2

Progressions

The chapter on progressions essentially yields common-sense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems, since there is such a lot of commonality of logic between the problems associated with these two chapters.

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus each of the following series forms an arithmetic progression:

$$3, 7, 11, 15, \ldots$$

$$8, 2, -4, -10, \ldots$$

$$a, a + d, a + 2d, a + 3d, \ldots$$

The common difference is found by subtracting any term of the series from the next term.

That is, common difference of an *A.P.* = $(t_N - t_{N-1})$.

In the first of the above examples the common difference is 4; in the second it is –6; in the third it is d.

If we examine the series $a, a + d, a + 2d, a + 3d, \ldots$ we notice that *in any term the coefficient of d is always less by one than the position of that term in the series*.

Thus the rth term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If n be the number of terms, and if L denotes the last term or the nth term, we have

$$L = a + (n - 1)d$$

To Find the Sum of the given Number of Terms in an Arithmetic Progression

Let a denote the first term d, the common difference, and n the total number of terms. Also, let L denote the last term, and S the required sum; then

$$S = \frac{n(a + L)}{2} \quad (1)$$

$$L = a + (n - 1)d \quad (2)$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \quad (3)$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus a is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in A.P. take $a - d$, a and $a + d$ as the three numbers as this reduces one unknown thereby making the solution easier.

To Find the Arithmetic Mean between any Two given Quantities

Let a and b be two quantities and A be their arithmetic mean. Then since a, A, b, are in A.P. We must have

$$b - A = A - a$$

Each being equal to the common difference;

This gives us $A = \frac{(a + b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in A.P. The terms thus inserted are called the **arithmetic means**.

To Insert a given Number of Arithmetic Means between Two given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in A.P., of which a is the first, and b is the last term.

Let d be the common difference;

then b = the $(n + 2)$th term

$= a + (n + 1)d$

Hence, $d = \dfrac{(b-a)}{(n+1)}$

and the required means are

$$a+\frac{(b-a)}{n-1}, a+\frac{2(b-a)}{n+1}, \cdots a+\frac{n(b-a)}{n+1}$$

Till now we have studied A.P.s in their mathematical context. This was important for you to understand the basic mathematical construct of A.P.s. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis, (Especially under the time pressure that you are likely to face in the CAT and other aptitude exams). In such situations the mathematical processes for solving progressions based questions are likely to fail or at the very least, be very tedious. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one:

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the

A.P. 3, 7, 11…….

The conventional mathematical process for this question would involve using the formula.

$T_n = a + (n - 1) d$

Thus, for the 17th term we would do

$T_{17} = 3 + (17 - 1) \times 4 = 3 + 16 \times 4 = 67$

Most students would mechanically insert the values for a, n and d and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this:

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).

Similarly, in order to find the 37th term of the A.P. 3, 11 …, all you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.

(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.)

2. Average of an A.P. and Corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = (2 + 6 + 10 + 14 + 18 + 22)/6 = 12

Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.

(**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.)

We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (so that 1 + 6 = 7)
2nd and 5th (hence, 2 + 5 =7)
3rd and 4th (hence, 3 + 4 = 7)

Note: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.

For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

3. Process for finding the sum of an A.P.

Once you can find a pair of corresponding terms for any A.P., you can easily find the sum of the A.P. by using the property of averages:

i.e., Sum = Number of terms × Average.

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a+(n-1)d)$.

4. Finding the common difference of an A.P., given 2 terms of an A.P.

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualise this A.P. as
– , – , 8 , – , – , –, – , 28.

From the above figure, you can easily visualise that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

Illustration: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

Solution: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes 28 + 4 + 4 = 36.

Thus, the average of the A.P.

= Average of 8th and 10th terms

= (28 + 36)/2 = 32.

Hence, the required answer is sum of the A.P. = 17 × 32 = 544.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an AP is –30, you should realise that 5 times the common difference will be equal to –30. (Since 12 – 7 = 5).

Hence, $d = -6$.

Note: Replace this algorithmic thinking in lieu of the mathematical thinking of:

$$12^{th} \text{ term} = a + 11d$$
$$7^{th} \text{ term} = a + 6d$$
$$\text{Hence, difference} = -30 = (a + 11d) - (a + 6d)$$
$$-30 = 5d$$
$$\therefore \quad d = -6.$$

5. Types of A.P.s: Increasing and Decreasing A.P.s.

Depending on whether 'd' is positive or negative, an A.P. can be increasing or decreasing.

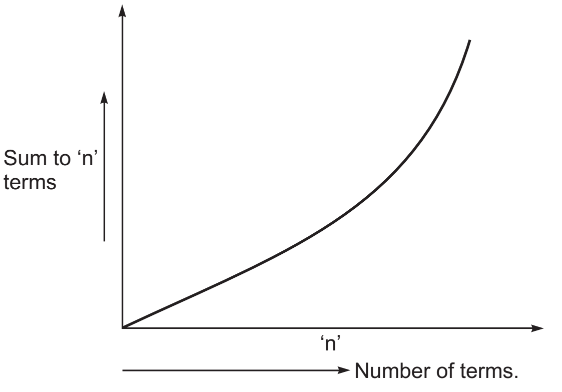
Let us explore these two types of A.P.s further:

(A) Increasing A.P.s:

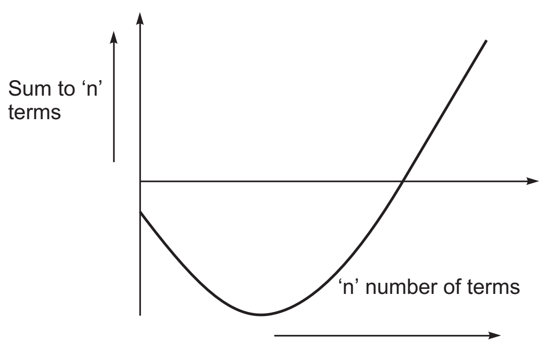
Every term of an increasing A.P. is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

Case 1: When the first term of the increasing A.P. is positive. In such a case the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below:



Case 2: When the first term of the increasing A.P. is negative. In such a case, the Sum of the A.P. plotted against the number of terms will give the following figure:



The specific case of the sum to n_1 terms being equal to the sum to n_2 terms.

In the series case 2 above, there is a possibility of the sum to 'n' terms being repeated for 2 values of 'n'. However, this will not necessarily occur.

This issue will get clear through the following example:

Consider the following series:

Series 1: –12, –8, –4, 0, 4, 8, 12

As is evident the sum to 2 terms and the sum to 5 terms in this case is the same. Similarly, the sum to 3 terms is the same as the sum to 4 terms. This can be written as:

$$S_2 = S_5 \text{ and } S_3 = S_4.$$

In other words the sum to n_1 terms is the same as the sum to n_2 terms.

Such situations arise for increasing A.P.s where the first term is negative. But as we have already stated that this does not happen for all such cases.

Consider the following A.P.s.

Series 2 : –8, –3, +2, + 7, + 12…

Series 3 : –13, –7, –1, + 5, + 11…

Series 4 : –12, –6, 0, 6, 12 …

Series 5 : –15, –9, –3, + 3, 9, 15 …

Series 6 : –20, –12, –4, 4 , 12, …

If you check the series listed above, you will realise that this occurrence happens in the case of Series 1, Series 4, Series 5 and Series 6 while in the case of Series 2 and Series 3 the same value is not repeated for the sum of the Series.

A clear look at the two series will reveal that this phenomenon occurs in series which have what can be called a balance about the number zero.

Another issue to notice is that in Series 4,

$$S_2 = S_3 \text{ and } S_1 = S_4$$

While in series 5,

$$S_1 = S_5 \text{ and } S_2 = S_4.$$

In the first case (where '0' is part of the series) the sum is equal for two terms such that one of them is odd and the other is even.

In the second case on the other hand (when '0' is not part of the series) the sum is equal for two terms such that both are odd or both are even.

Also notice that the sum of the term numbers which exhibit equal sums is constant for a given A.P.

Consider the following question which appeared in CAT 2004 and is based on this logic:

The sum to 12 terms of an A.P. is equal to the sum to 18 terms. What will be the sum to 30 terms for this series?

Solution: If $S_{12} = S_{18}$, $S_{11} = S_{19\ldots}$ and $S_0 = S_{30}$

But Sum to zero terms for any series will always be 0. Hence $S_{30} = 0$.

Note: The solution to this problem does not take more than 10 seconds if you know this logic

(B) Decreasing A.P.s.

Similar to the cases of the increasing A.P.s, we can have two cases for decreasing APs —

Case 1— Decreasing A.P. with first term negative.

Case 2— Decreasing A.P. with first term positive.

I leave it to the reader to understand these cases and deduce that whatever was true for increasing A.P.s with first term negative will also be true for decreasing A.P.s with first term positive.

GEOMETRIC PROGRESSION

Quantities are said to be in Geometric Progression when they increase or decrease by a constant factor.

The constant factor is also called the *common ratio* and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4,\ldots$ *we notice that in any term the index of r is always less by one than the number of the term in the series.*

If n be the number of terms and if l denote the last, or nth term, we have

$$l = ar^{n-1}$$

When three quantities are in geometrical progression, the middle one is called the geometric mean between the other two. While arbitrarily choosing three numbers in GP, we take a/r, a and ar. This makes it easier since we come down to two variables for the three terms.

To Find the Geometric Mean between Two Given Quantities

Let a and b be the two quantities; G the geometric mean. Then since a, G, b are in G.P.,

$$b/G = G/a$$

Each being equal to the common ratio

$$G^2 = ab$$

Hence $$G = \sqrt{ab}$$

To Insert a given Number of Geometric Means between Two Given Quantities

Let a and b be the given quantities and n the required number of means to be inserted. In all there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in G.P. of which a is the first and b the last.

Let r be the common ratio;

Then b = the $(n + 2)$th term $= ar^{n+1}$;

$$\therefore \quad r^{(n+1)} = \frac{b}{a}$$

$$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \quad (1)$$

Hence the required number of means are $ar, ar^2, \ldots ar^n$, where r has the value found in (1).

To Find the Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = \frac{a(r^n - 1)}{(r-1)} \quad (1)$$

If $r < 1$, then

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \qquad (2)$$

Note: It will be convenient to remember both forms given above for S. Number (2) will be used in all cases except when r is positive and greater than **one**.

Sum of an infinite geometric progression when $r < 1$

$$S_\infty = \frac{a}{(1 - r)}$$

Obviously, this formula is used only when the common ratio of the G.P. is less than one.

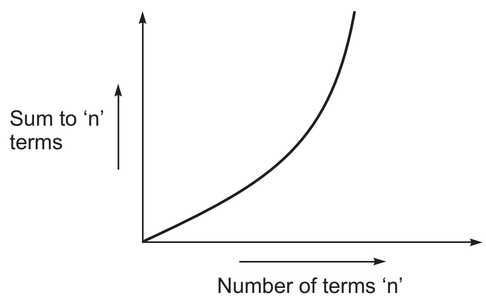
Similar to A.P.s, G.P.s can also be logically viewed. Based on the value of the common ratio and its first term a G.P. might have one of the following structures:

(1) Increasing G.P.s type 1:

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.,

e.g: 3, 6, 12, 24…(A G.P. with first term 3 and common ratio 2)

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



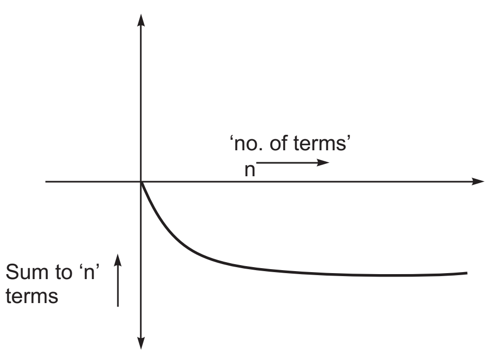
(2) Increasing G.P.s type 2:

A G.P. with first term negative and common ratio less than 1.

e.g: –8, –4,–2, –1, – ……

As you can see in this G.P. all terms are greater than their previous terms.

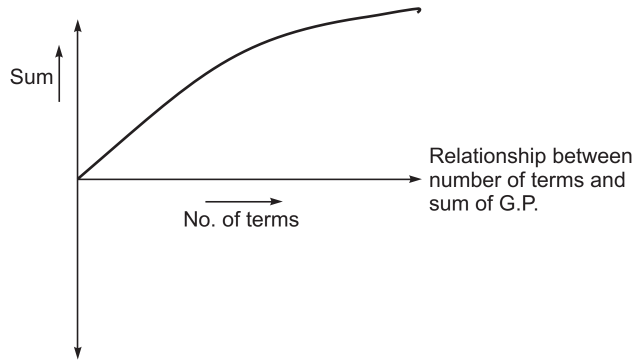
[The following figure will illustrate the relationship between the number of terms and the sum to 'n' terms in this case]



(3) Decreasing G.P.s type 1:

These G.P.s have their first term positive and common ratio less than 1.

e.g: 12, 6, 3, 1.5, 0.75 …..

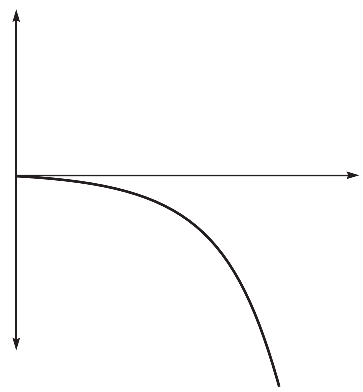


(4) Decreasing G.P.s type 2:

First term negative and common ratio greater than 1.

e.g: –2, –6, –18 ….

In this case the relationship looks like.



HARMONIC PROGRESSION

Three quantities a, b, c are said to be in Harmonic Progression when $a/c = \frac{(a-b)}{(b-c)}$.

In general, if a, b, c, d are in A.P. then $1/a$, $1/b$, $1/c$ and $1/d$ are all in H.P.

Any number of quantities are said to be in harmonic progression when every three consecutive terms are in harmonic progression.

The reciprocals of quantities in harmonic progression are in arithmetic progression. This can be proved as:

By definition, if a, b, c are in harmonic progression,

$$\frac{a}{c} = \frac{(a-b)}{(b-c)}$$

$\therefore \quad a(b - c) = c(a - b)$,

dividing every term by abc, we get

$$\left[\frac{1}{c}-\frac{1}{b}=\frac{1}{b}-\frac{1}{a}\right]$$

which proves the proposition.

