(d) None of these.

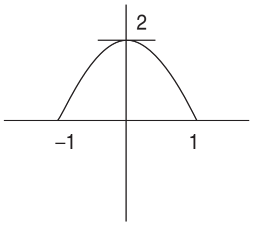
86. $f(x, y) = x^2 + y^2 - x - \frac{3y}{2} + 1$

When $f(x, y)$ is minimum then the value of $x + y$ = ?

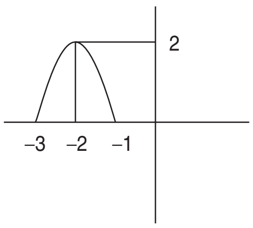
87. In the previous question, find the minimum value of $f(x, y)$

Directions for question number 88-89:

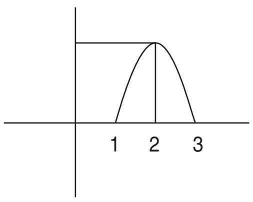
88. If the graph given below represents $f(x + 5)$ then which of the given options would represent the graph of $f(-2 - x)$?



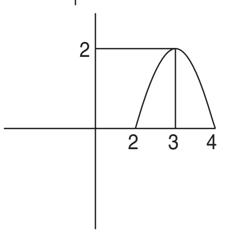
(a)



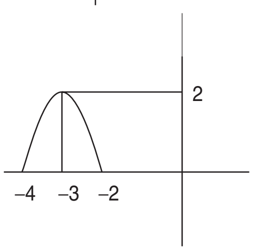
(b)



(c)

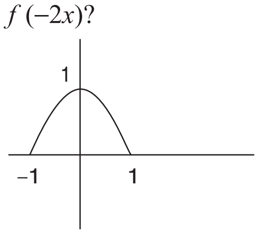


(d)

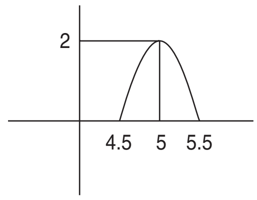


89. Which of the following options represents curve of $f(-2x)$?

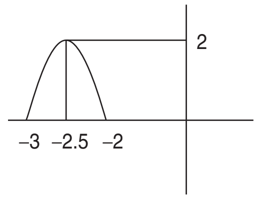
(a)



(b)



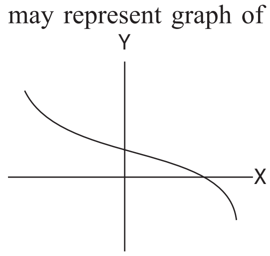
(c)



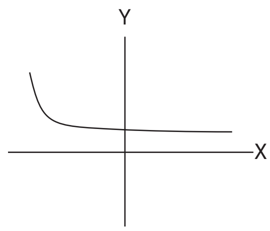
(d) None of these.

90. If x, y are real numbers and function $g(x)$ satisfies $\frac{(g(x+y)+g(x-y))}{2} = g(x)g(y)$ and $g(0)$ is a positive real number then which of the following options may represent graph of $g(x)$?

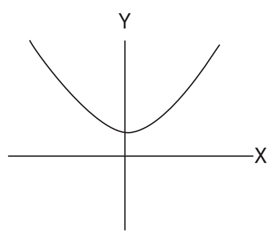
(a)



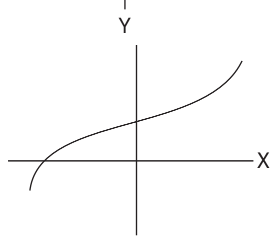
(b)



(c)



(d)



Space for Rough Work

LEVEL OF DIFFICULTY (III)

1. Find the domain of the definition of the function $y = 1/(x - |x|)^{1/2}$.
(a) $-\infty < x < \infty$ (b) $-\infty < x < 0$
(c) $0 < x < \infty$ (d) No where
2. Find the domain of the definition of the function $y = (x - 1)^{1/2} + 2(1 - x)^{1/2} + (x^2 + 3)^{1/2}$.
(a) $x = 0$ (b) $[1, \infty)$
(c) $[-1, 1]$ (d) $x = 1$
3. Find the domain of the definition of the function $y = \log_{10} [(x - 5)/(x^2 - 10x + 24)] - (x + 4)^{1/2}$.
(a) $x > 6$ (b) $4 < x < 5$
(c) Both *a* and *b* (d) None of these
4. Find the domain of the definition of the function $y = [(x - 3)/(x + 3)]^{1/2} + [(1 - x)/(1 + x)]^{1/2}$.
(a) $x > 3$ (b) $x < -3$
(c) $-3 \le x \le 3$ (d) Nowhere
5. Find the domain of the definition of the function $y = (2x^2 + x + 1)^{-3/4}$.
(a) $x \ge 0$ (b) All x except $x = 0$
(c) $-3 \le x \le 3$ (d) Everywhere
6. Find the domain of the definition of the function $y = (x^2 - 2x - 3)^{1/2} - 1/(-2 + 3x - x^2)^{1/2}$.
(a) $x > 0$ (b) $-1 < x < 0$
(c) x^2 (d) None of these
7. Find the domain of the definition of the function $y = \log_{10} [1 - \log_{10}(x^2 - 5x + 16)]$.
(a) (2, 3] (b) [2, 3)
(c) [2, 3] (d) None of these
8. If $f(t) = (t - 1)/(t + 1)$, then $f(f(t))$ will be equal to
(a) $1/t$ (b) $-1/t$
(c) t (d) $-t$
9. If $f(x) = e^x$ and $g(x) = \log_e x$ then value of *fog* will be
(a) x (b) 0
(c) 1 (d) e
10. In the above question, find the value of *gof*.
(a) x (b) 0
(c) 1 (d) e
11. The function $y = 1/x$ shifted 1 unit down and 1 unit right is given by
(a) $y - 1 = 1/(x + 1)$ (b) $y - 1 = 1/(x - 1)$
(c) $y + 1 = 1/(x - 1)$ (d) $y + 1 = 1/(x + 1)$
12. Which of the following functions is an even function?
(a) $f(t) = (a^t + a^{-t})/(a^t - a^{-t})$
(b) $f(t) = (a^t + 1)/(a^t - 1)$
(c) $f(t) = t.\ (a^t - 1)/(a^t + 1)$
(d) None of these
13. Which of the following functions is not an odd function?
(a) $f(t) = \log_2(t + \sqrt{t^2} + 1)$
(b) $f(t) = (a^t + a^{-t})/(a^t - a^{-t})$
(c) $f(t) = (a^t + 1)/(a^t - 1)$
(d) All of these
14. Find *fof* if $f(t) = t/(1 + t^2)^{1/2}$.
(a) $1/(1+ 2t^2)^{1/2}$ (b) $t/(1 + 2t^2)^{1/2}$
(c) $(1 + 2t^2)$ (d) None of these
15. At what integral value of x will the function $\dfrac{(x^2 + 3x + 1)}{(x^2 - 3x + 1)}$ attain its maximum value?
(a) 3 (b) 4
(c) –3 (d) None of these
16. Inverse of $f(t) = (10^t - 10^{-t})/(10^t + 10^{-t})$ is
(a) $1/2 \log \{(1 - t)/(1 + t)\}$
(b) $0.5 \log \{(t - 1)/(t + 1)\}$
(c) $1/2 \log_{10} (2^t - 1)$
(d) None of these
17. If $f(x) = |x - 2|$, then which of the following is always true?
(a) $f(x) = (f(x))^2$ (b) $f(x) = f(-x)$
(c) $f(x) = x - 2$ (d) None of these

