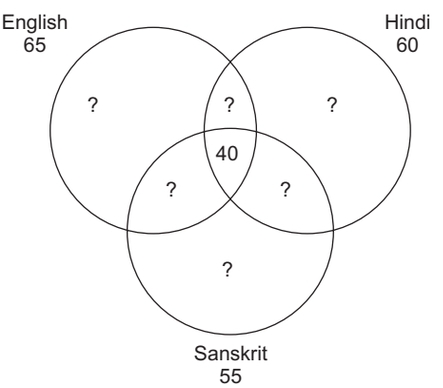
   This question again has to be dealt with from the perspective of extra counting. In this question, 80 students in the class are counted 65 + 60 + 55 = 180 times — an extra count of 100. If we put 50 people in the all three categories as shown below, we would get the maximum number of students who study all three languages.



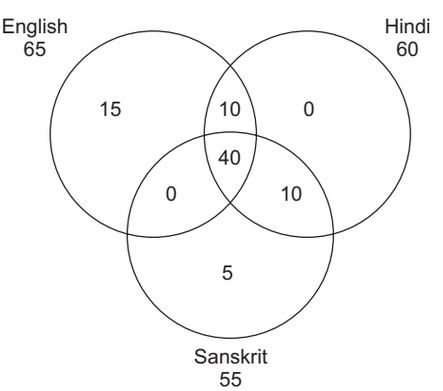
2. Find the minimum number of people who study all three languages.

   In order to think about how many students are necessarily in the 'study all three languages' area of the figure (this thinking would lead us to the answer to the minimum number of people who study all three languages) we need to think about how many people we can shift out of the 'study all three category' for the previous question. When we try to do that, the following thought process emerges:

   Step 1: Let's take a random value for the all three categories (less than 50 of course) and see whether the numbers can be achieved. For this purpose we try to start with the value as 40 and see what happens. Before we move on, realise the basic situation in the question remains the same — 80 students have been counted 180 times — which means that there is an extra count of 100 students & also realise that when you put an individual student in the all three categories, you get an extra count of 2, while at the same time when you put an individual student into the 'exactly two languages category', he/she is counted twice — hence an extra count of 1.
   The starting figure we get looks something like this:



At this point, since we have placed 40 people in the all three categories, we have taken care of an extra count of $40 \times 2 = 80$. This leaves us with an extra count of 20 more to manage and as we can see in the above figure we have a lot of what can be described as 'slack' to achieve the required numbers. For instance, one solution we can think of from this point is as below:

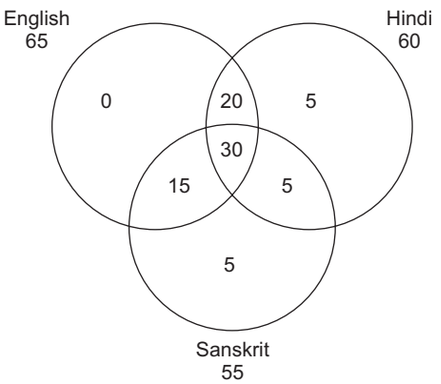


One look at this figure should tell you that the solution can be further optimised by reducing the middle value in

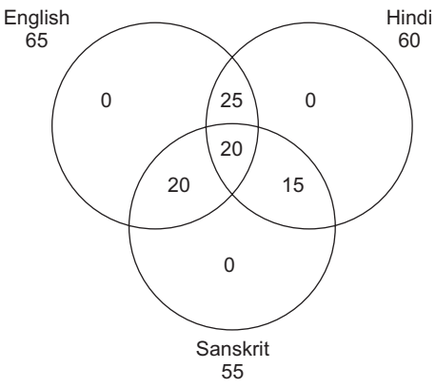
the figure since there is still a lot of 'slack' in the figure — in the form of the number of students in the 'exactly one language category'. Also, you can easily see that there are many ways in which this solution could have been achieved with 40 in the middle. Hence, we go in search of a lower value in the middle.