There is no general formula for the sum of any number of quantities in harmonic progression. Questions in H.P. are generally solved by inverting the terms, and making use of the properties of the corresponding A.P.

To Find the Harmonic Mean between Two Given Quantities

Let a, b be the two quantities, H their harmonic mean; then $1/a$, $1/H$ and $1/b$ are in A.P.;

$$\therefore \quad \frac{1}{H}-\frac{1}{a}=\frac{1}{b}-\frac{1}{H}$$

$$\frac{2}{H}=\frac{1}{a}+\frac{1}{b}$$

i.e. $$H=\frac{2ab}{(a+b)}$$

THEOREMS RELATED WITH PROGRESSIONS

If A, G, H are the arithmetic, geometric, and harmonic means between a and b, we have

$$A=\left(\frac{a+b}{2}\right) \quad (1)$$

$$G=\sqrt{ab} \quad (2)$$

$$H=\frac{2ab}{(a+b)} \quad (3)$$

Therefore, $A \times H=\frac{(a+b)}{2}\times\frac{2ab}{(a+b)}=ab=G^2$

that is, *G is the geometric mean between A and H.*
From these results we see that

$$A-G=\frac{a+b}{2}-\sqrt{ab}=\frac{(a+b-2\sqrt{ab})}{2}$$

$$=\left[\frac{(\sqrt{a}-\sqrt{b})}{\sqrt{2}}\right]^2$$

which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive quantities is greater than their geometric mean.

Also from the equation $G^2 = AH$, we see that G is intermediate in value between A and H; and it has been proved that $A > G$, therefore $G > H$ and $A > G > H$.

The arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

As we have already seen in the Back to school section of this block there are some number series which have a continuously decreasing value from one term to the next — and such series have the property that they have what can be defined as the sum of infinite terms. Questions on such series are very common in most aptitude exams. Even though they cannot be strictly said to be under the domain of progressions, we choose to deal with them here.

Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\ldots\ldots.$$

(a) 27/14 (b) 21/13
(c) 49/27 (d) 256/147

Solution: Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately, this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series the values of the terms are:

First term = 1
Second term = 4/7 = 0.57
Third term = 9/63 = 0.14
Fourth term = 16/343 = 0.04
Fifth term = 25/2401 = 0.01

Addition upto the fifth term is approximately 1.76.

Options (b) and (d) are smaller than 1.76 in value and hence cannot be correct.

That leaves us with options 1 and 3.

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms)

Looking at the pattern we can predict that the sixth term will be

$$36/7^5 = 36/16807 = 0.002 \text{ (approx.)}$$

And the seventh term would be $49/7^6 = 49/117649 = 0.0004$ (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualised that the residual terms in the series are highly insignificant. Based on this judgement you realise that the answer will not reach 1.92 and will be restricted to 1.81. Hence the answer will be option 3.

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

Useful Results

1. If the same quantity be added to, or subtracted from, all the terms of an A.P., the resulting terms will form an A.P., but with the same common difference as before.
2. If all the terms of an A.P. be multiplied or divided by the same quantity, the resulting terms will form an A.P., but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
3. If all the terms of a G.P. be multiplied or divided by the same quantity, the resulting terms will form a G.P. with the same common ratio as before.
4. If $a, b, c, d, \ldots$ are in G.P., they are also in continued proportion, since, by definition,

   $$a/b = b/c = c/d = \ldots = 1/r$$

   Conversely, a series of quantities in continued proportion may be represented by $x, xr, xr^2, \ldots$
5. If you have to assume 3 terms in A.P., assume them as

   $$a - d, a, a + d \quad \text{or} \quad \text{as } a, a + d \text{ and } a + 2d$$

   For assuming 4 terms of an A.P. we use: $a - 3d$, $a - d$, $a + d$ and $a + 3d$

   For assuming 5 terms of an A.P., take them as:

   $$a - 2d, a - d, a, a + d, a + 2d.$$

   These are the most convenient in terms of problem solving.
6. For assuming three terms of a G.P. assume them as

   $$a, ar \text{ and } ar^2 \quad \text{or} \quad \text{as } a/r, a \text{ and } ar$$
7. To find the sum of the first n natural numbers

   Let the sum be denoted by S; then

   $S = 1 + 2 + 3 + \ldots + n$, is given by

   $$S = \frac{n(n+1)}{2}$$
8. To find the sum of the squares of the first n natural numbers

   Let the sum be denoted by S; then

   $$S = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

Contd

Useful Results (*Contd*)

This is given by : $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first n natural numbers.

   Let the sum be denoted by S; then

   $$S = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

   $$S = \left[\frac{n(n+1)}{2}\right]^2$$

   Thus, the sum of the cubes of the first n natural numbers is equal to the square of the sum of these numbers.
10. To find the sum of the first n odd natural numbers.

    $$S = 1 + 3 + 5 + \ldots + (2n - 1) \rightarrow n^2$$
11. To find the sum of the first n even natural numbers.

    $$S = 2 + 4 + 6 + \ldots + 2n \rightarrow n(n + 1) = n^2 + n$$
12. To find the sum of odd numbers $\le n$ where n is a natural number:

    Case A: If n is odd $\rightarrow [(n + 1)/2]^2$

    Case B: If n is even $\rightarrow [n/2]^2$
13. To find the sum of even numbers $\le n$ where n is a natural number:

    Case A: If n is even $\rightarrow \{(n/2)[(n/2) + 1]\}$

    Case B: If n is odd $\rightarrow [(n - 1)/2][(n + 1)/2]$
14. Number of terms in a count:
    - If we are counting in steps of 1 from n_1 to n_2 including both the end points, we get $(n_2 - n_1) + 1$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 including only one end, we get $(n_2 - n_1)$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

    Example: Between 16 and 25 both included there are 9 + 1 = 10 numbers.

    Between 100 and 200 both excluded there are 100 – 1 = 99 numbers.
    - If we are counting in steps of 2 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.
    - If we are counting in steps of 2 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/2]$ numbers.
    - If we are counting in steps of 2 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.

Contd

Useful Results (*Contd*)

- If we are counting in steps of 3 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/3]$ numbers.
- If we are counting in steps of 3 from n_1 to n_2, excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

Example: Number of numbers between 100 and 200 divisible by three.

Solution: The first number is 102 and the last number is 198. Hence, answer = (96/3) + 1 = 33 (since both 102 and 198 are included).
Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.
Hence, 201 – 99 = 102 → 102/3 = 34 → Answer = 34 – 1 = 33 (Since both ends are not included.)

In General

- If we are counting in steps of x from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.

Contd

Useful Results (*Contd*)

- If we are counting in steps of "x" from n_1 to n_2 including only one end, we get $(n_2 - n_1)/x$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

  For instance, if we have to find how many terms are there in the series 107, 114, 121, 128 ... 254, then we have

  (254 – 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22 terms in the series

  Of course, an appropriate adjustment will have to be made when n_2 does not fall into the series. This will be done as follows:

  For instance, if we have to find how many terms of the series 107, 114, 121, 128 ... are below 258, then we have by the formula:

  (258 – 107)/7 + 1 = 151/7 + 1 = 21.57 + 1. = 22.57. This will be adjusted by taking the lower integral value = 22. → The number of terms in the series below 258.

The student is advised to try and experiment on these principles to get a clear picture.

Space for Notes

WORKED-OUT PROBLEMS

Problem 2.1 Two persons—Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of ₹ 500 and ₹ 640, respectively with annual increments of ₹ 25 and ₹ 20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

Solution The current difference between the salaries of the two is ₹ 140. The annual rate of reduction of this dfference is ₹ 5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year he will earn more.

This problem should be solved while reading and the thought process should be 140/5 = 28. Hence, answer is 29th year.

Problem 2.2 Find the value of the expression

1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms

(a) –250 (b) –500

(c) –450 (d) –300

Solution The series (1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms) can be rewritten as:

⇒ (1 + 2 + 3 + … to 50 terms) – (6 + 7 + 8 + … to 50 terms)

Both these are *AP*'s with values of a and d as →

$a = 1$, $n = 50$ and $d = 1$ and $a = 6$, $n = 50$ and $d = 1$, respectively.

Using the formula for sum of an A.P. we get:

$\rightarrow 25(2 + 49) - 25(12 + 49)$

$\rightarrow 25(51 - 61) = -250$

Alternatively, we can do this faster by considering (1 – 6), (2 – 7), and so on as one unit or one term.

1 – 6 = 2 – 7 = … = –5. Thus the above series is equivalent to a series of fifty –5's added to each other.

So, (1 – 6) + (2 – 7) + (3 – 8) + … 50 terms = –5 × 50 = **–250**

Problem 2.3 Find the sum of all numbers divisible by 6 in between 100 to 400.

Solution Here 1st term = a = 102 (which is the 1st term greater than 100 that is divisible by 6.)

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the *AP;* 102, 108, 114…396 is given by [(396 – 102)/6] +1= 50 numbers.

Common difference = d = 6

So, $S = 25\ (204 + 294) = 12450$

Problem 2.4 If x, y, z are in G.P., then $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in:

(a) A.P. (b) G.P.

(c) H.P. (d) Cannot be said

Solution Go through the options.

Checking option (a), the three will be in A.P. if the 2nd expression is the average of the 1st and 3rd expressions. This can be mathematically written as

$$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$$

$$= \frac{[1 + (1 + \log_{10} x) + 1 + (1 + \log_{10} z)]}{[(1 + \log_{10} x)(1 + \log_{10} z)}$$

$$= \frac{[2 + \log_{10} xz]}{(1 + \log_{10} x)\ (1 + \log_{10} z)}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b),

$$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)]\ [1/(1 + \log_{10}z)]$$

$$= [1/(1 + \log_{10}(x + z) + \log_{10}xz)]$$

Again we are trapped and any solution is not in sight.

Checking option (c),

$1/(1 + \log_{10}x\ )$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP* then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in A.P.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in A.P.

Hence, $2 \log_{10}y = \log_{10}x + \log_{10}z$

⇒ $y^2 = xz$ which is given.

So, **(c)** is the correct option.

Alternatively, you could have solved through the following process.

x, y and z are given as logarithmic functions.

Assume $x = 1$, $y = 10$ and $z = 100$ as x, y, z are in G.P.

So, $1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

⇒ Thus we find that since 1, 2 and 3 are in A.P., we can assume that

$1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in A.P.

⇒ Hence, by definition of an H.P. we have that $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y\ )$ and $1/(1 + \log_{10}z)$ are in H.P. Hence, option (c) is the required answer.

Author's Note: In my experience I have always found that the toughest equations and factorisations get solved very easily when there are options, by assuming values in place of the variables in the equation. The values of the variables should be taken in such a manner that the basic restrictions put on the variables should be respected. For example, if an expression in three variables *a*, *b* and *c* is given and it is mentioned that $a + b + c = 0$ then the values that you assume for *a*, *b* and *c* should satisfy this restriction. Hence, you should look at values like 1, 2 and –3 or 2, –1, –1, etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of maths are then not required. This process will be very beneficial for students who are weak at Mathematics.

Problem 2.5 Find t_{10} and S_{10} for the following series:

1, 8, 15,…

Solution This is an A.P. with first term 1 and common difference 7.

$$t_{10} = a + (n - 1)\,d = 1 + 9 \times 7 = 64$$

$$S_{10} = \frac{n[2a + (n-1)d]}{2}$$

$$= \frac{10\,[2(1) + (10-1)7]}{2} = 325$$

Alternatively, if the number of terms is small, you can count it directly.

Problem 2.6 Find t_{18} and S_{18} for the following series:

2, 8, 32, …

Solution This is a G.P. with first term 2 and common ratio 4.

$$t_{18} = ar^{n-1} = 2.4^{17}$$

$$S_{18} = \frac{a(r^n - 1)}{r - 1} = \frac{2(4^{18} - 1)}{(4-1)}$$

Problem 2.7 Is the series 1, 4,… to *n* terms an A.P., or a GP, or an HP, or a series which cannot be determined?

Solution *To determine any progression, we should have at least three terms*.

If the series is an A.P. then the next term of this series will be 7

Again, if the next term is 16, then this will be a GP series (1, 4, 16 …)

So, *we cannot determine* the nature of the progression of this series.

Problem 2.8 Find the sum to 200 terms of the series 1 + 4 + 6 + 5 + 11 + 6 +…

(a) 30,200 (b) 29,800
(c) 30,200 (d) None of these

Solution Spot that the above series is a combination of two A.P.s.

The 1st A.P. is (1 + 6 + 11 +…) and the 2nd A.P. is (4 + 5 + 6 +…)

Since the terms of the two series alternate, $S = (1 + 6 + 11 + \ldots \text{ to } 100 \text{ terms}) + (4 + 5 + 6 + \ldots \text{ to } 100 \text{ terms})$

$$= \frac{100[2\times1 + 99\times5]}{2} + \frac{100[2\times4 + 99\times1]}{2} \rightarrow \text{(Using the}$$

formula for the sum of an *AP*)

$= 50[497 + 107] = 50[604] = 30200$

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

$(1 + 4) + (6 + 5) + (11 + 6) \ldots \rightarrow 5, 11, 17\ldots$

Now, $a = 5$, $d = 6$ and $n = 100$

Hence the sum of the given series is

$$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$$

$$= 50[604] = 30200$$

Problem 2.9 How many terms of the series –12, –9, –6,… must be taken that the sum may be 54?

Solution Here $S = 54$, $a = -12$, $d = 3$, *n* is unknown and has to be calculated. To do so we use the formula for the sum of an *AP* and get.

$$54 = \frac{[2(-12) + (n-1)3]n}{2}$$

or $108 = -24n - 3n + 3n^2$ or $3n^2 - 27n - 108 = 0$

or $n^2 - 9n - 36 = 0$, or $n^2 - 12n + 3n - 36 = 0$

$n(n - 12) + 3(n - 12) = 0 \Rightarrow (n + 3)(n - 12) = 0$

The value of *n* (the number of terms) cannot be negative. Hence –3 is rejected.

So we have $n = 12$

Alternatively, we can directly add up individual terms and keep adding manually till we get a sum of 54. We will observe that this will occur after adding 12 terms. (In this case, as also in all cases where the number of terms is mentally manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation based process.