Directions for Questions 18 to 20: Read the instructions below and solve:

$f(x) = f(x - 2) - f(x - 1)$, x is a natural number
$f(1) = 0, f(2) = 1$

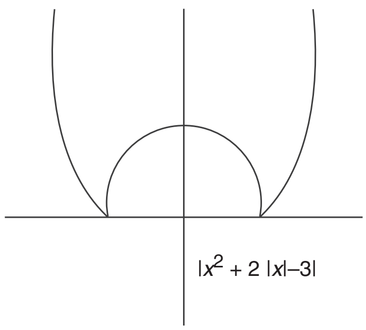
18. The value of $f(x)$ is negative for
(a) All $x > 2$
(b) All odd $x (x > 2)$
(c) For all even $x (x > 0)$
(d) $f(x)$ is always positive
19. The value of $f[f(6)]$ is
(a) 5 (b) – 1
(c) – 3 (d) – 2
20. The value of $f(6) - f(8)$ is
(a) $f(4) + f(5)$ (b) $f(7)$
(c) $-\{f(7) + f(5)\}$ (d) $-f(5)$

21. Which of the following is not an even function?
    (a) $f(x) = e^x + e^{-x}$ (b) $f(x) = e^x - e^{-x}$
    (c) $f(x) = e^{2x} + e^{-2x}$ (d) None of these
22. If $f(x)$ is a function satisfying $f(x).\ f(1/x) = f(x) + f(1/x)$ and $f(4) = 65$, what will be the value of $f(6)$?
    (a) 37 (b) 217
    (c) 64 (d) None of these

Directions for Questions 23 to 34:

Mark (a) if $f(-x) = f(x)$,
Mark (b) if $f(-x) = -f(x)$
Mark (c) if neither (a) nor (b) is true
Mark (d) if $f(x)$ does not exist at at least one point of the domain.

23.



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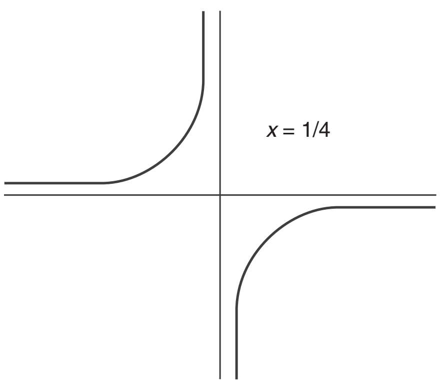

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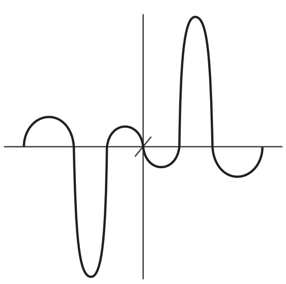

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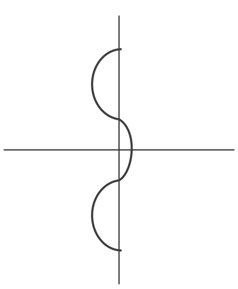

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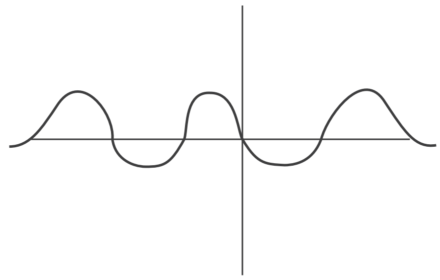

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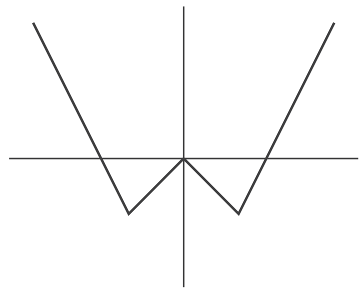

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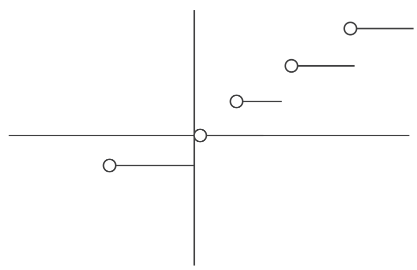

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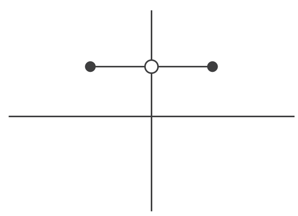

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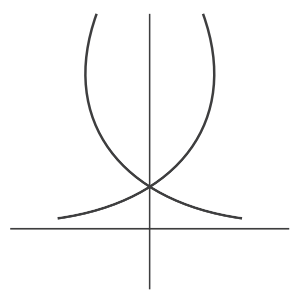
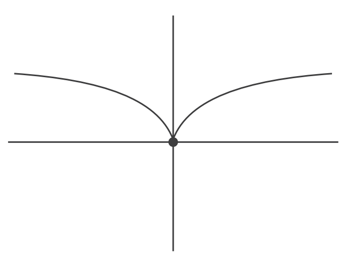
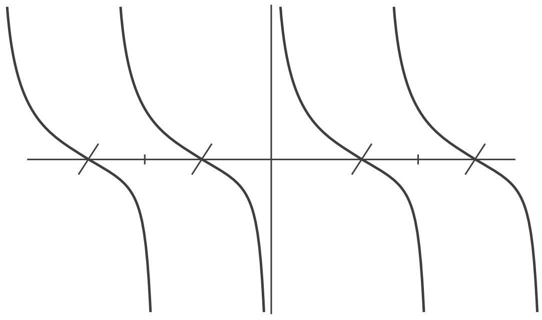
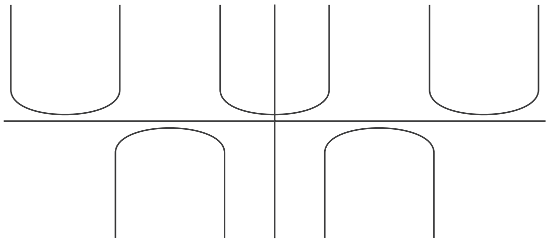


32.



33. 

34. 

Directions for Questions 35 to 40: Define the functions:

$A(x, y, z)$ = Max (max (x, y), min (y, z) min (x, z))
$B(x, y, z)$ = Max (max (x, y), min (y, z) max (x, z))
$C(x, y, z)$ = Max (min (x, y), min (y, z) min (x, z))
$D(x, y, z)$ = Min (max (x, y), max (y, z) max (x, z))
Max (x, y, z) = Maximum of x, y and z.
Min (x, y, z) = Minimum of x, y and z.

Assume that x, y and z are distinct integers.