So, we try to take an arbitrary value of 30 to see whether this is still achievable.

In this case we see the following as one of the possible ways to achieve this (again there is a lot of slack in this solution as the 'only Hindi' or the 'only Sanskrit' areas can be reallocated):



Trying the same solution for 20 in the middle we get the optimum solution:



We realise that this is the optimum solution since there is no 'slack' in this solution and hence, there is no scope for re-allocating numbers from one area to another.

Author's note: You might be justifiably thinking how do you do this kind of a random trial and error inside the exam? That's not the point of this question at this place. What I am trying to convey to you is that this is a critical thought structure which you need to have in your mind. Learn it here and do not worry about how you would think inside the exam — remember you would need to check only the four options to choose the best one. We are talking about a multiple choice test here.

Illustration 9

In a group of 120 athletes, the number of athletes who can play Tennis, Badminton, Squash and Table Tennis is 70, 50, 60 and 30 respectively. What is the maximum number of athletes who can play none of the games?

In order to think of the maximum number of athletes who can play none of the games, we can think of the fact that since there are 70 athletes who play tennis, fundamentally there are a maximum of 50 athletes who would be possibly in the 'can play none of the games'. No other constraint in the problem necessitates a reduction of this number and hence the answer to this question is 50.

LEVEL OF DIFFICULTY (I)

Directions for Questions 1 and 2: Refer to the data below and answer the questions that follow:

In the Indian athletic squad sent to the Olympics, 21 athletes were in the triathlon team; 26 were in the pentathlon team; and 29 were in the marathon team. 14 athletes can take part in triathlon and pentathlon; 12 can take part in marathon and triathlon; 15 can take part in pentathlon and marathon; and 8 can take part in all the three games.

1. How many players are there in all?
   (a) 35 (b) 43
   (c) 49 (d) none of these
2. How many were in the marathon team only?
   (a) 10 (b) 14
   (c) 18 (d) 15

Directions for Questions 3 and 4: Refer to the data below and answer the questions that follow.

In a test in which 120 students appeared, 90 passed in History, 65 passed in Sociology and 75 passed in Political Science. 30 students passed in only one subject and 55 students in only two. 5 students passed no subjects.

3. How many students passed in all the three subjects?
   (a) 25 (b) 30
   (c) 35 (d) Data insufficient
4. Find the number of students who passed in at least two subjects.
   (a) 85 (b) 95
   (c) 90 (d) Data insufficient

Directions for Questions 5 to 8: Refer to the data below and answer the questions that follow.

5% of the passengers who boarded Guwahati- New Delhi Rajdhani Express on 20th February, 2002 do not like coffee, tea and ice cream and 10% like all the three. 20% like coffee and tea, 25% like ice cream and coffee and 25% like ice cream and tea. 55% like coffee, 50% like tea and 50 % like ice cream.

5. The number of passengers who like only coffee is greater than the passengers who like only ice cream by
   (a) 50% (b) 100%
   (c) 25% (d) 0
6. The percentage of passengers who like both tea and ice cream but not coffee is
   (a) 15 (b) 5
   (c) 10 (d) 25
7. The percentage of passengers who like at least 2 of the 3 products is
   (a) 40 (b) 45
   (c) 50 (d) 60
8. If the number of passengers is 180, then the number of passengers who like ice cream only is
   (a) 10 (b) 18
   (c) 27 (d) 36

Directions for Questions 9 to 15: Refer to the data below and answer the questions that follow.