Problem 2.10 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + …

(a) $n(n + 1)(n + 2)$
(b) $(n(n + 1)/12)(3n^2 + 19n + 26)$
(c) $((n + 1)(n + 2)(n + 3))/4$
(d) $(n^2 (n + 1)(n + 2)(n + 3))/3$

Solution In order to solve such problems in the examination, the option-based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put $n = 1$, we should get the sum as 1.2.4 = 8. By substituting $n = 1$ in each of the four options we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6
Option (b) gives a value of: 8
Option (c) gives a value of: 6
Option (d) gives a value of: 8

From this check we can reject the options (a) and (c).

Now put $n = 2$. You can see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38.

The correct option should also give 38 if we put $n = 2$ in the expression. Since, (a) and (c) have already been rejected, we only need to check for options (b) and (d).

Option (b) gives a value of 38.
Option (d) gives a value of 80.

Hence, we can reject option (d) and get (b) as the answer.

Note: **The above process is very effective for solving questions having options. The student should try to keep an eye open for the possibility of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).**

Space for Rough Work

LEVEL OF DIFFICULTY (I)

1. There is an AP 11, 13, 15.... Which term of this AP is 65?
   (a) 25th (b) 26th
   (c) 27th (d) 28th
2. Find the 25th term of the sequence 50, 45, 40, ...
   (a) −55 (b) −65
   (c) −70 (d) −75
3. If Ajit saves Rs. 400 more each year than he did the year before and if he saves Rs. 2000 in the first year, after how many years will his savings be more than Rs.100000 altogether?
   (a) 19 years (b) 20 years
   (c) 21 years (d) 18 years
4. The 6th and 20th terms of an AP are 8 and −20 respectively. Find the 30th term.
   (a) −34 (b) −40
   (c) −32 (d) −30
5. How many terms are there in the AP 10, 15, 20, 25,... 120?
   (a) 21 (b) 22
   (c) 23 (d) 24
6. Find the number of terms of the series 1/27, 1/9, 1/3,... 729.
   (a) 10 (b) 11
   (c) 12 (d) 13
7. If the fifth term of a G.P. is 80 and first term is 5, what will be the 4th term of the G.P.?
   (a) 20 (b) 15
   (c) 40 (d) 25
8. Binay was appointed to Mindworkzz in the pay scale of 12000–1500–22,500. Find how many years he will take to reach the maximum of the scale.
   (a) 7 years (b) 8 years
   (c) 9 years (d) 10 years
9. How many natural numbers between 100 to 500 are multiples of 9?
   (a) 44 (b) 48
   (c) 47 (d) 50
10. The sum of the first 20 terms of an AP whose first term and third term are 25 and 35, respectively is
    (a) 1200 (b) 1250
    (c) 1400 (d) 1450
11. A number 39 is divided into three parts which are in A.P. and the sum of their squares is 515. Find the largest number.
    (a) 17 (b) 15
    (c) 13 (d) 11
12. Sushil agrees to work at the rate of 10 rupee on the first day, 20 rupees on the second day, 40 rupees on the third day and so on. How much will Sushil get if he starts working on the 1st of April and finishes on the 20th of April?
    (a) 10.2^{20} (b) $10.2^{20} - 10$
    (c) $10.2^{20} - 1$ (d) 2^{19}
13. Find the sum of all numbers in between 1–100 excluding all those numbers which are divisible by 7. (Include 1 and 100 for counting.)
    (a) 4315 (b) 4245
    (c) 4320 (d) 4160
14. The 3rd and 8th term of a GP are 1/3 and 81, respectively. Find the 2nd term.
    (a) 3 (b) 1
    (c) 1/27 (d) 1/9
15. The sum of 5 numbers in AP is 35 and the sum of their squares is 285. Which of the following is the third term?
    (a) 5 (b) 7
    (c) 6 (d) 8
16. The number of terms of the series 26 + 24 + 22 +... such that the sum is 182 is
    (a) 13 (b) 14
    (c) Both a and b (d) 15
17. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.
    (a) 5 (b) 10
    (c) 15 (d) (a), (b) & (c)
18. Find the general term of the GP with the third term 1 and the seventh term 8.
    (a) $(2^{3/4})^{n-3}$ (b) $(2^{3/2})^{n-3}$
    (c) $(2^{3/4})^{3-n}$ (d) $(2^{3/4})^{2-n}$
19. The sum of the first and the third term of a geometric progression is 15 and the sum of its first three terms is 21. Find the progression.
    (a) 3,6,12… (b) 12, 6, 3...
    (c) Both of these (d) None of these
20. Ishita's salary is Rs.5000 per month in the first year. She has joined in the scale of 5000-500-10000. After how many years will her expenses be 64,800?
    (a) 8 years (b) 7 years
    (c) 6 years (d) Cannot be determined
21. A sum of money kept in a bank amounts to Rs. 1500 in 5 years and Rs. 2000 in 10 years at simple interest. Find the sum.
    (a) Rs. 1250 (b) Rs. 1200
    (c) Rs. 1150 (d) Rs. 1000
22. The sum of three numbers in a G.P. is 13 and the sum of their squares is 91. Find the smallest number.
    (a) 1 (b) 3
    (c) 4 (d) 12

23. Find the 1st term of an AP whose 8th and 12th terms are respectively 60 and 80.
(a) 15 (b) 20
(c) 25 (d) 30
24. The first term of an arithmetic progression is 13 and the common difference is 4. Which of the following will be a term of this AP?
(a) 4003 (b) 10091
(c) 7881 (d) 13631
25. Anuj receives Rs. 600 for the first week and Rs. 30 more each week than the preceding week. How much does he earn by the 30th week?
(a) 31050 (b) 32320
(c) 32890 (d) 32900
26. A number of squares are described whose areas are in G.P. Then their sides will be in
(a) A.P. (b) G.P.
(c) H.P. (d) Nothing can be said
27. How many terms are there in the G.P. 5, 10, 20, 40,... 1280?
(a) 6 (b) 8
(c) 9 (d) 10
28. The least value of n for which the sum of the series 5 + 10 + 15... n terms is not less than 765 is
(a) 17 (b) 18
(c) 19 (d) 20
29. Four geometric means are inserted between 5 and 160. Find the 2^{nd} geometric mean.
(a) 80 (b) 40
(c) 10 (d) 20
30. The seventh term of a GP is 4 times the 5th term. What will be the first term when its 4th term is 40?
(a) 4 (b) 5
(c) 3 (d) 2
31. How many terms are identical in the two A.P.s 21, 23, 25,... up to 120 terms and 23, 26, 29,... up to 80 terms?
(a) 39 (b) 40
(c) 41 (d) None of these.
32. The sum of the first four terms of an A.P. is 56 and sum of the first eight terms of the same A.P. is 176. Find the sum of the first 16 terms of the A.P.?
(a) 646 (b) 640
(c) 608 (d) 536
33. X and Y are two numbers whose A.M. is 41 and G.M. is 9. Which of the following may be a value of X?
(a) 125 (b) 49
(c) 81 (d) 25
34. Two numbers A and B are such that $A > B$ and their G.M. is 40% lower than their A.M. Find the ratio between the numbers.
(a) 4 : 3 (b) 9 : 1
(c) : 1 (d) 3 : 1
35. A man saves Rs. 1000 in January 2015 and increases his saving by Rs. 500 every month over the previous month. What is the annual saving for the man in the year 2015?
(a) Rs. 40000 (b) Rs .45000
(c) Rs. 42000 (d) Rs. 41000
36. Find the 23rd term of the sequence: 1, 4, 5, 8, 9, 12, 13, 16, 17, ...
(a) 33 (b) 39
(c) 45 (d) 43
37. If log$a$, log$b$, log$c$ are in A.P., then the GM of a & c is
(a) b (b) b^2
(c) b^4 (d) None of these.
38. Each of the series 1 + 3 + 5 + 7 +.... and 4 + 7 + 10 +... is continued to 1000 terms. Find how many terms are identical between the two series.
(a) 335 (b) 334
(c) 332 (d) 333
39. Find the sum of the series till 23rd terms for the series: 1, 4, 5, 8, 9, 12, 13, 16, 17, ...
(a) 585 (b) 560
(c) 540 (d) 520
40. What is the maximum sum of the terms in the arithmetic progression 25, 24, 23, 22……?
(a) 325 (b) 345
(c) 332.5 (d) 350
41. If 8^{th} term of an A.P. is the geometric mean of the 1^{st} and 22^{nd} terms of the same A.P. Find the common difference of the A.P., given that the sum of the first twenty-two terms of the A.P. is 770.
(a) Either 1 or 1/2 (b) 2
(c) 1 (d) Either 1 or 2
42. How many terms of the series 1 + 3 + 5 + 7 + …. amount to 1234567654321?
(a) 1110111 (b) 1111011
(c) 1011111 (d) 1111111
43. Tom and Jerry were playing mathematical puzzles with each other. Jerry drew a square of sides 32 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jerry asked Tom to determine the sum of the areas of all the squares that she drew. If Tom answered correctly then what would be his answer?
(a) 2048 (b) 1024
(c) 512 (d) 4096
44. The sum of the first two terms of an infinite geometric series is 36. Also, each term of the series is equal to the sum of all the terms that follow. Find the sum of the series
(a) 48 (b) 54
(c) 72 (d) 96

45. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the areas of all the equilateral triangles, if the side of the largest equilateral triangle is 8 units.
    (a) $32\sqrt{3}$ units (b) $64\sqrt{3}$ units
    (c) 64 units (d) $64/\sqrt{3}$ units
46. After striking a floor a rubber ball rebounds (5/6)th of the height from which it has fallen. Find the total distance (in metres) that it travels before coming to rest, if it is gently dropped from a height of 210 metres.
    (a) 2960 (b) 2310
    (c) 2080 (d) 2360

For questions 47 to 57, there are no options. Kindly solve these and put down your answer to the question asked.

47. In an infinite geometric progression, each term is equal to 3 times the sum of the terms that follow. If the first term of the series is 4, find the product of first three terms of the series?
48. A student takes a test consisting of 100 questions with differential marking is told that each question after the first is worth 5 marks more than the preceding question. If the 5^{th} question of the test is worth 25 marks, What is the maximum score that the student can obtain by attempting 90 questions?
49. In Narora nuclear power plant a technician is allowed an interval of maximum 100 minutes. A timer with a bell rings at specific intervals of time such that the minutes when the timer rings are not divisible by 2, 3, 5 and 7. The last alarm rings with a buzzer to give time for decontamination of the technician. How many times will the bell ring within these 100 minutes and what is the value of the last minute when the bell rings for the last time in a 100 minute shift?
50. The internal angles of a plane polygon are in AP. The smallest angle is 100° and the common difference is 10°. Find the number of sides of the polygon.
51. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean of a and b then find the value of n.
52. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the harmonic mean of a and b then find the value of n.
    (a) −1 (b) 0
    (c) 1 (d) None of these.
53. If a, b are two numbers such that $a, b > 0$. If harmonic mean of a, b is equals to geometric mean of a, b then what can be said about the relationship between a and b.
54. Product of 36 positive integers is 1. Their sum is ≥
55. If we have two numbers a, b. A.M. of a, b is 12 and H.M. is 3. Find the value of ab
56. If $x+\frac{1}{yz}, y+\frac{1}{zx}, z+\frac{1}{xy}$ are in A.P. then x, y, z are in:

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. If a times the a^{th} term of an A.P. is equal to b times the b^{th} term, find the $(a + b)^{th}$ term.
   (a) 0 (b) $a^2 - b^2$
   (c) $a - b$ (d) 1
2. A number 28 is divided into four parts that are in AP such that the product of the first and fourth is to the product of the second and third is 5: 6. Find the smallest part.
   (a) 2 (b) 4
   (c) 8 (d) 6
3. Find the value of the expression: 1 – 3 + 5 − 7… to 100 terms.
   (a) −150 (b) −100
   (c) −50 (d) 75
4. If a clock strikes once at 12 A.M., twice at 1 A.M., thrice at 2 A.M. and so on, how many times will the clock be struck in the course of 3 days? (Assume a 24 hour clock)
   (a) 756 (b) 828
   (c) 678 (d) 1288
5. What will be the maximum sum of 54, 52, 50, ... ?
   (a) 702 (b) 704
   (c) 756 (d) 700
6. Find the sum of the integers between 100 and 300 that are multiples of 7.
   (a) 10512 (b) 5586
   (c) 10646 (d) 10546
7. If x > 1, y > 1, z > 1 are in G.P., then $\frac{1}{1+log\ x}$, $\frac{1}{1+log\ y}$, $\frac{1}{1+log\ z}$ are in
   (a) A.P. (b) H.P.
   (c) G.P. (d) None of the above
8. Find the sum of all odd numbers lying between 1000 and 2000.
   (a) 7,50,000 (b) 7,45,000
   (c) 7,55,000 (d) 7,65,000
9. Find the sum of all integers of 3 digits that are divisible by 11.
   (a) 49,335 (b) 41,338
   (c) 44,550 (d) 47,300
10. The first and the last terms of an A.P. are 113 and 253. If there are six terms in this sequence, find the sum of sequence.
    (a) 980 (b) 910
    (c) 1098 (d) 920
11. Find the value of 1 − 2 − 3 + 2 − 3 − 4 +... + upto 100 terms.
    (a) −694 (b) −626
    (c) −624 (d) −660
12. What will be the sum to n terms of the series 7 + 77 + 777 +...?
    (a) $7(10^n - 9n)/81$ (b) $7(10^{n+1} - 10 - 9n)/81$
    (c) $7(10^{n-1} - 10)$ (d) $7(10^{n+1} - 10)$
13. If log a, log b, log c are in A.P., then a, b, c are in
    (a) A.P. (b) G.P.
    (c) H.P. (d) None of these
14. After striking the floor, a rubber ball rebounds to 3/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 20 metres.
    (a) 40 metres (b) 60 metres
    (c) 80 metres (d) 120 metres
15. If x be the first term, y be the nth term and p be the product of n terms of a G.P., then the value of p^2 will be
    (a) $(xy)^{n-1}$ (b) $(xy)^n$
    (c) $(xy)^{1-n}$ (d) $(xy)^{n/2}$
16. The sum of an infinite G.P. whose common ratio is positive and is numerically less than 1 is 36 and the sum of the first two terms is 32. What will be the third term?
    (a) 1/3 (b) 4/3
    (c) 8 /3 (d) 2
17. What will be the value of $2^{1/3}.2^{1/6}.2^{1/12}$... to infinity.
    (a) 2^2 (b) $2^{2/3}$
    (c) $2^{3/2}$ (d) 8
18. In an infinite G.P. the first term is A and the infinite sum 5, then A belongs to
    (a) $A < -10$ (b) $0 < A < 10$
    (c) $0 < A \leq 10$ (d) None of these
19. Determine the fourth term of the geometric progression, the sum of whose first term and third term is 50 and the sum of the second term and fourth term is 150.
    (a) 120 (b) 125
    (c) 135 (d) 45
20. What is the 13^{th} term of 2/9, 1/4, 2/7, 1/3 ………..?
    (a) –2 (b) 1
    (c) –3/13 (d) –2/3
21. The sum of the third and the fourth term of an A.P. is 19 and that of the first and the seventh term is 22. Find the 9^{th}term.
    (a) 26 (b) 17
    (c) 15 (d) 16
22. How many terms of an A.P. must be taken for their sum to be equal to 200 if its third term is 16 and the difference between the 6th and the 1^{st}term is 30?
    (a) 6 (b) 9
    (c) 7 (d) 8
23. Four numbers are inserted between the numbers 4 and 34 such that an A.P. results. Find the smallest of these four numbers.