35. For what condition will $A(x, y, z)$ be equal to Max (x, y, z)?
    (a) When x is maximum (b) When y is maximum
    (c) When z is maximum (d) Either (a) or (b)
36. For what condition will $B(x, y, z)$ be equal to Min (x, y, z)?
    (a) When y is minimum (b) When z is minimum
    (c) Either (a) or (b) (d) Never
37. For what condition will $A(x, y, z)$ not be equal to $B(x, y, z)$?
    (a) $x > y > z$ (b) $y > z > x$
    (c) $z > y > x$ (d) None of these
38. Under what condition will $C(x, y, z)$ be equal to $B(x, y, z)$?
    (a) $x > y > z$ (b) $z > y > x$
    (c) Both *a* and *b* (d) Never
39. Which of the following will always be true?
    (I) $A(x, y, z)$ will always be greater than Min (x, y, z)
    (II) $B(x, y, z)$ will always be lower than Max (x, y, z)
    (III) $A(x, y, z)$ will never be greater than $B(x, y, z)$
    (a) I only (b) III only
    (c) Both *a* and *b* (d) All the three
40. The highest value amongst the following will be
    (a) Max/Min (b) A/B
    (c) C/D (d) Cannot be determined

Directions for Questions 41 to 49: Suppose x and y are real numbers. Let $f(x, y) = |x + y|$ $F(f(x, y)) = -f(x, y)$ and $G(f(x, y)) = -F(f(x, y))$

41. Which one of the following is true?
    (a) $F(f(x, y)).G(f(x, y)) = -F(f(x, y)).\ G.(f(x, y))$
    (b) $F(f(x, y)).G(f(x, y)) \leq -F(f(x, y)).\ G.(f(x, y))$
    (c) $G(f(x, y)).f(x, y) = F(f(x, y)).\ (f(x, y)$
    (d) $G(f(x, y)).F(f(x, y)) = f(x, y).\ f(x, y)$
42. Which of the following has a^2 as the result?
    (a) $F(f(a, -a)).G(f(a, -a))$
    (b) $-F(f(a, a).G(f(a, a))/4$
    (c) $F(f(a, a)).G(f(a, a))/2^2$
    (d) $f(a, a).f(a, a)$
43. Find the value of the expression.
    $$\frac{G(f(3, 2)) + F(f(-1, 2))}{f(2, -3) + G(f(1, 2))} \ldots$$
    (a) 3/2 (b) 2/3
    (c) 1 (d) 2
44. Which of the following is equal to
    $$\frac{G(f(32, 13)) + F(f(15, -5))}{f(2, 3) + G(f(1.5, 0.5))}?$$
    (a) $\dfrac{2G(f(1, 2)) + (f(-3, 1))}{G(f(2, 6) + F(f(-8, 2))}$
    (b) $\dfrac{3.G(f(3, 4)) + F(f(1, 0))}{f(1, 1) + G(f(2, 0))}$
    (c) $\dfrac{(f(3, 4)) + F(f(1, 2))}{G(f(1, 1))}$
    (d) None of these

Now if $A(f(x, y) = f(x, y)$
$B(f(x, y) = -f(x, y)$
$C(f(x, y) = f(x, y)$
$D(f(x, y) = -f(x, y)$ and similarly
$Z(f(x, y) = -f(x, y)$

Now, solve the following:

45. Find the value of $A(f(0, 1)) + B(f(1, 2)) + C(f(2, 3)) + \ldots + Z(f(25, 26))$.
    (a) –50 (b) –52
    (c) –26 (d) None of these
46. Which of the following is true?
    (i) $A(f(0, 1)) < B(f(1, 2)) < C(f(2, 3)\ldots$
    (ii) $A(f(0, 1)).\ B(f(1, 2)) > B(f(1, 2)).C(f(2, 3)) > C(f(2, 3)).\ D(f(3, 4))$
    (iii) $A(f(0, 0)) = B(f(0, 0)) = C(f(0, 0)) = \ldots = Z(f(0, 0))$
    (a) only (i) and (ii) (b) only (ii) and (iii)
    (c) only (ii) (d) only (i)

47. If max (x, y, z) = maximum of x, y and z
Min (x, y, z) = minimum of x, y and z
$f(x, y) = |x + y|$
$F(f(x, y)) = -f(x, y)$
$G(f(x, y) = -F(f(x, y))$
Then find the value of the following expression:
Min (max $[f(2, 3), F(f(3, 4)), G(f(4, 5))]$, min $[f(1, 2), F(f(-1, 2)), G(f(1, -2))]$, max $[f(-3\ -4), f(-5\ -1), G(f(-4, -6))]$)
(a) –1 (b) –7
(c) –6 (d) –10

48. Which of the following is the value of
Max. $[f(a, b), F(f(b, c), G(f(c, d))]$
for all $a > b > c > d$?
(a) Anything but positive
(b) Anything but negative
(c) Negative or positive
(d) Any real value

49. If another function is defined as $P(x, y) = \dfrac{F(f(x, y))}{(x.\ y)}$
which of the following is second lowest in value?
(a) Value of $P(x, y)$ for $x = 2$ and $y = 1$
(b) Value of $P(x, y)$ for $x = 3$ and $y = 4$
(c) Value of $P(x, y)$ for $x = 3$ and $y = 5$
(d) Value of $P(x, y)$ for $x = 3$ and $y = 2$

50. If $f(s) = (b^s + b^{-s})/2$, where $b > 0$. Find $f(s + t) + f(s - t)$.
(a) $f(s) - f(t)$ (b) $2 f(s).f(t)$
(c) $4 f(s).f(t)$ (d) $f(s) + f(t)$

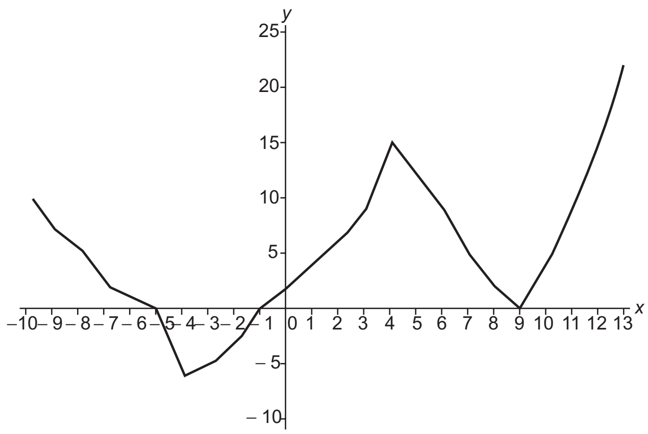
Questions 51 to 60 are all actual questions from the XAT exam.

51. A_0, A_1, A_2,… is a sequence of numbers with $A_0 = 1$, $A_1 = 3$, and $A_t = (t + 1)A_{(t-1)} - tA_{(t-2)}$, where t = 2,3,4…
Conclusion I. A8 = 77
Conclusion II. A10 = 121
Conclusion III. A12 = 145
(a) Using the given statement, only Conclusion I can be derived.
(b) Using the given statement, only Conclusion II can be derived.
(c) Using the given statement, only Conclusion III can be derived.
(d) Using the given statement, Conclusion I, II and III can be derived.
(e) Using the given statement, none of the three Conclusions I, II and III can be derived.