In a survey among students at all the IIMs, it was found that 48% preferred coffee, 54% liked tea and 64% smoked. Of the total, 28% liked coffee and tea, 32% smoked and drank tea and 30% smoked and drank coffee. Only 6% did none of these. If the total number of students is 2000 then find

9. The ratio of the number of students who like only coffee to the number who like only tea is
   (a) 5:3 (b) 8:9
   (c) 2:3 (d) 3:2
10. Number of students who like coffee and smoking but not tea is
   (a) 600 (b) 240
   (c) 280 (d) 360
11. The percentage of those who like coffee or tea but not smoking among those who like at least one of these is
   (a) more than 30 (b) less than 30
   (c) less than 25 (d) None of these
12. The percentage of those who like at least one of these is
   (a) 100 (b) 90
   (c) Nil (d) 94
13. The two items having the ratio 1:2 are
   (a) Tea only and tea and smoking only.
   (b) Coffee and smoking only and tea only.
   (c) Coffee and tea but not smoking and smoking but not coffee and tea.
   (d) None of these
14. The number of persons who like coffee and smoking only and the number who like tea only bear a ratio
   (a) 1:2 (b) 1:1
   (c) 5:1 (d) 2:1
15. Percentage of those who like tea and smoking but not coffee is
   (a) 14 (b) 14.9
   (c) less than 14 (d) more than 15
16. 30 monkeys went to a picnic. 25 monkeys chose to irritate cows while 20 chose to irritate buffaloes. How many chose to irritate both buffaloes and cows?
   (a) 10 (b) 15
   (c) 5 (d) 20

Directions for Questions 17 to 20: Refer to the data below and answer the questions that follow.

In the CBSE Board Exams last year, 53% passed in Biology, 61% passed in English, 60% in Social Studies, 24% in Biology & English, 35% in English & Social

Studies, 27% in Biology and Social Studies and 5% in none.

17. Percentage of passes in all subjects is
    (a) Nil (b) 12
    (c) 7 (d) 10
18. If the number of students in the class is 200, how many passed in only one subject?
    (a) 48 (b) 46
    (c) more than 50 (d) less than 40
19. If the number of students in the class is 300, what will be the % change in the number of passes in only two subjects, if the original number of students is 200?
    (a) more than 50% (b) less than 50%
    (c) 50% (d) None of these
20. What is the ratio of percentage of passes in Biology and Social Studies but not English in relation to the percentage of passes in Social Studies and English but not Biology?
    (a) 5:7 (b) 7:5
    (c) 4:5 (d) None of these

Directions for Questions 21 to 25: Refer to the data below and answer the questions that follow.

In the McGraw-Hill Mindworkzz Quiz held last year, participants were free to choose their respective areas from which they were asked questions. Out of 880 participants, 224 chose Mythology, 240 chose Science and 336 chose Sports, 64 chose both Sports and Science, 80 chose Mythology and Sports, 40 chose Mythology and Science and 24 chose all the three areas.

21. The percentage of participants who did not choose any area is
    (a) 23.59% (b) 30.25%
    (c) 37.46% (d) 27.27%
22. Of those participating, the percentage who choose only one area is
    (a) 60% (b) more than 60%
    (c) less than 60% (d) more than 75%
23. Number of participants who chose at least two areas is
    (a) 112 (b) 24
    (c) 136 (d) None of these
24. Which of the following areas shows a ratio of 1:8?
    (a) Mythology & Science but not Sports: Mythology only
    (b) Mythology & Sports but not Science: Science only
    (c) Science: Sports
    (d) None of these
25. The ratio of students choosing Sports & Science but not Mythology to Science but not Mythology & Sports is
    (a) 2:5 (b) 1:4
    (c) 1:5 (d) 1:2

Directions for Questions 26 to 30: Refer to the data below and answer the questions that follow.

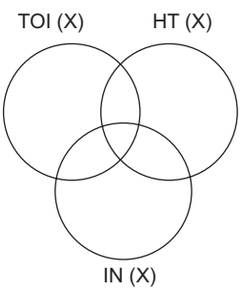
The table here gives the distribution of students according to professional courses.