(a) 11.5 (b) 11
(c) 12 (d) 10

24. Find the sum of all three-digit natural numbers, which on being divided by 7, leave a remainder equal to 6.
(a) 70,208 (b) 70,780
(c) 70,680 (d) 71,270

25. The sum of the first three terms of the arithmetic progression is 24 and the sum of the squares of the first term and the second term of the same progression is 80. Find the 8^{th} term of the progression if its fifth term is known to be exactly divisible by 10.
(a) 32 (b) 36
(c) 40 (d) 42

26. Anita and Babita set out to meet each other from two places 200 km apart. Anita travels 20 km the first day, 19 km the second day, 18 km the third day and so on. Babita travels 8 km the first day, 10 km the second day, 12 km the third day and so on. After how many days will they meet?
(a) 9 days (b) 8 days
(c) 7 days (d) 6 days

27. If a man saves Rs. 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?
(a) Rs 21,478 (b) Rs 21,578
(c) Rs 22,578 (d) Rs 22,478

28. If sum to n terms of a series is given by $(2n + 7)$ then its second term will be given by
(a) 10 (b) 9
(c) 8 (d) 2

29. If A is the sum of the n terms of the series 2 + 1/2 + 1/8 + ... and B is the sum of $2n$ terms of the series 2 + 1 + 1/2 + ..., then find the value of B/A.
(a) 1/3 (b) 2
(c) 2/3 (d) 3/2

30. Aman receives a pension starting with Rs.1000 for the first year. Each year he receives 80% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.
(a) 4000 (b) 5000
(c) 1500 (d) 4900

31. The sum of the series $\frac{1}{1\times5} + \frac{1}{5\times9} + \frac{1}{9\times13}.....+ \frac{1}{221\times225}$ is
(a) 28/221 (b) 56/221
(c) 56/225 (d) None of these

32. The sum of the series $\frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}}+\ldots+ \frac{1}{\sqrt{224}+\sqrt{225}}$ is:
(a) $15-\sqrt{3}$ (b) $\sqrt{15}-2$
(c) 12 (d) None of these

33. Find the infinite sum of the series 1/1 + 1/3 + 1/6 + 1/10 + 1/15 ….
(a) 2 (b) 2.25
(c) 3 (d) 4

34. The sum of the series 3 × 5 + 5 × 7 + 7 × 9 +…. upto n terms will be:
(a) $\frac{n(4n^2+18n+23)}{3}$ (b) $\frac{n(4n^2+18n+23)}{6}$
(c) $n(4n^2+18n+23)$ (d) None of these.

35. The sum of the series: 1/3 + 4/15 + 4/35 + 4/63 + ... upto 6 terms is:
(a) 12/13 (b) 13/14
(c) 14/13 (d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms:
(a) 1.1 (b) 1
(c) 14/13 (d) None of these

Directions for Questions 37 to 39: Answer these questions based on the following information. There are 250 integers $a_1, a_2,\ldots a_{250}$, not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers a_1 through a_{124} form sequence A and the rest form sequence B. Each member of A is less than or equal to each member of B.

37. All values in A are changed in sign, while those in B remain unchanged. Which of the following statements is true?
(a) Every member of A is greater than or equal to every member of B.
(b) Max is in A.
(c) If all numbers originally in A and B had the same sign, then after the change of sign, the largest number of A and B is in A.
(d) None of these

38. Elements of A are in ascending order, and those of B are in descending order. a_{124} and a_{125} are interchanged. Then which of the following statements is true?
(a) A continues to be in ascending order.
(b) B continues to be in descending order.
(c) A continues to be in ascending order and B in descending order.
(d) None of the above

39. Every element of A is made greater than or equal to every element of B by adding to each element of A an integer x. Then, x cannot be less than:
(a) 2^{10} (b) the smallest value of B
(c) the largest value of B
(d) (Maximm-Minimum)

40. Rahul drew a rectangular grid of 625 cells, arranged in 25 Rows and 25 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The $6^{th}$and the 20^{th}numbers of the fifth row were 37 and 73 respectively, while the $6^{th}$and the 20^{th}numbers of the 25^{th} row were 63 and 87, respectively. What is the sum of all the numbers in the grid?
    (a) 32798 (b) 65596
    (c) 52900 (d) None of these
41. How many four digit numbers have the property that their digits taken from left to right form an Arithmetic or a Geometric Progression?
    (a) 15 (b) 21
    (c) 20 (d) 23

Directions for Questions 42 and 43: These questions are based on the following data. At Goli - Vadapav—a famous fast food centre in Andheri in Mumbai, vadapavs are made only on an automatic vadapav making machine. The machine continuously makes different sorts of vadapavs by adding different sorts of fillings on a common bread. The machine makes the vadapavs at the rate of 1 vadapav per half a minute. The various fillings are added to the vadapavs in the following manner. The 1st, 3^{rd}, 5th, 7th,…vadapavs are filled with a chicken patty; the 1^{st} , 5^{th} , 9th, …..vadapavs with vegetable patty; the 1st, 8th, 17th, …..vadapavs with mushroom patty; and the rest with plain cheese and tomato fillings. The machine makes exactly 500 vadapavs per day.

42. How many vadapavs per day are made with cheese and tomato as fillings?
43. How many vadapavs are made with all three fillings Chicken, vegetable and mushroom?
44. An arithmetic progression P consists of terms. From the progression three different progressions P_1, P_2 and P_3 are created such that P_1 is obtained by the 1st, 4th ,7th terms of P, P_2 has the 2nd, 5th, 8th, terms of P and P_3 has the 3rd, 6th, 9th, terms of P. It is found that of P_1, P_2 and P_1 two progressions have the property that their average is itself a term of the original Progression P. Which of the following can be a possible value of n?
    (a) 20 (b) 26
    (c) 36 (d) Both (a) and (b)
45. For the above question, if the Common Difference between the terms of P_1 is 6, what is the common difference of P?
    (a) 2 (b) 3
    (c) 6 (d) Cannot be determined

Direction for question number 46 to 48:
If $S = a, b, b, c, c, c, d, d, d, d, …..z,z,z$.

46. Find the number of terms in the above series:
47. Find 144^{th} term of the above series:
48. If a = 1, b = 3, c = 5, d = 7, …..,z = 51 then find the sum of all terms of S:
49. If $f(4x) = 8x + 1$. Then for how many positive real values of x , $f(2x)$ will be G.M. of $f(x)$ and $f(4x)$:
50. If x, y, z, w are positive real numbers such that x, y, z, w form an increasing A.P. and x, y, w form an increasing G.P. then w/x =?
    (a) 1 (b) 2
    (c) 3 (d) 4
51. If x, y, z are the mth, nth and pth terms, respectively of a G.P. then $(n – p) \log x + (p – m) \log y + (m – n) \log z$ =?
52. Find the sum of first n groups of 1 + (1 + 2) + (1 + 2 + 3) + …..
    (a) $\frac{n(n+1)(n+2)}{6}$
    (b) $\frac{n(n+1)(n+2)}{12}$
    (c) $\frac{n(n+1)(n+2)(n+3)}{6}$
    (d) None of these.
53. If $A= 1 + x + x^2+ x^3+ …..$& $B = 1 + y + y^2 + y^3 + …..$ and $0 < x, y < 1$, then the value of $1 + \underline{xy} + x^2y^2 + x^3y^3 + …..$is:
    (a) $AB/(A + B)$ (b) $AB/(A + B – 1)$
    (c) $(AB – 1)/(A + B)$ (d) AB
54. If all the angles of a quadrilateral are in G.P. and all the angles and the common ratio are natural numbers. Exactly two angles are acute and two are obtuse then find the largest angle.
55. The sum to 16 groups of the series (1)+ (1 + 3) + (1 + 3 + 5) + (1 +3 + 5 + 7)+ …..
56. Sum of 16 terms of the series 1 + 1 + 3 + 1 + 3 + 5 + 1 + 3 + 5 + 7 + …..
57. If the sum of n terms of a progression is $2n^2 + 3$. Then which term is equals to 78?
58. Sum of 17 terms of the series $1^2 – 2^2 + 3^2 – 4^2 + 5^2 – 6^2 + …..$
59. Find the sum of 20 terms of the series 3 + 6 + 10 + 15 + …..
60. If $1^n + 2^n + 3^n + … + x^n$ is always divisible by $1 + 2 + 3 + ….+ x$ then n is
    (a) Even (b) odd
    (c) Multiple of 2 (d) None of these.
61. Find the 12^{th} term of the series 3, 14, 61, 252, …..

LEVEL OF DIFFICULTY (III)

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to –36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to –16, then what will be first term of this series?
   (a) 16 (b) 20
   (b) –16 (d) –20
2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.
   (a) 15 (b) 5
   (c) 8 (d) 10
3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the ninth term by the fourth term of the progression gives quotient as 11 and the remainder as 10. Find the first term of the progression.
   (a) – 52 (b) – 42
   (c) – 56 (d) – 66
4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 10 metres between two consecutive saplings. Mithilesh, the country's greatest forester, can carry only one sapling at a time and has to move back to the original point to get the next sapling. In this manner he covers a total distance of 1.32 kms. How many saplings does he plant in the process if he ends at the starting point?
   (a) 15 (b) 14
   (c) 13 (d) 12
5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.
   (a) P_2/P_1 (b) P_1/P_2
   (c) $P_2 + P_1/P_1$ (d) $P_2 + P_1/P_2$
6. The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. Find the common ratio.
   (a) $(S_1/S_2)^{1/10}$ (b) $-(S_1/S_2)^{1/10}$
   (c) $\pm\sqrt[10]{S_2/S_1}$ (d) $(S_1/S_2)^{1/5}$
7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.
   (a) 2.25 or 25 (b) 2.5
   (c) 1.5 (d) 3.25
8. If (2 + 4 + 6 +… 50 terms)/(1 + 3 + 5 +… n terms) = 51/2, then find the value of n .
   (a) 12 (b) 13
   (c) 9 (d) 10
9. (666…. n digits)^2 + (888… n digits) is equal to
   (a) $(10^n - 1) \times \frac{4}{9}$ (b) $(10^{2n} - 1) \times \frac{4}{9}$
   (c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$ (d) $\frac{4(10^n + 1)}{9}$
10. The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.
    (a) 7 (b) 8
    (c) 9 (d) 10
11. Find the sum to n terms of the series 11 + 103 + 1005 + …
    (a) $\frac{10(10^n - 1)}{9} + 1$ (b) $\frac{10(10^n - 1)}{9} + n$
    (c) $\frac{10(10^n - 1)}{9} + n^2$ (d) $\frac{10(10^n + 1)}{11} + n^2$
12. The sum of the first term and the fifth term of an AP is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this AP.
    (a) 110 (b) 114
    (c) 112 (d) 116
13. The sum of the third and the ninth term of an AP is 10. Find a possible sum of the first 11 terms of this AP.
    (a) 55 (b) 44
    (c) 66 (d) 48
14. The sum of the squares of the fifth and the eleventh term of an AP is 3 and the product of the second and the fourteenth term is equal it P. Find the product of the first and the fifteenth term of the AP.
    (a) $(58P - 39)/45$ (b) $(98P + 39)/72$
    (c) $(116P - 39)/90$ (d) $(98P + 39)/90$
15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.
    (a) 4/9 or 9/4 (b) 2/3 or 3/2
    (c) 2/5 or 5/2 (d) None of these
16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots + 100.\ 2^{100}$.