52. A, B, C be real numbers satisfying A < B < C, A + B + C = 6 and AB + BC + CA = 9
Conclusion I. 1 < B < 3
Conclusion II. 2 < A < 3
Conclusion III. 0 < C < 1
(a) Using the given statement, only Conclusion I can be derived.
(b) Using the given statement, only Conclusion II can be derived.
(c) Using the given statement, only Conclusion III can be derived.
(d) Using the given statement, Conclusion I, II and III can be derived.
(e) Using the given statement, none of the three Conclusions I, II and III can be derived.

53. If $F(x, n)$ be the number of ways of distributing "x" toys to "n" children so that each child receives at the most 2 toys, then $F(4, 3)$ = ___?
(a) 2 (b) 6
(c) 3 (d) 4
(e) 5

54. The figure below shows the graph of a function $f(x)$. How many solutions does the equation $f(f(x)) = 15$ have?
(a) 5 (b) 6
(c) 7 (d) 8
(e) Cannot be determined from the given graph



55. In the following question, a question is followed by two statements. Mark your answer as:
(a) If the question can be answered by the first statement alone but cannot be answered by the second statement alone;
(b) If the question can be answered by the second statement alone but cannot be answered by the first statement alone;
(c) If the question can be answered by both the statements together but cannot be answered by any one of the statements alone;
(d) If the question can be answered by the first statement alone as well as by the second statement alone;
(e) If the question cannot be answered even by using both the statements together.

A sequence of positive integers is defined as $A_{n+1} = A_n^2 + 1$ for each $n \geq 1$. What is the value of the Greatest Common Divisor of A_{900} and A_{1000}?

I. $A_0 = 1$ II. $A_1 = 2$

56. A manufacturer produces two types of products— A and B, which are subjected to two types of operations, viz., grinding and polishing. Each unit of product A takes 2 hours of grinding and 3 hours of polishing whereas product B takes 3 hours of grinding and 2 hours of polishing. The manufacturer has 10 grinders and 15 polishers. Each grinder operates for 12 hours/day and each polisher 10 hours/day. The profit margin per unit of A and B are ₹ 5/- and ₹ 7/- respectively. If the manufacturer utilises all his resources for producing these two types of items, what is the maximum profit that the manufacturer can earn?
(a) ₹ 280/- (b) ₹ 294/-
(c) ₹ 515/- (d) ₹ 550/-
(e) None of the above

57. Consider a function $f(x) = x^4 + x^3 + x^2 + x + 1$, where x is a positive integer greater than 1. What will be the remainder if $f(x^5)$ is divided by $f(x)$?
(a) 1 (b) 4
(c) 5 (d) A monomial in x
(e) A polynomial in x

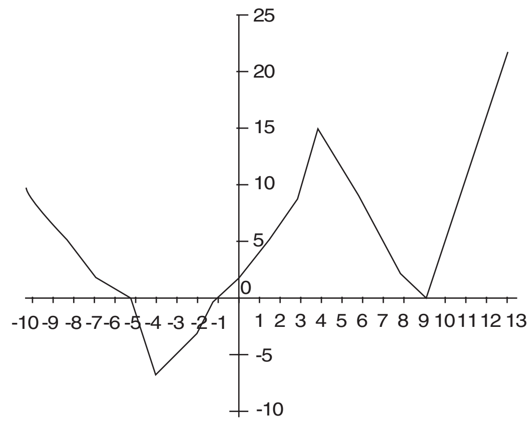
58. For all real numbers x, except $x = 0$ and $x = 1$, the function F is defined by $F\left(\frac{x}{x-1}\right) = \frac{1}{x}$
If $0 < \alpha < 90°$ then $F((\text{cosec } \alpha)^2) =$
(a) $(\sin \alpha)^2$ (b) $(\cos \alpha)^2$
(c) $(\tan \alpha)^2$ (d) $(\cot \alpha)^2$
(e) $(\sec \alpha)^2$

59. $F(x)$ is a fourth order polynomial with integer coefficients and with no common factor. The roots of $F(x)$ are –2, –1, 1, 2. If p is a prime number greater than 97, then the largest integer that divides $F(p)$ for all values of p is:
(a) 72 (b) 120
(c) 240 (d) 360
(e) None of the above.

60. If $x = (9 + 4\sqrt{5})^{48} = [x] + f$, where $[x]$ is defined as integral part of x and f is a fraction, then $x(1 - f)$ equals–
(a) 1
(b) Less than 1
(c) More than 1
(d) Between 1 and 2
(e) None of the above

61. If $3f(x+2) + 4f\left(\frac{1}{x+2}\right) = 4x$, $x \neq -2$, then $f(4) =$
(a) 7 (b) $-52/7$
(c) 8 (d) None of the above.

62. The figure below shows the graph of a function f(x). How many solutions does the equation $f(f(x)) = 15$ have for the span of the graph shown?
(a) 5 (b) 6
(c) 7 (d) 8



63. If $f(x) = (x-6), g(x) = \frac{(x-9)(x-1)}{(x-7)(x-3)}$
How many real values of x satisfy the equation $[f(x)]^{g(x)} = 1$

64. A continuous function $f(x)$ is defined for all real values of x, such that $f(x) = 0$, only for two distinct real values of x. Only for two distinct real values of x. It is also known that $f(4) + f(6) = 0$, $f(5).f(7) > 0$, $f(4).f(8) < 0$, $f(1) > 0$ & $f(2) < 0$
Which of the following statement must be true.
(a) $f(1)f(2)f(4) < 0$ (b) $f(5)f(6)f(7) < 0$
(c) $f(1)f(3)f(4) > 0$ (d) None of these

65. $f(x) = 7[x] + 4\{x\}$
where $[x]$ = Greatest integer less than or equals to x.
$\{x\} = x - [x]$
How many real values of x satisfy the equation $f(x) = 12 + x$
(a) 0 (b) 1
(c) 2 (d) none of these

66. How many non-negative integer solutions (x, y) are possible for the equation $x^2 - xy + y^2 = x + y$ such that $x \geq y$.
(a) 1 (b) 2
(c) 3 (d) 4

67. A function 'g' is defined for all natural numbers $n \geq 2$ as $\frac{g(n-1)}{g(n)} = \frac{n}{n-1}$
If $g(1) = 2$ then what is the value of
$$\frac{\left[\frac{1}{g(1)} \times \frac{1}{g(2)} \times \frac{1}{g(3)} \times \ldots \frac{1}{g(8)}\right]}{\left[\frac{1}{g(1)} + \frac{1}{g(2)} + \frac{1}{g(3)} + \ldots \frac{1}{g(8)}\right]}$$
(a) $8!/2^8$ (b) $8!/(2^8.18)$
(c) $8!/18.2^7$ (d) $8!/3^7.24$

68. $f(x)$ is a polynomial of degree 77 which when divided by $(x - 1)$, $(x - 2)$, $(x - 3)$, $(x - 4)$,…, $(x - 77)$ it leaves 1, 2, 3, …, 77 respectively as the remainders. Find the value of $f(0) + f(78)$?

(a) 77 (b) 78
(c) −77 (d) 78!