Courses	*STUDENTS*			
	English		*Maths*	
	MALES	*FEMALES*	*MALES*	*FEMALES*
Part-time MBA	30	10	50	10
Full-time MBA only	150	8	16	6
CA only	90	10	37	3
Full time MBA & CA	70	2	7	1

26. What is the percentage of Math students over English students?
    (a) 50.4 (b) 61.4
    (c) 49.4 (d) None of these
27. The average number of females in all the courses is (count people doing full-time MBA and CA as a separate course)
    (a) less than 12 (b) greater than 12
    (c) 12 (d) None of these
28. The ratio of the number of girls to the number of boys is
    (a) 5:36 (b) 1:9
    (c) 1:7.2 (d) None of these
29. The percentage increase in students of full-time MBA only over CA only is
    (a) less than 20 (b) less than 25
    (c) less than 30 (d) more than 30
30. The number of students doing full-time MBA or CA is
    (a) 320 (b) 80
    (c) 160 (d) None of these.

Directions for Questions 31 to 34: Refer to the data below and answer the questions that follow:

A newspaper agent sells The TOI, HT and IN in equal numbers to 302 persons. Seven get HT & IN, twelve get The TOI & IN, nine get The TOI & HT and three get all the three newspapers. The details are given in the Venn diagram:



31. How many get only one paper?
    (a) 280 (b) 327
    (c) 109 (d) None of these
32. What percent get The TOI or The HT or both (but not The IN)?
    (a) more than 65% (b) less than 60%
    (c) $\cong$ 64% (d) None of these.

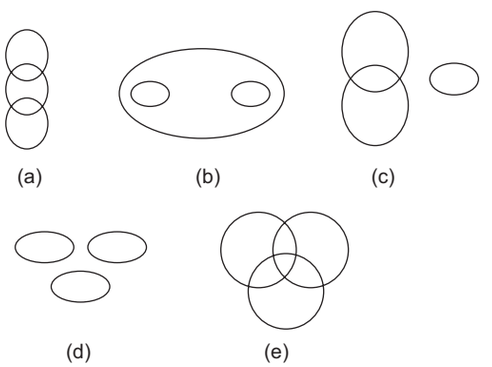
33. The number of persons buying The TOI and The HT only, The TOI and The IN only and The HT and The IN only are in the ratio of
    (a) 6:4:9 (b) 6:9:4
    (c) 4:9:6 (d) None of these
34. The difference between the number reading The HT and The IN only and HT only is
    (a) 77 (b) 78
    (c) 83 (d) None of these.
35. A group of 78 people watch Zee TV, Star Plus or Sony. Of these, 36 watch Zee TV, 48 watch Star Plus and 32 watch Sony. If 14 people watch both Zee TV and Star Plus, 20 people watch both Star Plus and Sony, and 12 people watch both Sony and Zee TV find the ratio of the number of people who watch only Zee TV to the number of people who watch only Sony.
    (a) 9:4 (b) 3:2
    (c) 5:3 (d) 7:4

Directions for Questions 36 and 37: Answer the questions based on the following information.

The following data was observed from a study of car complaints received from 180 respondents at Colonel Verma's car care workshop, viz., engine problem, transmission problem or mileage problem. Of those surveyed, there was no one who faced exactly two of these problems. There were 90 respondents who faced engine problems, 120 who faced transmission problems and 150 who faced mileage problems.

36. How many of them faced all the three problems?
    (a) 45 (b) 60
    (c) 90 (d) 20
37. How many of them faced either transmission problems or engine problems?
    (a) 30 (b) 60
    (c) 90 (d) 40

Directions for Questions 38 to 42: given below are five diagrams one of which describes the relationship among the three classes given in each of the five questions that follow. You have to decide which of the diagrams is the most suitable for a particular set of classes.



38. Elephants, tigers, animals
39. Administrators, Doctors, Authors
40. Platinum, Copper, Gold
41. Gold, Platinum, Ornaments
42. Television, Radio, Mediums of Entertainment
43. Seventy percent of the employees in a multinational corporation have VCD players, 75 percent have microwave ovens, 80 percent have ACs and 85 percent have washing machines. At least what percentage of employees has all four gadgets?
    (a) 15 (b) 5
    (c) 10 (d) Cannot be determined

Space for Rough Work