(a) $100.2^{101} + 2$ (b) $99.2^{100} + 2$
(c) $99.2^{101} + 2$ (d) None of these

17. The sequence $[x_n]$ is a *GP* with $x_2/x_4 = 1/4$ and $x_1 + x_4 = 108$. What will be the value of x_3?
(a) 42 (b) 48
(c) 44 (d) 56

18. If x, y, z are in GP and a^x, b^y and c^z are equal, then a, b, c are in
(a) *AP* (b) *GP*
(c) *HP* (d) None of these

19. Find the sum of all possible whole number divisors of 720.
(a) 2012 (b) 2624
(c) 2210 (d) 2418

20. Sum to n terms of the series $\log m + \log m^2/n + \log m^3/n^2 + \log m^4/n^3 \ldots$ is
(a) $\log\left(\frac{m^{n+1}}{n^{n-1}}\right)^{\frac{n}{2}}$ (b) $\log\left(\frac{n^{n-1}}{m^{n+1}}\right)^{\frac{n}{2}}$
(c) $\log\left(\frac{m^{n}}{n^{n}}\right)^{\frac{n}{2}}$ (d) $\log\left(\frac{m^{1-n}}{n^{1-m}}\right)^{\frac{n}{2}}$

21. The sum of first 20 and first 50 terms of an *AP* is 420 and 2550. Find the eleventh term of a *GP* whose first term is the same as the *AP* and the common ratio of the *GP* is equal to the common difference of the *AP*.
(a) 560 (b) 512
(c) 1024 (d) 2048

22. If three positive real numbers x, y, z are in *AP* such that $xyz = 4$, then what will be the minimum value of y?
(a) $2^{1/3}$ (b) $2^{2/3}$
(c) $2^{1/4}$ (d) $2^{3/4}$

23. If a_n be the nth term of an *AP* and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is
(a) 3 (b) 3/2
(c) 7 (d) 0

24. If $a_1, a_2, a_3 \ldots a_n$ are in *AP*, where $a_i > 0$, then what will be the value of the expression
$1/(\sqrt{a_1} + \sqrt{a_2}) + 1/(\sqrt{a_2} + \sqrt{a_3}) + 1/(\sqrt{a_3} + \sqrt{a_4}) + \ldots$ to n terms?
(a) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$
(b) $(n - 1)/(\sqrt{a_1} + \sqrt{a_n})$
(c) $(n - 1)/(\sqrt{a_1} - \sqrt{a_n})$
(d) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

25. If the first two terms of a *HP* are 2/5 and 12/13, respectively, which of the following terms is the largest term?
(a) 4th term (b) 5th term
(c) 6th term (d) 2nd term

26. One side of a staircase is to be closed in by rectangular planks from the floor to each step. The width of each plank is 9 inches and their height are successively 6 inches, 12 inches, 18 inches and so on. There are 24 planks required in total. Find the area in square feet.
(a) 112.5 (b) 107
(c) 118.5 (d) 105

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated infinitely. If the perimeter and area of the outer triangle are *P* and *A* respectively, what will be the sum of perimeters of triangles thus formed?
(a) $2P$ (b) P^2
(c) $3P$ (d) $p^2/2$

28. In Problem 27, find the sum of areas of all the triangles.
(a) $\frac{4}{5}$A (b) $\frac{4}{3}$A
(c) $\frac{3}{4}$A (d) $\frac{5}{4}$A

29. A square has a side of 40 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimeter of all the squares thus formed.
(a) $160(1 + \sqrt{2})$ (b) $160(2 + \sqrt{2})$
(c) $160(2 - \sqrt{2})$ (d) $160(1 - \sqrt{2})$

30. In problem 29, find the area of all the squares thus formed.
(a) 1600 (b) 2400
(c) 2800 (d) 3200

31. The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first $3n$ terms of the progression to the sum of its first n terms.
(a) 5 (b) 6
(c) 7 (d) 8

32. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours.
(a) $2^9 - 1$ (b) 2^{10}
(c) 2^9 (d) $2^{10} - 1$

33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.

(a) 49637 (b) 39767
(c) 49634 (d) 39770

34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3)\ /abc$ equals
(a) $(ab)^{1/2}/c$ (b) 1
(c) a^2c/b (d) None of these

35. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is
(a) Positive (b) Negative
(c) Non-positive (d) Non-negative

36. If $S = \left[1+\left(-\frac{1}{3}\right)\right]\left[1+\left(-\frac{1}{3}\right)^2\right]\left[1+\left(-\frac{1}{3}\right)^4\right][1+\left(-\frac{1}{3}\right)^8 \ldots..$ till n terms. Then S = ?
(a) $4(10^{2n} - 1)$ (b) $4/3(10^n - 1)$
(c) $2/3(10^n - 1)$ (d) None of these

37. The number 7777….77 (total 133 digits) is:
(a) divisible by 3
(b) a composite number
(c) None of these

38. 1st term of an A.P. of consecutive integers is $n^2 + 1$ (n is a positive integer). Sum of 1st $2n$ terms of the series will be.
(a) $n(2n^2 + 2n + 1)$ (b) $(2n^2 + 2n + 3)$
(c) $n(2n^2 + 2n + 3)$ (d) None of these.

39. $S = \frac{1}{1!+2!} + \frac{1}{2!+3!} + \frac{1}{3!+4!} + \ldots.. + \frac{1}{19!+20!}$ Then S =?
(a) $\frac{1}{2!} - \frac{1}{21!}$ (b) $\frac{1}{2!} - \frac{1}{20!}$
(c) None of these.

40. a, b, c are in H.P. and $n > 1$ then $a^n + c^n$ is:
(a) Less than $2b^n$ (b) Less than or equals to $2b^n$
(c) More than $2b^n$ (d) More than or equals to $2b^n$

41. The sum to 17 terms of the series
$\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \ldots.$ is:

42. Find the value of S if
$S = \frac{4}{11} + \frac{44}{11^2} + \frac{444}{11^3} + \frac{4444}{11^4} + \ldots\infty$

43. Find the value of S if $S = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots.. + \infty$
(a) 1/3 (b) 1/2
(c) 1/6 (d) None of these.

44. $A = a + A_1 + A_2 + \ldots.. + A_N + b$
$B = a + G_1 + G_2 + \ldots. + G_N + b$
A is the sum of $n + 2$ terms of an A.P. with first term a & last term b.
B is the sum of $n + 2$ terms of a G.P. with first term a &last term b.
Then, what can be said about the relative values of A & B?

45. $\frac{9}{2} + \frac{25}{6} + \frac{49}{12} + \ldots. + \frac{9801}{2450}$ =?

46. Two series X (x_1, x_2, x_3, x_4, ….x_n) and Y (y_1, y_2, y_3, y_4, ….., y_n) are in A.P., such that $x_n - y_n = n - 2$. It is also known that $x_3 = b_5$. Find the value of $x_{99} - y_{197}$.
(a) 47 (b) 48
(c) 49 (d) 50

47. From the 1st 12 natural numbers how many Arithmetic Progressions of 4 terms can be formed such that the common difference is a factor of the 4th term?

48. The product of 1st five terms of an increasing A.P. is 3840. If the 1st, 2nd and 4th terms of the A.P. are in G.P. Find 10th term of the series.

49. If
$S = \left[1+\left(-\frac{1}{3}\right)\right]\left[1+\left(-\frac{1}{3}\right)^2\right]\left[1+\left(-\frac{1}{3}\right)^4\right]\left[1+\left(-\frac{1}{3}\right)^8 \ldots..\right]$
then S =

50. Let the positive numbers a, b, c, d be in A.P. Then the type of progression for the numbers abc, abd, acd, bcd is:

51. Sum of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots.$ to 10 terms:

ANSWER KEY

Level of Difficulty (I)

1. (d)	2. (c)	3. (a)	4. (b)
5. (c)	6. (a)	7. (c)	8. (a)
9. (a)	10. (d)	11. (b)	12. (b)
13. (a)	14. (d)	15. (b)	16. (c)
17. (d)	18. (a)	19. (c)	20. (d)
21. (d)	22. (a)	23. (c)	24. (c)
25. (a)	26. (b)	27. (c)	28. (a)
29. (d)	30. (b)	31. (b)	32. (c)
33. (c)	34. (b)	35. (b)	36. (c)
37. (a)	38. (d)	39. (c)	40. (a)
41. (b)	42. (d)	43. (a)	44. (a)
45. (d)	46. (b)	47. 1	48. (d)
49. 22,97	50. 8	51. 0	52. −1
53. a = b	54. 36	55. 36	56. A.P.

Level of Difficulty (II)

1. (a)	2. (b)	3. (b)	4. (b)
5. (c)	6. (b)	7. (b)	8. (a)
9. (c)	10. (c)	11. (b)	12. (b)
13. (b)	14. (c)	15. (b)	16. (c)
17. (b)	18. (b)	19. (c)	20. (d)
21. (a)	22. (d)	23. (d)	24. (a)

25. (a)	26. (c)	27. (b)	28. (d)
29. (d)	30. (b)	31. (c)	32. (a)
33. (a)	34. (a)	35. (d)	36. (b)
37. (d)	38. (a)	39. (d)	40. (d)
41. (d)	42. 214	43. 18	44. (d)
45. (a)	46. 351	47. q	48. 12051
49. 0	50. (d)	51. 0	52. (a)
53. (b)	54. 1920	55. 1496	56. 56
57. 20	58. 153	59. 1770	60. (b)
61. $4^{12}-12$			

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (d)
5. (a)	6. (c)	7. (b)	8. (d)
9. (b)	10. (c)	11. (c)	12. (c)
13. (a)	14. (c)	15. (a)	16. (c)
17. (b)	18. (b)	19. (d)	20. (a)
21. (d)	22. (b)	23. (d)	24. (b)
25. (d)	26. (a)	27. (a)	28. (b)
29. (b)	30. (d)	31. (b)	32. (c)
33. (b)	34. (d)	35. (d)	36. (d)
37. (c)	38. (a)	39. (a)	40. (c)
41. $\frac{323}{324}$	42. 22/5	43. (c)	44. A > B
45. 9849/50	46. (b)	47. 13	48. 24
49. ¾	50. reciprocals are in H.P		51. 126.5

Space for Rough Work

Solutions and Shortcuts

Level of Difficulty (I)

1. The number of terms in a series are found by:
$\frac{\text{Difference between first and last terms}}{\text{Common Difference}} + 1 =$
$\frac{65-11}{2} + 1 = 27 + 1 = 28^{th}$ term. Option (d) is correct.
2. The first term is 50 and the common difference is −5, thus the 25th term is: 50 + 24 x (−5) = −70. Option (c) is correct.
3. We need the sum of the series 2000 + 2400 + 2800 to cross 100000. Trying out the options, we can see that in 20 years the sum of his savings would be: 2000 + 2400 + 2800 +…+ 9600. The sum of the series would be 20 × 5800 = 116000. If we remove the 20th year we will get the saving for 19 years. The series would be 2000 + 2400 + 2800 + … + 9200. Sum of the series would be 116000 − 9600 = 106400. If we remove the 19th year's savings the savings would be 106400 − 9200 which would go below 100000. Thus, after 19 years his savings would cross 100000. Option (a) is correct.
4. $a + 5d = 8$ and $a + 19d = -20$. Solving we get $14d = -28 \rightarrow d = -2$. 30th term = 20th term + $10d$ = −20 + 10 × (−2) = −40. Option (b) is correct.
5. In order to count the number of terms in the AP, use the shortcut:
[(last term − first term)/ common difference] + 1. In this case it would become:
[(120 − 10)/5] + 1 = 23. Option (c) is correct.
6. $r = 3$. $729 = \frac{1}{27}(3)^{n-1}$, $n - 1 = 9$ or $n = 10$ option (a) is correct.
7. $5r^4 = 80 \rightarrow r^4 = 80/5 \rightarrow r = 2$. Thus, 4^{th} term = ar^3 = $5 \times (2)^3 = 40$. Option (c) is correct.
8. 12000 – 1500 – 22500 means that the starting scale is 12000 and there is an increment of 1500 every year. Since, the total increment required to reach the top of his scale is 10500, the number of years required would be 10500/1500 = 7. Option (a) is correct.
9. The series will be 108, 117, 126,…. 495.
Hence, Answer = $\frac{495-108}{9} + 1$ = 44. Option (a) is correct.
10. $a = 25$, $a + 2d = 35$ means $d = 5$. The 20th term would be $a + 19d = 25 + 95 = 120$. The sum of the series would be given by: [20/2] × [25 + 120] = 1450. Option (d) is correct.
11. The three parts are 11, 13 and 15 since $11^2 + 13^2 + 15^2 = 515$. Since, we want the largest number, the answer would be 15. Option (b) is correct.
12. Sum of a G.P. with first term 10 and common ratio 2 and no. of terms 20. $\frac{10 \times (2^{20} - 1)}{2-1} = 10(2^{20} - 1)$. Option (b) is correct.
13. The answer will be given by:
[1 + 2 + 3 +…+ 100]— [7 + 14 + 21 +… + 98]
= 50 × 101 − 7 × 105
= 5050 − 735 = 4315. Option (a) is correct.
14. 3rd term $ar^2 = 1/3$, 8th term $ar^7 = 81$
r^5=243 Gives us: $r = 3$.
Hence, the second term will be given by (3^{rd} term/r) = 1/3 1/3 = 1/9. Option (d) is correct.
[Note: To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]
15. Since the sum of 5 numbers in AP is 35, their average would be 7. The average of 5 terms in an AP is also equal to the value of the 3rd term (logic of the middle term of an AP). Hence, the third term's value would be 7. Option (b) is correct.
16. Use trial and error by using various values from the options.
If you find the sum of the series till 13 terms the value is 182. The 14^{th} term of the given series is 0, so also for 14 terms the value of the sum would be 182. Option (c) is correct.
17. Trying Option (a),
We get least term 5 and largest term 30 (since the largest term is 6 times the least term).
The average of the A.P becomes (5 + 30)/2 = 17.5
Thus, 17.5 × n = 105 gives us:
to get a total of 105 we need $n = 6$ i.e. 6 terms in this A.P. That means the A.P. should look like: 5, _, _, _, _, 30.
It can be easily seen that the common difference should be 5. The A.P, 5, 10, 15, 20, 25, 30 fits the situation.
The same process used for option (b) gives us the A.P. 10, 35, 60. (10 + 35 + 60 = 105) and in the third option 15, 90 (15 + 90 = 105).
Hence, all the three options are correct.
18. Go through the options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 7$. Only option (a) satisfies both conditions.
19. The answer to this question can be seen from the options. Both 3, 6, 12 and 12, 6, 3 satisfy the required conditions— viz, GP with sum of first and third terms as 15. Thus, option (c) is correct.
20. The answer to this question cannot be determined because the question is talking about income and asking about expenses. You cannot solve this unless you know the value of the expenditure she incurs over the years. Thus, "Cannot be Determined" is the correct answer.