69. A function $f(n)$ is defined as $f(n - 1)[2 - f(n)] = 1$ for all natural numbers 'n'. If $f(1) = 3$, then find the value of $f(21)$
(a) 42/41 (b) 45/43
(c) 43/41 (d) 47/45

Directions for question number 70&71: If $f(x) = 10[x] + 22\{x\}$, where $[x]$ denotes the largest integer less than or equal to x and $\{x\} = x - [x]$, (i.e. the fractional part of x) then answer the following questions.

70. How many solutions does the equation $f(x) = 250$ have?
(a) 0 (b) 1
(c) 2 (d) 3

71. Sum of all possible values of x is

72. If $f(x + 1) = f(x) - f(x-1)$ and $f(5) = 6$ and $f(17) = 2f(16)$ then $f(17) = ?$
(a) 5 (b) 6
(c) 16 (d) 18

73. If $h(x)$ is a positive valued function and
$$\frac{h(x)}{h(x-1)} = \frac{h(x-2)}{h(x+1)} \text{ for all } x \geq 0$$
If $h(56) = 16$ and $h(52) = 4$ then $h(54) = ?$

74. $f(x) = 1 - \dfrac{2}{(x+1)}$
If $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, $f^4(x) = f(f(f(f(x))))$ and so on then find $f^{802}(x)$ at $x = -1/2$

Directions for question number 75 – 76: If $\log_3(x + y) + \log_3(x - y) = 3$, where x and y are positive integers then answer the following questions:

75. If $y > 0$ then how many different pairs of (x, y) are possible?

76. The Maximum value of $x + y$ = ____

Direction for 77&78:

$f(x) = |x + 2|$
$g(x) = x^2 - 7x + 10$
$h(x) = \min(f(x), g(x))$

77. For how many positive integer values of x, is $h(x) \leq 0$?

78. Find the sum of all integer values of x for which $h(x) < 0$.

79. If $[x]$ denotes the greatest integer less than or equal to x. If p and q are two distinct real numbers and $[2p - 3] = q + 7$, $[3q + 1] = p + 6$ then the value of $p^2 \times q^2$ is:

80. If $f(a) = 3^a$ and $f(a + 1) = 3^{(a+1)} + 4$, where 'a' is an odd number, what is the value of:
$$\frac{1}{4}[f(1) + f(2) + f(3) + f(4) + \ldots + f(72)]$$
(a) $\frac{3}{8}(3^{72} - 1) + 36$
(b) $\frac{3}{8}(3^{72} - 1) - 36$
(c) $\frac{3}{8}(3^{72} + 1) + 36\frac{3}{8}(3^{72} + 1) + 36$
(d) None of these.

Directions for question number 81 and 82:

$f(x) = \dfrac{x^2}{4}$ and $g(x) = 2x^{[3x]} + 2$ where $[x]$ is the greatest integer less than or equal to 'x'. Then answer the following questions:

81. Which of the following statement is true about $g(f(x))$?
(a) $g(f(x))$ is neither even nor odd.
(b) $g(f(x))$ is maximum for $x = 11$
(c) $g(f(x))$ will have its' minimum for a value of x that obeys $\dfrac{3x^2}{4} \leq 1$
(d) None of these

82. Which of the following is the value of $g(f(x))$ at $x = 2$?
(a) 66 (b) 34
(c) 18 (d) 64

83. The area bounded between $|x + y| = 2$ and $|x - y| = 2$ is:
(a) 2 (b) 4
(c) 6 (d) 8

Directions for question number 84 to 86:
If $f(x) = |x| + |x + 4| + |x + 8| + |x + 12| + \ldots + |x + 4n|$, where x is an integer and n is a positive integer.

84. If $n = 8$, what is the minimum value of $f(x)$?

85. If $n = 7$, then for how many values of x, $f(x)$ is minimum.

86. For $n = 9$ which of the following statements is true?
(a) $f(x)$ will be minimum for a total 5 values of x.
(b) $f(-17) = f(-19)$
(c) Minimum value of $f(x)$ is 100
(d) All of these

87. Find the area enclosed by the graph $|x| + |y| = 3$

88. Find the area enclosed by curve $|x - 2| + |y - 3| = 3$

Directions for question numbers 89 and 90:

$8\{x\} = x + 2[x]$
$\{x\}$ denotes the fractional part of x.
$[x]$ denotes the greatest integer less than or equals to x.

89. For how many positive values of x, is the given equation true?

90. Find the difference of the greatest and the least value of x for which the given equation is true? (till two digits after the decimal point)

Directions for question numbers 91 and 92:

If $f(x)=\dfrac{4^{x-1}}{4^{x-1}+1}$ and $g(x) = 2x$, then answer the following questions.

91. $fog\left(\dfrac{1}{4}\right)+fog\left(\dfrac{3}{4}\right)=?$

92. $fog\left(\dfrac{1}{2}\right)+fog\left(\dfrac{1}{4}\right)+fog\left(\dfrac{1}{8}\right)+f\left(\dfrac{1}{16}\right)+f\left(\dfrac{3}{4}\right)+fog\left(\dfrac{7}{8}\right)+fog\left(\dfrac{15}{16}\right)=?$

Directions for questions numbers 93 – 96:

If for a positive integer x, $f(x + 2) = f(x) + 2(x + 1)$, when x is even and $f(x + 2) = f(x) + 1$, when x is odd. If $f(1) = 1$ and $f(2) = 5$. Then answer the following questions.

93. $f(24) = ?$

94. $\left[\dfrac{f(14)}{f(11)}\right]=?$, where [ ] denotes the greatest integer function

95. Which of the following statement is true?
 (a) For even value of x value of $f(x)$ is also even
 (b) For odd value of x, value of $f(x)$ is odd.
 (c) For even value of x, value of $f(x)$ is odd.
 (d) None of these

96. Value of $f(f(f(f(3)))) + f(f(f(2))) = ?$

Direction for question numbers 97 to 98:

$F(x)$ is a 6^{th} degree polynomial of x. It is given that $F(0) = 0$, $F(1) = 1$, $F(2) = 2$, $F(3) = 3$, $F(4) = 4$, $F(5) = 5$, $F(6) = 7 =$

97. Find the value of $F(8) =$

98. If x is a negative integer then the minimum value of $F(x) = ?$

99. If $g(x + y) = g(x).g(y)$ and $g(1) = 5$, then find the value of $g(1) + g(2) + g(3) + g(4) + g(5)$.

100. In the previous question if

$$\sum_{p=1}^{n} g(q+p)=\frac{1}{4}\left(5^{p+3}-125\right)$$

Where 'p' is a positive integer then $q =$

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (d)	3. (a)	4. (c)
5. (a)	6. (d)	7. (b)	8. (a)
9. (c)	10. (b)	11. (d)	12. (b)
13. (d)	14. (a)	15. (a)	16. (c)
17. (a)	18. (d)	19. (b)	20. (c)
21. (b)	22. (b)	23. (a)	24. (c)
25. (a)	26. (d)	27. (c)	28. (c)
29. (c)	30. (d)	31. (a)	32. (a)
33. (a)	34. (a)	35. (b)	36. (c)
37. (d)	38. (d)	39. (c)	40. (a)
41. (a)	42. (d)	43. (d)	44. (a)
45. (a)	46. (c)	47. (d)	48. (d)
49. (d)	50. (d)	51. (b)	52. (a)
53. (a)	54. (b)	55. (d)	56. (d)
57. (c)	58. (b)	59. (d)	60. (c)
61. (a)	62. (a)	63. (c)	64. 3
65. 1	66. 1	67. 7.86	68. 13.75
69. 12	70. 1.5	71. (d)	72. 2
73. (d)	74. 8	75. (d)	76. (c)
77. 0.14	78. 2	79. (b)	80. (b)
81. 9	82. 10	83. 0	84. (c)
85. –2	86. (b)	87. (c)	88. (a)
89. (b)	90. (b)	91. (c)	92. (c)
93. 3	94. (c)	95. (c)	