21. The difference between the amounts at the end of 5 years and 10 years will be the simple interest on the initial capital for 5 years.
    Hence, (2000 − 1500)/5 = 100 (simple interest.)
    Also, the Simple Interest for 5 years when added to the sum gives 1500 as the amount.
    Hence, the original sum must be 1000. Option (d) is correct.
22. Visualising the squares below 91, we can see that the only way to get the sum of 3 squares as 91 is: $1^2 + 3^2 + 9^2 = 1 + 9 + 81 = 91$. The smallest number is 1. Option (a) is correct.
23. Since the 8th and the 12th terms of the AP are given as 60 and 80, respectively, the difference between the two terms would equal 4 times the common difference. Thus we get $4d = 80 - 60 = 20$. This gives us $d = 5$. Also, the 8th term in the AP is represented by $a + 7d$, we get:
    $a + 7d = 60 \rightarrow a + 7 \times 5 = 60 \rightarrow a = 25$. Option (c) is correct.
24. The series would be given by: 13, 17, 21... which essentially means that all the numbers in the series are of the form $4n + 13$ or $4k + 1$(Where $k = n + 3$). Only the value in option (c) is a $4k + 1$ number and is hence the correct answer.
25. His total earnings would be 600 + 630 + 660 + … + 1470 = Rs 31050. Option (a) is correct.
26. If we take the square of the side we get the area of the squares. Thus, if the side of the respective squares are a_1, a_2, a_3, a_4... their areas would be a_1^2 a_2^2, a_3^2, a_4^2. Since the areas are in GP, the sides would also be in GP**.**
27. $1280 = 5.\ 2^{n-1}$ or $n - 1 = 8$ or $n = 9$. Thus, there are total of 9 terms in the series. Option (c) is correct.
28. Solve this question through trial and error by using values of n from the options:
    For 16 terms, the series would be 5 + 10 + 15 + … + 80 which would give us a sum for the series as 8 × 85 = 680. The next term (17th term of the series) would be 85. Thus, 680 + 85 = 765 would be the sum to 17 terms. It can thus be concluded that for 17 terms the value of the sum of the series is not less than 765. Option (a) is correct.
29. $5 \times r^5 = 160 \rightarrow r^5 = 32 \Rightarrow r = 2$.
    Thus, the series would be 5, 10, 20, 40, 80, 160. The second geometric mean between 5 and 160 in this case would be 20. Option (d) is correct.
30. In the case of a G.P. the 7th term is derived by multiplying the 5th term twice by the common ratio. (Note: this is very similar to what we had seen in the case of an A.P.) Since, the seventh term is derived by multiplying the 5th term by 4, the relationship. $r^2 = 4$ must be true.
    Hence, $r = 2$
    If the 4th term is 40, the series in reverse from the 4th to the first term will look like:
    40, 20, 10, 5. Hence, option (b) is correct.
31. The first common term is 23, the next will be 29 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.)
    Thus, the common terms will be given by the A.P 23, 29, 35 ....., last term. To find the answer you need to find the last term that will be common to the two series.
    The first series is 23, 25, 27 ... 259
    While the second series is 23, 26, 29 ..... 260.
    Hence, the last common term is 257.
    Thus our answer becomes $\frac{257-23}{6}+1=40$. Option (b) is correct.
32. Think like this:
    The average of the first 4 terms is 14, while the average of the first 8 terms must be 22.
    Now visualise this:
    1st $\underbrace{\text{2nd}\quad\text{3rd}}_{\text{average}=14}$ $\underbrace{\text{4th}\quad\text{5th}}_{\text{average}=22}$ 6th 7th 8th
    Hence, $d = 8/2 = 4$ [Note: understand this as a property of an A.P.]
    Hence, the average of the 8th and 9th term = 22 + 4.4 = 38 But this 38 also represents the average of the 16 term A.P.
    Hence, required answer = 16 × 38 = 608. Option (c) is correct.
33. AM = 41 means that their sum is 82 and GM = 9 means their product is 81. The numbers can only be 81 and 1. Option (c) is correct.
34. Trial and error gives us that for option (b):
    With the ratio 9:1, the numbers can be taken as $9x$ and $1x$. Their AM would be $5x$ and their GM would be $3x$. The GM can be seen to be 40% lower than the AM. Option (b) is thus the correct answer.
35. The total savings would be given by the sum of the series: 1000 + 1500 + 2000 +…..+ 6500 = 12 × 3750 = 45000. Option (b) is correct.
36. The 23rd term of the sequence would be the 12th term of the sequence 1, 5, 9, 13, ...
    The 12th term of the sequence would be 1 + 4 × 11 = 45. Option (c) is correct.
37. log a, log b, log c are in A.P then 2log b = log a+ log c or log b^2 = log ac or $b^2 = ac$
    So G.M. of a and c is b.
    Option (a) is correct.

38. The two series till their 1000th terms are 1, 3, 5, 7....1999 and 4, 7, 10...3001. The common terms of the series would be given by the series 7, 13, 19,...1999. The number of terms in this series of 333. Option (d) is correct.
39. The sum to 23 terms of the sequence would be:
The sum to 12 terms of the sequence 1, 5, 9, 13, ….. + The sum to 11 terms of the sequence 4, 8, 12, 16,….
The required sum would be
$\frac{12}{2}(2.1+(12-1)4)+\frac{11}{2}[2.4+(11-1)4]$ = 6 × 46 + 11 × 24 = 276 + 264 = 540. Option (c) is correct.
40. The maximum sum would occur when we take the sum of all the positive terms of the series. The series 25, 24, 23,… 1, 0 has 26 terms. The sum of the series would be given by:
n × average = 26 × 12.5 = 325
Option (a) is correct.
41. Since the sum of 22 terms of the AP is 770, the average of the numbers in the AP would be 770/22 = 35. This means that the sum of the first and last terms of the AP would be 2 × 35= 70. Trial and error gives us the terms of the required GP as 14, 28, 56. Thus, the common difference of the $AP = \frac{28-14}{7} = 2$.
42. It can be seen that for the series the average of first two terms is 2, for first 3 terms the average is 3 and so on. Thus, the sum of first 2 terms is 2^2, of first 3 terms it is 3^2 and so on. For 1111111 terms it would be 1111111^2 = 1234567654321. Option (d) is correct.
43. The area of the first square would be 1024 sq cm. the second square would give 512, the third one 256 and so on. The infinite sum of the geometric progression 1024 + 512 + 256 + 128… = 2048. Option (a) is correct.
44. $a = \frac{ar}{1-r}$ or 1 − r = r or r = 1/2 a + $\underline{ar}$ = 36 or a = 24
Required sum $= \frac{24}{1-\frac{1}{2}} = 48.$ Option (a) is correct.
45. The side of the first equilateral triangle being 8 units, the first area is $16\sqrt{3}$ square units. The second area would be 1/4 of area of largest triangle and so on.
$16\sqrt{3}$, $4\sqrt{3}$, $\sqrt{3}$, $\frac{\sqrt{3}}{4}$, $\frac{\sqrt{3}}{16}$,…..
The infinite sum of this series — $16\sqrt{3}$ /(1 − 1/4) = $64/\sqrt{3}$ square units.
Option (d) is correct.
46. The sum of the total distance it travels would be given by the infinite sum of the series:
$$210+2\left[210\times\left(\frac{5}{6}\right)+210\times\left(\frac{5}{6}\right)^2+210\times\left(\frac{5}{6}\right)^3+\ldots..\right]$$
$$210+\ 2\cdot 210\cdot\left(\frac{5}{6}\right)\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+\ldots..\right]$$
$$210 + 350\left[\frac{1}{1-\frac{5}{6}}\right] = 210 + 350 \times 6 = -210 + 2100$$
= 2310 metres.
Option (b) is correct.
47. Let the series be a, ar, ar^2, ar^3,….
According to the question a= 3ar/(1 − r) or r = 1/4. The series would be 4, 4/4, 4/16,….. and so on. The product of first three terms of the series would be $4\cdot 1\cdot\frac{1}{4} = 1$.
48. 5^{th} term= 25. 1^{st} term = 25 – (4 × 5) = 5, 11^{th} term = 5 + 10 × 5 = 55, 100^{th} term = 5 + 99 × 5 = 500
Student will score maximum marks if he attempts question 11 to 100. The maximum score would be the sum of the series 55 + 60 + … + 495 + 500 = (90 × 555)/2 = 24975.
49. In order to find how many times the alarm rings we need to find the number of numbers below 100, which are not divisible by 2, 3, 5 or 7. This can be found by:
100 — (numbers divisible by 2) – (numbers divisible by 3 but not by 2) — (numbers divisible by 5 but not by 2 or 3) — (numbers divisible by 7 but not by 2 or 3 or 5).
Numbers divisible by 2 up to 100 would be represented by the series 2, 4, 6, 8, 10…100 → A total of 50 numbers.
Numbers divisible by 3 but not by 2 up to 100 would be represented by the series 3, 9, 15, 21...99 → A total of 17 numbers. Note use short cut for finding the number of number in this series :
[(last term — first term)/ common difference] + 1 = [(99 — 3)/6] + 1 = 16 + 1 = 17.
Numbers divisible by 5 but not by 2 or 3: Numbers divisible by 5 but not by 2 up to 100 would be represented by the series 5, 15, 25, 35...95 → A total of 10 numbers. But from these numbers, the numbers 15, 45 and 75 are also divisible by 3. Thus, we are left with 10 — 3 = 7 new numbers which are Divisible by 5 but not by 2 and 3.
Numbers divisible by 7, but not by 2, 3 or 5: numbers divisible by 7 but not by 2 upto 100 would be represented by the series 7, 21, 35, 49, 63, 77, 91 → A total of 7 numbers. But from these numbers we should not count 21, 35 and 63 as they are divisible by either 3 or 5. Thus a total of 7 — 3 = 4 numbers are divisible by 7 but not by 2, 3 or 5.

Thus we get a total of 100 − 78 = 22 times. Also, the last time the bell would ring would be in the 97^{th} minute (as 98, 99 and 100 are divisible by at least one of the numbers).

50. Smallest interior angle = 100, largest exterior angle= 180 − 100 =80

Similarly other exterior angles are 70, 60, 50, ….

Sum of all the exterior angles = 360

So $\frac{n(2.80+(n-1)(-10))}{2}$ = 360 or $n(17 - n) = 72$

We can see that the above equation is true for both $n = 8, 9$. But for $n = 9$, the 9^{th} exterior angle must be 0 which is not possible so only 8 is possible.

51. $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

For n =0 the above equality is true. So n must be 0.

52. $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{2ab}{a+b}$

For $n = -1$ the above equality is true so $n = -1$.

53. According to the question $\frac{2ab}{a+b}=\sqrt{ab}$, this equality will be true only for $a = b$. Hence, $a = b$.

54. A.M. of n positive integers is always greater than or equals to G.M. of the numbers. Then according to the question,

Sum of the numbers ≥ n (product of n positive integers) $^{1/n}$

Sum of the numbers ≥ n

55. $G.M.^2$ = A.M. × H.M. = 12 × 3 =36.

56. We can solve this problem by checking for values. If we assume that x, y, z are in A.P. & $x = 1$, $y = 2$, $z = 3$.

$$x+\frac{1}{yz}=1+\frac{1}{6}=\frac{7}{6}$$

$$y+\frac{1}{zx}=2+\frac{1}{3}=\frac{7}{3}$$

$$z+\frac{1}{xy}=3+\frac{1}{2}=\frac{7}{2}$$

We can see if x, y, z are in A. P. then $x+\frac{1}{yz},y+\frac{1}{zx},z+\frac{1}{xy}$ are also in A.P.

Level of Difficulty (II)

1. Identify an AP which satisfies the given condition.

Suppose we are talking about the second and third terms of the AP.

Then an AP with second term 3 and third term 2 satisfies the condition.

A times the $\underline{a}^{th}$ term = b times the b^{th} term.

In this case the value of $a = 2$ and $b = 3$.

Hence, for the $(a + b)^{th}$ term, we have to find the 5^{th} term.

It is clear that the 5^{th} term of the AP must be zero. Check the other three options to see whether any option gives 0 when $a = 2$ and $b = 3$.

Since none of the options b, c or d gives zero for this particular value, option (a) is correct.

2. Since the four parts of the number are in AP and their sum is 28, the average of the four parts must be 7. Looking at the options for the smallest part, only the value of 4 fits in, as it leads us to think of the AP 4, 6, 8, 10. In this case, the ratio of the product of the first and fourth (4 × 10) to the product of the first and second (6 × 8) are in 5: 6 ratio.

3. View: 1 − 3 + 5 − 7 + 9 − 11 ..... 100 terms as (1 − 3) + (5 − 7) + (9 − 11) ...... 50 terms. Hence, −2 + −2 + −2 … 50 terms = 50 × −2 = −100. Option (b) is correct.

4. In a period of 1 day or 24 hours the clock would strike 1 + 2 + 3 + ….+ 23 = 276 times. In the course of 3 days the clock would strike 276 × 3 = 828 times. Option (b) is correct.

5. Since this is a decreasing A.P. with first term positive, the maximum sum will occur upto the point where the progression remains non-negative. 54, 52, 50, ….. 0 Hence, 28 terms × 27 = 756. Option (c) is correct.

6. The sum of the required series of integers would be given by 105 + 112 + 119 + ...294 = 28 × 199.5 = 5586. Option (b) is correct.

7. $y^2 = xz$, 1 + log x, 1 + log y, 1 + log z are in A.P. if 2 log y= log x + log z

or $y^2 = xz$, Option (b) is right.

8. 1001 + 1003 + 1005 + ... 1999 = 1500 × 500 = 750000.

9. The required sum would be given by the sum of the series 110, 121, 132, ... 990. The number of terms in this series = (990 − 110)/11 + 1 = 80 + 1 = 81. The sum of the series = 81 × 550 (average of 110 and 990) = 44550. Option (c) is correct.

10. 6 × average of 113 and 253 = 6 × 183 = 1098. Option (c) is correct.

11. The first 100 terms of this (1 − 2 − 3) + (2 − 3 − 4) + … + (33 − 34 − 35) + 34

The first 33 terms of the above series (indicated inside the brackets) will give an AP.:

−4, −5, −6.... −36 = 33 × −20 = −660 (sum of this A.P.). The required answer would be

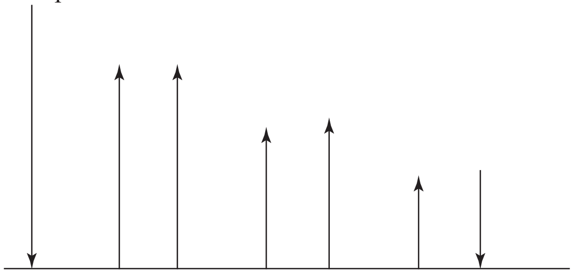
–660 + 34 = –626.

12. Solve this one through trial and error. For $n = 2$ terms the sum upto 2 terms is equal to 84. Putting n in the options it can be seen that for option (b) the sum to two terms would be given by

7 × (1000 − 10 − 18)/81 = 7 × 972/81 = 7 × 12 = 84.