Level of Difficulty (II)

1. (a)	2. (d)	3. (a)	4. (d)
5. (d)	6. (c)	7. (c)	8. (b)
9. (c)	10. (c)	11. (a)	12. (b)
13. (b)	14. (c)	15. (c)	16. (a)
17. (d)	18. (c)	19. (a)	20. (c)
21. (c)	22. (d)	23. (b)	24. (a)
25. (d)	26. (d)	27. (c)	28. (a)
29. (c)	30. (d)	31. (b)	32. (d)
33. (b)	34. (a)	35. (c)	36. (d)
37. (c)	38. (a)	39. (c)	40. (b)
41. (c)	42. (b)	43. (c)	44. (b)
45. (c)	46. (c)	47. (b)	48. (c)
49. (d)	50. (a)	51. (a)	52. (c)
53. (c)	54. (b)	55. (b)	56. (c)
57. (b)	58. (c)	59. (c)	60. (a)
61. (d)	62. (a)	63. (b)	64. (c)
65. (d)	66. (b)	67. –8/3	68. 1
69. 30.8	70. 4	71. 440/3	72. 1054
73. 1999997	74. 0	75. (c)	76. (b)
77. (a)	78. 14	79. 625	80. 216
81. (a)	82. 12	83. (c)	84. (a)
85. (d)	86. 1.25	87. 3/16	88. (d)
89. (c)	90. (c)		

Level of Difficulty (III)

1. (d)	2. (d)	3. (c)	4. (d)
5. (d)	6. (d)	7. (d)	8. (b)
9. (a)	10. (a)	11. (c)	12. (c)
13. (a)	14. (b)	15. (a)	16. (b)
17. (d)	18. (b)	19. (c)	20. (b)
21. (b)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (b)	28. (a)
29. (c)	30. (a)	31. (d)	32. (a)
33. (d)	34. (a)	35. (d)	36. (d)
37. (c)	38. (d)	39. (c)	40. (d)
41. (b)	42. (b)	43. (c)	44. (b)
45. (c)	46. (b)	47. (a)	48. (b)
49. (b)	50. (b)	51. (e)	52. (a)
53. (b)	54. (e)	55. (d)	56. (b)
57. (c)	58. (b)	59. (d)	60. (a)
61. (b)	62. (c)	63. 3	64. (c)
65. (b)	66. (d)	67. (b)	68. (b)
69. (c)	70. (d)	71. 73.36	72. (b)
73. 8	74. 2	75. 2	76. 27
77. 4	78. 7	79. 784	80. (a)
81. (c)	82. (b)	83. (d)	84. 80
85. 5	86. (d)	87. 18	88. 18
89. 2	90. 2.86	91. 1	92. 3.5
93. 291	94. 16	95. (c)	96. 4
97. 36	98. 0	99. 3905	100. 2

Solutions and Shortcuts

Level of Difficulty (I)

1. $y = |x|$ will be defined for all values of x. From = $-\infty$ to $+\infty$
   Hence, option (b).
2. For $y = \sqrt{x}$ to be defined, x should be non-negative. i.e. $x \geq 0$.
3. Since the function contains *a* $\sqrt{x}$ in it, $x \geq 0$ would be the domain.
4. For $(x - 2)^{1/2}$ to be defined $x \geq 2$.
   For $(8 - x)^{1/2}$ to be defined $x \leq 8$.
   Thus, $2 \leq x \leq 8$ would be the required domain.
5. $(9 - x^2) \geq 0 \Rightarrow -3 \leq x \leq 3$.
6. The function would be defined for all values of x except where the denominator viz: $x^2 - 4x + 3$ becomes equal to zero.
   The roots of $x^2 - 4x + 3 = 0$ being 1, 3, it follows that the domain of definition of the function would be all values of x except $x = 1$ and $x = 3$.
7. $f(x) = x$ and $g(x) = (\sqrt{x})^2$ would be identical if $\sqrt{x}$ is defined.
   Hence, $x \geq 0$ would be the answer.
8. $f(x) = x$ is defined for all values of x.
   $g(x) = x^2/x$ also returns the same values as $f(x)$ except at $x = 0$ where it is not defined.

Hence. option (a).

9. $f(x) = \sqrt{x^3} \Rightarrow f(3x) = \sqrt{(3x)^3} = 3\sqrt{3x^3}$.
   Option (c) is correct.
10. $7\, f(x) = 7\, e^x$.
11. While $\log x^2$ is defined for $-\infty < x < \infty$, $2 \log x$ is only defined for $0 < x < \infty$. Thus, the two functions are identical for $0 < x < \infty$.
12. y – axis by definition.
13. Origin by definition.
14. x^{-8} is even since $f(x) = f(-x)$ in this case.
15. $(x + 1)^3$ is not odd as $f(x) \neq -f(-x)$.
16. $dy/dx = 2x + 10 = 0 \Rightarrow x = -5$.
17. Required value $= (-5)^2 + 10(-5) + 11$
    $= 25 - 50 + 11 = -14$.
18. Since the denominator $x^2 - 3x + 2$ has real roots, the maximum value would be infinity.
19. The minimum value of the function would occur at the minimum value of $(x^2 - 2x + 5)$ as this quadratic function has imaginary roots.
    For $y = x^2 - 2x + 5$
    $dy/dx = 2x - 2 = 0 \Rightarrow x = 1$
    $\Rightarrow \quad x^2 - 2x + 5 = 4$.
    Thus, minimum value of the argument of the log is 4.
    So minimum value of the function is $\log_2 4 = 2$.
20. $y = 1/x + 1$
    Hence, $y - 1 = 1/x$
    $\Rightarrow \quad x = 1/(y - 1)$
    Thus $f^{-1}(x) = 1/(x - 1)$.

21–23.

$$f(1) = 0, f(2) = 1,$$
$$f(3) = f(1) - f(2) = -1$$
$$f(4) = f(2) - f(3) = 2$$
$$f(5) = f(3) - f(4) = -3$$
$$f(6) = f(4) - f(5) = 5$$
$$f(7) = f(5) - f(6) = -8$$
$$f(8) = f(6) - f(7) = 13$$
$$f(9) = f(7) - f(8) = -21$$

21. 13
22. $-8 + 2 = -6$
23. $0 + 1 - 1 + 2 - 3 + 5 - 8 + 13 - 21 = -12$.
24. For any ${}^nC_r$, n should be positive and $r \geq 0$.
    Thus, for positive x, $5 - x \geq 0$
    $\Rightarrow x = 1, 2, 3, 4, 5$.