13. 2log b = log a+ log c or log b^2 = log ac or $b^2 = ac$, so a, b, c are in GP. Option (b) is correct.

14. The path of the rubber ball is:



In the figure above, every bounce is 3/5th of the previous drop.

In the above movement, there are two infinite G.Ps (The GP representing the falling distances and the GP representing the rising distances.)

The required answer: (Using a/(1 − r) formula)

$\frac{20}{2/5}+\frac{12}{2/5}=80$ metres. Option (c) is correct.

15. Solve this for a sample GP. Let us say we take the GP as 2, 6, 18, 54. x, the first term is 2, let n = 3 then the 3rd term y = 18 and the product of 3 terms $p = 2 \times 6 \times 18 = 216 = 6^3$. The value of $p^2 = 216 \times 216 = 6^6$.

Putting these values in the options we have:

Option (a) gives us $(xy)^{n-1} = 36^2$ which is not equal to the value of p^2 we have from the options

Option (b) gives us $(xy)^n = 36^3 = 6^6$ which is equal to the value of p^2 we have from the options.

It can be experimentally verified that the other options yield values of p^2 which are different from 6^6 and hence we can conclude that option (b) is correct.

16. Trying to plug in values we can see that the infinite sum of the GP 24, 8, 8/3... is 36. Hence the third term is 8/3.

17. The expression can be written as $2^{1/3+1/6+1/12+....} = 2^{\text{INFINTE SUM OF THE GP}} = 2^{2/3}$. Option (b) is correct.

18. $\frac{A}{1-r} = 5$ then $r = 1 - \frac{A}{5}$

Now since it is an infinite G.P. |r|<|, implies

$-1 < 1 - \frac{A}{5} < 1$ or $0 < A < 10$

19. From the facts given in the question it is self evident that the common ratio of the GP must be 3 (as the sum of the 2nd and 4th term is thrice the sum of the first and third term).

$a + ar^2 = 50$ or $a = 50/(1 + 9) = 5$

Largest term = $5.(3)^3 = 135$. Option (c) is correct.

20. 2/9, 1/4, 2/7, 1/3 ………………………..

This is an HP series. The corresponding AP will be:

9/2, 4/1, 7/2, 3/1, ……………………

or 4.5, 4, 3.5, 3 ……………………..

i.e., this is an AP with first term 4.5 and common difference –0.5.

Hence $T_{13} = 4.5 + 12\ (-0.5) = -1.5$

The corresponding T_{13} HP is 1/–1.5 = 1 × –2/3 = –2/3

21. Third term = $a + 2d$, Fourth term = $a + 3d$; 1st term = a, seventh term = $a + 6d$.

Thus $2a + 5d$=19 and $2a + 6d = 22 \rightarrow d = 3$ and a= 2.

The 9th term = $a + 8d$ = 2 + 24= 26. Thus, option (a) is correct.

22. If the difference between the 6th and the 1st term is 30, it means that the common difference is equal to 6. Since, the third term is 16, the AP would be 4, 10, 16, 22, 28, 34, 40, 46 and the sum to 8 terms for this AP would be 200. Thus, option (d) is correct.

23. $5d = 30 \rightarrow d = 6$. Thus, the numbers are 4, 10, 16, 22, 28, 34. The smallest number is 10. Option (d) is correct.

24. Find sum of the series: 104, 111, 118,…993

Average × $n = \frac{104+993}{2} \times 128 = 70208$. Option (a) is correct.

25. Since the sum of the first three terms of the AP is 24, the average of the AP till 3 terms would he 24/3 = 8. Value of the second term would be equal to this average and hence the second term is 8. Using the information about the sum of squares of the first and second terms being 80, we have that the first term must be 4. Thus, the AP has a first term of 4 and a common difference of 4. The seventh term would be 32. Thus option (a) is correct.

26. The combined travel would be 28 on the first day, 29 on the second day, 30 on the third day, 31 on the fourth day, 32 on the fifth day and 33 on the sixth day, 34 on 7th day. They meet after 7 days. Option (c) is correct.

27. This is an intensive calculation, problem and you are not supposed to know how to do the calculations in this question mentally. The problem has been put here to test your concepts about whether you recogsize how this is a question of GPs. If you feel like, you can use a calculator/computer spreadsheet to get the answer to this question.

The logic of the question would hinge on the fact that the value of the investment of the fifteenth year would be 1000. At the end of the 15th year, the investment of the 14th year would be equal to 1000 × 1.05, the 13th year's investment would amount to 1000×1.05^2 and so on till the first year's investment which would amount to 1000×1.05^{14} after 15 years. Thus, you need to calculate the sum of the GP: 1000, 1000×1.05, 1000×1.05^2, 1000×1.05^3 for 15 terms.

28. Since, sum to n terms is given by $(2n + 7)$,

Sum to 1 terms = 9

Sum to 2 terms = 11

Thus, the 2nd term must be 2,

29. Solve this question by looking at hypothetical values for n and $2n$ terms. Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms ($2n$ = 2) of the second series we would get B/A as 3/2

For n = 2 and $2n$ = 4 we get A =5/2 and B = 15/4 and B/A = 15/4/5/2 = 3/2

Thus, we can conclude that the required ratio is always constant at 3/2 and hence the correct option is (d).

30. We need to find the infinite sum of the GP: 1000, 800, 640….. (first term = 1000 and common ratio = 0.8) We get: infinite sum of the series as 1000/(1 − 0.8) = 5000, Thus option (b) is correct.

31. Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum upto the first term is: 1/5. Upto the second term it is 2/9 and upto the third term it is 3/12. It can be easily seen that for the first term, second term and third term the numerators are 1, 2 and 3 respectively. Also, for 1/5 — the 5 is the second value in the denominator of 1/1 × 5 (the first term); for 2/9 also the same pattern is followed— as 9 comes out of the denominator of the second term of series and tar 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence the correct answer would be 56/225.

32. Solve this on the same pattern as Question 31 and you can easily see that for the first term sum of the series is $2-\sqrt{3}$, for 2 terms we have the sum as $\sqrt{5}-\sqrt{3}$ and so on. For the given series of 120 terms the sum would be $\sqrt{225}-\sqrt{3}=15-\sqrt{3}$. Option (a) is correct.

33. If you look for a few more terms in the series, the series is: 1, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28, 1/36, 1/45, 1/55, 1/66, 1/78, 1/91, 1/105, 1/120, 1/136. 1/153 and so on. If you estimate the values of the individual terms it can be seen that the sum would tend to 2 and would not be good enough to reach even 2.25. Thus, option (a) is correct.

34. Solve this using trial and error. For 1 term the sum should be 15 and we get 15 only from the first option when we put is n = 1. Thus, option (a) is correct.

35. For this question too you would need to read the pattern of the values being followed. The given sum has 6 terms.

It can be seen that the sum to 1 term = 1/3

Sum to 2 terms = 3/5

Sum to 3 terms = 5/7

Hence, the sum to 6 terms would be 11/13.

36. The sum to infinite terms would tend to 1 because we would get (infinity)/(infinity + 2).

37. All members of A are smaller than all members of B. In order to visualise the effect of the change in sign in k assume that A is {1, 2, 3...124} and B is {126, 12.7...250}. It can be seen that for this assumption of values neither options (a), (b) or (c) is correct.

38. If elements of A are in ascending order a_{124} would be the largest value in A. Also a_{125} would be the largest value in B. On interchanging a_{124} and a_{125}, A continues to be in ascending order, but B would lose its descending order arrangement since a_{124} would be the least value in B. Hence, option (a) is correct.

39. Since the minimum is in A and the maximum is in B, the value of x cannot be less than Maximum − Minimum.

40. It is evident that the whole question is built around Arithmetic progressions. The 5th row has an average of 55, while the 25th row has an average of 75. Since even column wise each column is arranged in an AP we can conclude the following:

1st row - average 51 - total = 25 × 51

2nd row - average 52 - total 25 × 52.....

25th row - average 75 - total 25 × 75

The overall total can be got by using averages as: 25 (51 + 52 + 53 + …..+ 75)= 25.1575 = 39375

41. The numbers forming an AP would be:

1234, 1357, 2345, 2468, 3210, 3456, 3579, 4321, 4567, 5432, 5678, 6543, 6420, 6789, 7654, 7531, 8765, 8642, 9876, 9753, 9630.

A total of 21 numbers.

If we count the GPs we get: 1248, 8421-a total of 2 numbers.

Hence, we have a total of 23, 4-digit numbers why the digits are either APs or GPs.

Thus, option (d) is correct.

42. Total vadapavs made = 500

Vadapavs with chicken and mushroom patty =250 (Number of terms in the series 1,3, 5, 7, 9...499) out of which half of the vadapavs also have vegetable petty.

Vadapavs with only mushroom patty = 36 (Number of terms of the series 8, 22, 36, ….)

Vadapavs with chicken, mushroom and vegetable patty = 18 (Number of terms in the series 1, 29, 57... .

Required answer = 500 − 250 – 36 = 214.

43. From the above question, we have 18 such vadapavs.

44. The key to this question is what you understand from the statement— 'for two progressions out of P_1, P_2 and P_3 the average is itself a term of the original progression P.' For option (a) which tells us that the Progression P has 20 terms, we can see that P_1 would have 7 terms, P_2 would have 7 terms and P_3 would have 6 terms. Since, both P_1 and P_2 have an odd number of terms we can see that for P_1 and P_2 their 4th terms (being the middle terms for an AP with 7 terms) would be equal to their average. Since, all the terms of P_1, P_2 and P_3 have been taken out of the original AP

P, we can see that for P_1 and P_2 their average itself would be a term of the original progression P. This would not occur for P_3 as P_3, has an even number of terms. Thus, 20 is a correct value for n.

Similarly, if we go for $n = 26$ from the second option we get:

P_1, P_2 and P_3 would have 9, 9 and 8 terms, respectively and the same condition would be met here too.

For $n = 36$ from the third option, the three progressions would have 12 terms each and none of them would have an odd number of terms.

Thus, option (d) is correct as both options (a) and (b) satisfy the conditions given in the problem.

45. Since, P_1 is formed out of every third term of P, the common difference of P_1 would be three times the common difference of P. Thus, the common difference of P would be 2.

46. S consists one a, two b's, three c's and so on. So total number of terms = 1 + 2 + 3 + …..+ 26 = $\frac{26}{2}(1+26) = 13 \times 27 = 351.$

47. For 16^{th} alphabet total number of terms of $S = 136$. So 144^{th} term of S will be 17^{th} alphabet, which is q.

48. Let S' be the sum of all terms of the series S then according to the question:

$S' = 1a + 2b + 3c + 4d + 5e + \ldots + 26z = 1.1 + 2.3 + 3.5 + 4.7 + \ldots + 26.\ 51 = \sum_{n=26} n(2n-1) = \sum_{n=26} 2n^2 - n = 12051$

49. If $f(4x) = 8x + 1$ then $f(x) = 2x + 1$ & $f(2x) = 4x + 1$

$(4x + 1)^2 = (8x + 1)(2x + 1)$

$x = 0$

So for no positive value of x, $f(2x)$ is the G.M. of $f(x)$, $f(4x)$.

50. If $y = x + d$, $z = x + 2d$, $w = x + 3d$ then

$(x + d)^2 = x(x + 3d)$ or $d = x$

$w/x = 4x/x = 4$

51. Let the G.P. be 1, 3, 9, 27, 81,…

Let $m = 2$, $n = 3$, $p = 5$ then $(n - p) \log x + (p - m) \log y + (m - n) \log z = (3 - 5)\log 3 + (5 - 2)\log 9 + (2 - 3)\log 81 = -2\log 3 + 6 \log 3 - 4\log 3 = 0$

52. Go through options.

Option (a) $\frac{n(n+1)(n+2)}{6}$ for $n = 1$, sum = 1

For $n = 2$, Sum = 4

So option (a) is correct.

53. $A = 1/(1 - x)$ & $B = 1/(1 - y)$

$x = (A - 1)/A$ & $y = (B - 1)/B$

$1 + xy + x^2y^2 + x^3y^3 + \ldots =$

$$\frac{1}{1-xy} = \frac{1}{1-\frac{A-1}{A}.\frac{B-1}{B}} = \frac{AB}{A+B-1}$$

54. Let the angles are x, xr, xr^2, xr^3

$x + xr + xr^2 + xr^3 = 360^0$.

The angles are 24^0, 48^0, 96^0,192^0. Largest angle = 192^0

55. $S = 1 + 4 + 9 + 16 + \ldots$

$$S_n = \sum n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$S_{16} = \frac{1}{6}.16.17.33 = 1496$$

56. $S_{16} = 1 + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + (1 + 3 + 5 + 7 + 9) + 1$

$$S_{16} = \frac{1}{6}5(5+1)(2.5+1)+1 = 56$$

57. Going through the trial and error = $2(20)^2 + 3 - 2(19)^2 - 3 = 78$

58. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots = (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + \ldots + (15 - 16)(15 + 16) + 17^2 = -(1 + 2 + 3 + 4 + \ldots + 16) + 17^2 = 289 - 136 = 153$

59. 3 + 6 + 10 + 15 + ….

(1 + 2) + (1 + 2 + 3)+ (1 + 2 + 3 + 4)+ ….. = 1 + (1 + 2) + (1 + 2 + 3)+ (1 + 2 + 3 + 4)+ ….. –1

Required sum = Sum of 21 terms of series 1 + (1 + 2) + (1 + 2 + 3)+ ,…. –1 = $\sum_{n=21} \frac{(n+1)n}{2} - 1$ = 1771 – 1 = 1770

60. If we take $x = 4$ and $n = 1$ then $1^1 + 2^1 + 3^1 = 6$ is divisible by 1 + 2 + 3 =6. But for $n = 2$

$1^2 + 2^2 + 3^2 = 14$ is not divisible by 6 again for $n = 3$, $1^n + 2^n + 3^n$ divisible by 6 and so on. So for every odd value of n, $1^n + 2^n + 3^n + \ldots + x^n$ is always divisible by $1 + 2 + 3 + \ldots + x$.