Directions for Questions 25 to 38: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things. Either the function is returning two values for one value of x. (as in questions 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).

We see even functions in: 25, 31, 32, 33 and 34, [Symmetry about the y axis].

We see odd functions in question 35.

While the figures in Questions 27, 28, 29 and 36 are neither odd nor even.

39. {[(3@4)! (3 #2)] @ [(4!3) @ (2 # 3)]}
    {[(3.5) ! (5)] @ [(0.5) @ (–5)]}
    {[–0.75] @ [–2.25]} = –1.5.
40. (7) @ (– 0.5) = 3.25.
41. 0 @ 0.5 = 0.25. Thus, a
42. $b = (1)\,(4) = 4$.
    $$C = \frac{(16)}{(1)\,(4)}$$
    $16/4 = 4$
    Hence, both (b) and (c).
43. (a) will always be true because $(a + b)/2$ would always be greater then $(a - b)/2$ for the given value range.
    Further, $a^2 - b^2$ would always be less than $a^3 - b^3$.
    Thus, option (d) is correct.

44 – 48.

44. Option $a = (a - b)\,(a + b) = a^2 - b^2$
45. Option $a = (a^2 - b^2) + b^2 = a^2$.
46. $3 - 4 \times 2 + 4/8 - 2 = 3 - 8 + 0.5 - 2 = -6.5$
    (using BODMAS rule)
47. The maximum would depend on the values of a and b. Thus, cannot be determined.
48. The minimum would depend on the values of a and b. Thus, cannot be determined.
49. Any of $(a + b)$ or a/b could be greater and thus we cannot determine this.
50. Again $(a + b)$ or a/b can both be greater than each other depending on the values we take for a and b. E.g. for $a = 0.9$ and $b = 0.91$, $a + b > a/b$.
    For $a = 0.1$ and $b = 0.11$, $a + b < a/b$
51. Given that $F(n - 1) = \dfrac{1}{(2 - F(n))}$, we can rewrite the expression as $F(n) = (2F(n-1) - 1)/((F(n-1))$.
    For $n = 2$: $F(2) = \dfrac{6-1}{3} \Rightarrow F(2) = \dfrac{5}{3}$.
    The value of $F(3)$ would come out as 7/5 and $F(4)$ comes out as 9/7 and so on. What we realise is that for each value of n, after and including $n = 2$, the value of $F(n) = \dfrac{2n+1}{2n-1}$.
    This means that the greatest integral value of $F(n)$ would always be 1 for $n = 2$ to $n = 1000$.

Thus, the value of the given expression would turn out to be:

$3 + 1 \times 999 = 1002$. Option (b) is the correct answer.

52. From the solution to the previous question, we already know how the value of the given functions at $n = 1, 2, 3$ and so on would behave.

Thus, we can try to see what happens when we write down the first few terms of the expression:

$F(1) \times F(2) \times F(3) \times F(4) \times \ldots F(1000)$

$= 3\times\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\cdots\times\frac{2001}{1999} = 2001$.

53. Since $f(0) = 15$, we get $c = 15$.

Next, we have $f(3) = f(-3) = 18$. Using this information, we get:

$9a + 3b + c = 9a - 3b + c \rightarrow 3b = -3b$

$\therefore\quad 6b = 0 \rightarrow b = 0$.

Also, since

$f(3) = 9a + 3b + c = 18 \rightarrow$ we get: $9a + 15 = 18 \rightarrow a = 1/3$

The quadratic function becomes $f(x) = x^2/3 + 15$. $f(12) = 144/3 + 15 = 63$.

54. What you need to understand about $M(x^2\theta y^2)$ is that it is the square of the sum of two squares. Since $M(x^2\theta\ y^2) = 361$, we get $(x^2 + y^2)^2 = 361$, which means that the sum of the sqaures of x and y viz. $x^2 + y^2 = 19$. (Note it cannot be –19 as we are talking about the sum of two squares, which cannot be negative under any circumstance).

Also, from $M(x^2\psi y^2) = 49$,we get $(x^2 - y^2)^2 = 49$, $\rightarrow (x^2 - y^2) = \pm 7$

Based on these two values, we can solve for two distinct situations:

(a) When $x^2 + y^2 = 19$ and $x^2 - y^2 = 7$, we get $x^2 = 13$ and $y^2 = 6$

(b) When $x^2 + y^2 = 19$ and $x^2 - y^2 = -7$, we get $x^2 = 6$ and $y^2 = 13$

In both cases, we can see that the value of: $((x^2\, y^2) + 3)$ would come out as $13 \times 6 + 3 = 81$ and the square root of its value would turn out to ±9. Option (b) is correct.

55. The first thing you need to understand while solving this question is that, since $[m]$ will always be integral, hence $\Psi(4x + 5)$ will also be integral. Since $\Psi(4x + 5) = 5y + 3$, naturally, the value of $5y + 3$ will also be integral. By a similar logic, the value of x will also be an integer considering the second equation: $\Psi(3y + 7) = x + 4$.

Using, this logic we know that $\Psi(4x+5) = 4x+5$ (because, whenever m is an integer the value of $[m] = m$).

This leads us to two linear equations as follows:

$4x + 5 = 5y + 3$ …(i)

$3y + 7 = x + 4$ …(ii)

Solving simultaneously, we will get: $x = -3$ and $y = -2$. Thus, $x^2 \times y^2 = 9 \times 4 = 36$.

56. Since $f(128) = 4$, we can see that the product of $f(256).f(0.5) = f(256 \times 0.5) = f(128) = 4$.

Similarly, the products $f(1).f(128) = f(2).f(64) = f(4).f(32) = f(8).f(16) = 4$.

Thus, M = 4×4×4×4×4 = 1024.

Option (d) is the correct answer.

57. The only values of x and y that satisfy the equation $4x + 6y = 20$ are $x = 2$ and $y = 2$ (since, x, y are non negative integers). This gives us: $4 \le M/2^{2/3}$. M has to be greater than $2^{8/3}$ for this expression to be satisfied. Option (c) is correct.

58. $\theta\ (\Psi(-7)) = \theta(-2) = 14$. Option (b) is correct.

59. $F(2b) = F(b + b) = F(b).F(b) \div 2 = (F(b))^2 \div 2$

Similarly, $F(3b) = F(b + b + b) = F(b + b).F(b) \div 2 = \{F(b)^2 \div 2\}.\{F(b)\} \div 2 = (F(b))^3 \div 2^2$

Similarly, $F\ (4b) = (F(b))^4 \div 2^3$.

Hence, $F(12b) = (F(b))^{12} \div 2^{11}$. Option (d) is correct.

60. To test for a reflexive function as defined in the problem use the following steps:

Step 1: To start with, assume a value of 'b' and derive a value for 'a' using the given function.

Step 2: Then, insert the value you got for 'a' in the first step into the value of 'b' and get a new value of 'a'. This value of 'a' should be equal to the first value of 'b' that you used in the first step. If this occurs the function would be reflexive. Else it is not reflexive.