61. $3 = 4^1 - 1$

$14 = 4^2 - 2$

$61 = 4^3 - 3$

$252 = 4^4 - 4$

So 12^{th} term = $4^{12} - 12$

Level of Difficulty (III)

1. Since the difference between the tenth and the sixth terms is −16, the common difference would be −4. Using a trial and error approach with the options, we can see that if we take the first term as 16, we will get the series 16,12,8,4,0,–4,–8,–12,–16,–20. We can see that both the conditions given in the question are met by this series. Hence, the first term would be 16.
2. Any sub-part of an AP is also an AP. Thus, the third term would be the average of the first and the fifth term. Hence, the third term would be 5.
3. A factor search for factor pairs of 456 give us the following possibilities. 1,456; 2,228; 3,152; 4,114; 6,76; 8,57; 12,38 & 19,24. A check of the conditions given in the problem, tells us that if we take

12 as the 4th term and 38 as the 5th term, we would get the series till 9 terms as: −66,−40,−14,12,38,64, 90,116,142. In this series we can see that the division of the 9th term by the 4th term gives us a quotient of 11 and a remainder of 10. Hence, the required first term is −66.

4. The distances covered by him (to and fro) would be 20, 40, 60,80,100, 120, 140, 160,180, 200 &220 to get a total distance of 1.32 kms. There are 11 terms in this AP. However, he must also have planted one sapling at the starting point & hence the number of saplings planted would be 12.
5. The answer would directly be P_2/P_1. Assume a series having a few number of terms e.g. 1,2,4,8,16,32. The value of P_2 here = 42, while P_1 = 21. The common ratio can be seen to be $P_2/P_1 = 2$.
6. Solve on the same pattern as above. The correct answer is option (c).
7. Solve this question using options. The average of the sum of the first 5 terms of the AP can be used to get the value of the third term of the AP. If we try to use the options, in option 2, if the sum of the first 5 terms is 2.5, the third term must be 2.5/5 = 0.5. This means our AP is 2,1.25,0.5,−0.25,−1. The corresponding GP with the same 1st and 3rd terms is 2,1,0.5,0.25… The condition for the second term is also matched here.
8. Use the options to get the answer. For n=10, we get the required ratio as 51/2.
9. For 1 digit the sum would be $6^2 + 8$, for 2 digits the sum would be $66^2 + 88$ and so on. Checking the options gives us option (b) as the correct answer.
10. The sum of the interior angles of any polygon of n sides is given by
    $(n - 2) \times 180$. This needs to match the sum of the AP 120 + 125 + 130 +…n terms. For n = 9, we get the two sums equal and hence option (c) is correct.
11. Solve this one using options to check the correct answer.
12. Tracing the second and fourth terms through factor pairs of 160, we get the numbers that fit in the requirements of the problem as: 7, 10, 13, 16, 19, 22, 25. The sum of the series is 112.
13. The third and ninth terms of an 11 term AP are a pair of corresponding terms of the AP. Hence, their average would be the average of the AP. This gives us the required sum of the AP as 11 × 5 = 55.
14. $(a + 4d)^2 + (a + 10d)^2 = 3 \rightarrow a^2 + 14ad + 58d^2 = 1.5$. Also, $(a + d)(a + 13d) = P \rightarrow a^2 + 14ad + 13d^2 = P$. Further, we need to find the value of $a^2 + 14ad$ (product of the first and fifteenth terms of the AP). From the above two equations, we get that $45d^2 = \frac{3}{2} - P \rightarrow 13d^2 = \frac{(39-26P)}{90}$
    Thus, $a^2 + 14ad = P - \frac{(39-26P)}{90} = (116P - 39)/90$. Option (c) is correct.
15. Solve using options. The values in option (a) gives you the required 12:13 ratio between the HM and the GM. Hence, option (d) is correct.
16. Solve this based on pattern of the options. The given series has 100 terms. For n = 100, the options can be converted as:
    Option (a) = $n \times 2^{(n+1)} + 2$. This means that for 1 term, the sum should be $1 \times 2^2 + 2 = 6$. But we can see that for 1 term, the series has a sum of only 1 × 2 = 2. Hence, this option can be rejected. Option (c) satisfies the conditions.
    Option (b) = $(n - 1) \times 2^n + 2$. For 1 term, this gives us a value of 2. For 2 terms, this gives us a value of 6, which does not match the actual value in the question.Hence, this option can be rejected.
17. Since the ratio of the 2nd to the fourth term is given as ¼, we can conclude that the common ratio of the GP is 2. Also, $a + 8a = 108 \rightarrow 9a = 108 \rightarrow a = 12$. Thus, $x_3 = 48$.
18. You can try to fit in values to get the correct answer. a,b,c would be in GP. If we take x, y and z as 1,2,4 we get a,b,c as 4,2 and 1, respectively to keep a^x, b^y & c^z equal.
19. The prime factors of 720 are: $2^4 \times 3^2 \times 5^1$. The required sum of factors would be: $(1 + 2 + 2^2 + 2^3 + 2^4)(1 + 3 + 3^2)(1 + 5) = 31 \times 13 \times 6 = 2418$.
20. Check the options to get option (a) as the correct answer.
21. The first term of the given AP is 2 and the common difference is also 2. Thus, the 11th term of the GP = $2 \times 2^{10} = 2048$.
22. The minimum value of y would occur when all the three values are equal. Thus, $y^3 = 4 \rightarrow y = 2^{2/3}$.
23. For the product to be the maximum, since the sum of a_2, a_7 and a_{12} would be fixed; we would need to keep each of the three numbers as equal. Thus, the value of the common difference would be 0.
24. Solve based on patterns and options as discussed for question 16 above.
25. The corresponding AP would be 2.5, 1.0833,… This gives us a common difference of −1.4166. From the third term onwards, the AP and its reciprocal HP would both become negative. Hence, the largest term of the HP is the second term itself.
26. The series of plank sizes would be: 0.75 × 0.5, 0.75 × 1, 0.75 × 1.5….0.75 × 12. The sum of this AP is 112.5.
27. Each subsequent triangle would have the sum of sides halved from the previous triangle. Thus, the sum of the perimeters would be given by $P + P/2 + P/4 + P/8 + \ldots$ till infinite terms. Hence, the sum of all the perimeters of the infinite triangles would be $2P$.

28. The areas would be $A + A/4 + A/16 + \ldots$ till infinite terms. The infinite sum of all the perimeters would be $4A/3$.
29. The first perimeter is 160, the second one is $\frac{160}{\sqrt{2}}$, the third one would be 80 and so on till infinite terms. The infinite sum would be equal to $160(2+\sqrt{2})$.
30. The areas would consecutively get halved. So, the first area being 1600, the next one would be 800, then 400 and so on till infinite terms. Thus, the infinite sum would be 3200.
31. If the sum of the first n terms is 'x', the sum of the next n terms is given as '$2x$' (as defined in the problem). Naturally, the sum of the next n terms would be '$3x$' (When you add the same number of terms of an AP consecutively, you get another AP). Thus, the required ratio is $6x/x = 6$.
32. Since, the problem says that the cell breaks into two new cells, it means that the original cell no longer exists. Hence, after 1 hour there would be 2^1 cells, after 2 hours there would be 2^2 cells and so on. After 9 hours there would be 2^9 cells. Hence, option (c) is correct.
33. We need the sum of the AP: 101, 104,107,…497 = $133 \times 299 = 39767$.
34. Solve by taking values and checking with the options. If we take the numbers as 1 and 8, we would get $a = 4.5$ and b and c as 2 and 4 respectively. Then $(b^3 + c^3)/abc = 2$. None of the first three options gives us a value equal to 2. Hence, the correct answer is option (d).
35. This can be checked using any values of p, q, r. If we try with 1,2,3 we get the value as 0. If we try values of p, q, r as 2,3,4 we get the expression as positive. Hence, we conclude that the expression's value would always be non-negative.
36. For $n = 1$ sum = 2/3
For $n = 2$ sum = 20/27
For $n = 3$ sum = 1640/2187
None of the options matches these numbers and hence option (d) is correct.
37. $77777\ldots7777 = 7(10^{132}+\ldots+10^2+10^1+1) = \frac{7(10^{133}-1)}{10-1} = \frac{7(10^{133}-1)}{9}$
$10^{133} - 1$ is divisible by 9. Hence the given number is a composite number. Option (c) is correct.
38. $S = n^2 + 1 + n^2 + 2 + \ldots + n^2 + 2n = 2n^3 + n(2n + 1) = n(2n^2 + 2n + 1)$
Option (a) is correct.
Alternative Method: Suppose $n = 2$ then 1st term of the series will be $2^2 + 1 = 5$. Now we want to find the sum of first $2n = 4$ terms. First 4 terms of the series will be 5, 6, 7, 8. Sum = 26.
If we put $n = 2$ in the above options then only option (a) satisfies.
39. mth term of the series $= \frac{1}{m!+(m+1)!} = \frac{1}{m!(m+2)!}$
$= \frac{1}{(m+1)!} - \frac{1}{(m+2)!}$
$S = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \ldots + \frac{1}{20!} - \frac{1}{21!} = \frac{1}{2!} - \frac{1}{21!}$.
Option (a) is correct.
40. $\frac{a^n + c^n}{2} > \left(\frac{a+c}{2}\right)^n$, if n does not lie between 0 and 1.
But we know that A.M. > H.M.
b is the H.M. of a and c.
$\frac{a+c}{2} > b$
As $n > 1$ $\left(\frac{a+c}{2}\right)^n > b^n$
$\frac{a^n + c^n}{2} > \left(\frac{a+c}{2}\right)^n > b^n$
$a^n + c^n > 2b^n$. Option (c) is correct.
41.
$S = \frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \ldots$
$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \ldots + \left(\frac{1}{17^2} - \frac{1}{18^2}\right)$
$== 1 - \frac{1}{18^2} = \frac{323}{324}$
42. $S = \frac{4}{11} + \frac{44}{11^2} + \frac{444}{11^3} + \frac{4444}{11^4} + \ldots$
$\frac{S}{11} = \frac{4}{11^2} + \frac{44}{11^3} + \frac{444}{11^4} + \ldots$
$S - \frac{S}{11} = \frac{4}{11} + \frac{40}{11^2} + \frac{400}{11^3} + \ldots$
$\frac{10S}{11} = \frac{4}{11}\left(1 + \frac{10}{11} + \frac{100}{11^2} + \ldots\right)$
$\frac{10S}{11} = \frac{4}{11}\left(\frac{1}{1 - \frac{10}{11}}\right) = 4$
S = 44/10 = 22/5
43. $S = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \infty$
$S = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \ldots - \frac{1}{\infty}\right)$
$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

44. We can easily solve these problems by considering suitable values.
Let G.P. be 1, 2, 4, 8, 16, 32 and A.P. be 1, 7.2, 13.4, 19.6, 25.8, 32.
$A = 99$, $B = 63$. So $A > B$

45.

$$4+\frac{1}{2}+4+\frac{1}{6}+4+\frac{1}{12}+\ldots..4+\frac{1}{2450}$$
$$=4+\left(1-\frac{1}{2}\right)+4+\left(\frac{1}{2}-\frac{1}{3}\right)+4+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots..$$
$$+4+\left(\frac{1}{49}-\frac{1}{50}\right)$$
$$=49\times 4+\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\ldots..+\frac{1}{49}-\frac{1}{50}\right)$$
$$=196+\frac{49}{50}=\frac{9849}{50}$$

46. Let the common difference of the series X be d_1 and that of Y be d_2.
Since $x_n - y_n = n - 2$, $x_1 - y_1 = -1$ or $y_1 = x_1 + 1$
$x_3 = y_5$
$x_1 + 2d_1 = y_1 + 4d_2$
$x_1 + 2d_1 = x_1 + 1 + 4d_2$
$2d_1 - 4d_2 = 1$
$x_{99} - y_{197} = x_1 + 98d_1 - y_1 - 196d_2 = -1 + 49(2d_1 - 4d_2)$
$= -1 + 49 = 48$. Option (b) is correct.

47. If a be the 1st term and d be the common difference of the A.P. the 4th term of the series will be $a + 3d$. If $a + 3d$ is divisible by d then a should be divisible by d. hence the cases are:
$d = 1$, $a = 1, 2, 3, 4, 5, 6, 7, 8, 9$
$d = 2$, $a = 2, 4, 6$
$d = 3$, $a = 3$
So the required answer is 9 + 3 + 1 = 13

48. If $a - 2d$ be the first term and d be the common difference of A.P. then according to the question:
$(a - 2d)(a - d)a(a + d)(a + 2d) = 3840$ (1)
$$\frac{a-d}{a-2d}=\frac{a+d}{a-d}$$
$$d(3d-a)=0$$
$$a=3d$$
By putting $a = 3d$ in equation 1 we get:
$d \times 2d \times 3d \times 4d \times 5d = 3840$
By solving we get $d = 2$ & $a = 6$
10^{th} term = 6 + 9.2 = 24.

49. Let $x = -1/3$
$S = (1 + x)(1 + x^2)(1 + x^4)\ldots.$
$(1 - x)S = (1 - x)(1 + x)(1 + x^2)(1 + x^4)\ldots$
$(1 - x)S = (1 - x^2)(1 + x^2)(1 + x^4)(1 + x^8)\ldots..$
$(1 - x)S = (1 - x^4)(1 + x^4)(1 + x^8)\ldots..$
Since $x < 0$ & $|x| < 1$ so the value of RHS would be equals to 1.
$(1 - x)\ S = 1$ or $S = 1/(1 - x)$ or $1/(1 -(-1/3)) = 3/4$.

50. a, b, c, d are in A.P.
$$\frac{a}{abcd},\frac{b}{abcd},\frac{c}{abcd},\frac{d}{abcd}\ \text{are also in A.P.}$$
$$\frac{1}{bcd},\frac{1}{acd},\frac{1}{abd},\frac{1}{abc}\ \text{are also in A.P.}$$
Hence their reciprocals are in H.P.

51. nth term $=\dfrac{\sum n^3}{\sum(2n-1)}=\dfrac{1}{4}(n^2+2n+1)$
Sum of n terms of the given series =
$$\frac{1}{4}\left(\frac{1}{6}n(n+1)(2n+1)+n(n+1)+n\right)$$
For $n = 10$ the required sum $=\dfrac{1}{4}[505]=126.5$

Space for Rough Work