Checking for the expression in (i) if we take $b = 1$, we get:

$a = 8/1 = 8$. Inserting, $b = 8$ in the function gives us $a = 29/29 = 1$. Hence, the function given in (i) is reflexive.

Similarly checking the other two functions, we get that the function in (ii) is not reflexive while the function in (iii) is reflexive.

Thus, Option (c) is the correct answer.

61. $f(g(x)) = f(|3x-2|) = \frac{1}{|3x-2|}$

Option (a) is correct.

62. $f(g(x)) = f(|x|) = |x|^2 + \frac{1}{|x|^2}$. This function would take the same values when you try to use a positive value or a negative value of x. For instance, if you were to put x as 2 you would get the same answer as if you were to use x as –2. Hence, $f\,(g(x))$ is an even function.

63. For this question, you would have to go through each of the options checking them for their correctness in order to identify the correct answer. Thus,

For option (a): $g(x)+(g(x))^2 = |x|+|x|^2$

$\Rightarrow f(x) \neq g(x)+(g(x))^2$. Hence, option (*a*) is not correct.

For option (b): $f(x) = x^2 + \frac{1}{x^2}, f(g(x)) = |x|^2 + \frac{1}{|x|^2}$

$f(x) \neq -f(g(x))$. Hence, option (*b*) is not correct.

For option (*c*): $g(f(x)) = \left|x^2 + \frac{1}{x^2}\right|$

$f(g(x)) = |x|^2 + \frac{1}{|x|^2}$ which is the same as $\left|x^2 + \frac{1}{x^2}\right|$.

Hence $f(g(x)) = g(f(x))$

∵ Hence option (c) is correct.

64. $f(x) = f(-x)$

$g(x) = g(-x)$

$h(x) = -h(-x)$

$t(x) = t(-x)$

Therefore 3 functions are even.

65. $f(x) = f(-x)$

$\Rightarrow h(f(x)) = h(f(-x))$

$\Rightarrow$ Hence, $h(f(x))$ is an even function. So the correct answer is 1.

66. $t(x) = t(-x)$

Hence, $h(t(x)) = h(t(-x))$

$\Rightarrow h(t(x))$ is an even function. Correct answer is 1.

67. $f(2) = \frac{2^2+1}{2-1} = 5$

$f(f(2)) = f(5) = \frac{5^2+1}{5-1} = \frac{26}{4}$

$f(f(f(2))) = f\left(\frac{26}{4}\right) = \frac{\left(\frac{26}{4}\right)^2+1}{\frac{26}{4}-1} = \frac{\frac{676+16}{16}}{\frac{22}{4}}$

$= \frac{692}{16} \times \frac{4}{22} = 7.86$

68. $D(3,4) = \frac{3}{4} = 0.75$

$S(2, D(3,4)) = S(2, 0.75) = 2.75$

$P(S(2, D(3,4)), 5) = P(2.75, 5) = 2.75 \times 5 = 13.75$

69. $P(2, 3) = 2 \times 3 = 6$

$D(4, 2) = 4 \div 2 = 2$

$S(P(2,3), D(4,2)) = S(6,2) = 8$

$t(1,5) = |1-5| = 4$

$S(8,4) = 8+4 = 12$

Solution for 70 to 72:

70. $[(5\ P\ 6)Q(4\ Q\ 2)]S\ (3\ S\ 1)$

$= [(|5-6|)Q\ (4/2)]S\ (1/3)$

$= [1\ Q\ 2]S\left(\frac{1}{3}\right)$

$= [1\ R\ 2]S\left(\frac{1}{3}\right)$

$= [1 \times 2)S\left(\frac{1}{3}\right)$

$= 2\ \text{S}\ \frac{1}{3}$

$= \left(\frac{1}{2\times\frac{1}{3}}\right) = \frac{3}{2} = 1.5$

71. For this question, we would need to check each option and select the one that is true.

Checking option (*a*) we can see that:

$(4P2) = (4Q2) = 4/2 = 2$, $(2P4) = |2-4| = 2$

So, option (*a*) is incorrect.

Checking option (*b*) we get:

$(4Q2) = 4/2 = 2$, $2R4 = 2 \times 4 = 8$

Hence, Option (*b*) is incorrect.

Checking option (*c*) we get:

$(6Q3) = 6/3 = 2$

$2\ S\ (0.5) = 1/(2 \times 0.5) = 1$

Hence, Option (*c*) is incorrect.

Option (*d*) is correct.

72. $(5P3)\text{Q}(4S2) = (5\ Q\ 3)Q\left(\frac{1}{4.2}\right)$

$= \frac{5}{3}Q\frac{1}{8}$

$= \frac{40}{3}$

$20Q1.5 = 20 \div 1.5 = \frac{40}{3}$, therefore the operator Q should replace 'K' in the equation.

Solutions for 73 – 75

73. $f(3,4) = [3]+\{4\} = 3+4 = 7$

$g(3.5, 4.5) = [4.5]-\{3.5\} = 4-4 = 0$

$i(f(3,4), g(3.5,4.5)) = i(7,0) = -7$

Hence, option (*d*) is correct.

74. $a^3 = 64 \Rightarrow a = 4$

$b^2 = 16 \Rightarrow b = 4 or -4$

When $a = 4$, $b = 4$

$f(4,4) = [4]+\{4\} = 4+4 = 8$

$g(4,4) = [4]-\{4\} = 4-4 = 0$

But these values do not satisfy the condition in the problem that $8+f(a,b)=-g(a,b)$. Hence, we will try to use $a = 4$ and $b = -4$ to see whether that gives us the right set of values for the conditions to be matched.

When $a = 4$, $b = -4$

$$f(4,-4)=[4]+\{-4\}=4-4=0$$

$$g(4,-4)=[-4]-\{4\}=-4-4=-8$$

The given condition $8+f(a,b)=-g(a,b)$ is satisfied here. Hence, $a = 4$ & $b = -4$. Therefore $a - b = 4 - (-4) = 8$

75. $f(1.2,-2.3)+g(-1.2,2.3)$

$$=[1.2]+\{-2.3\}+[2.3]-\{-1.2\}$$

$=1 - 2 + 2 + 1$

$= 2$

$= i(a,-1.3)=\{-a-1.3\}$

For $a=-2.4 \Rightarrow i(-2.4,-1.3)=\{2.4-1.3\}=\{1.1\}=2$

Hence, option (*d*) is correct.

Solutions for 76 & 77:

76. Given: $xPy=\dfrac{1}{1+\dfrac{y}{x}}=\dfrac{x}{x+y}$ and $xQy=1+\dfrac{x}{y}=\dfrac{x+y}{y}$.

From this point you would need to read the options and check the one that gives you a value of $\dfrac{x}{y}$. It is easily evident here that:

$$(xPy)\times(xQy)=\frac{x}{x+y}\times\frac{x+y}{y}=\frac{x}{y}$$

Hence, Option (c) is correct.

77. $S(2,3)=(2P3)P(2Q3)$

$$=\left(\frac{2}{2+3}\right)P\left(\frac{2+3}{3}\right)$$

$$=\frac{2}{5}P\frac{5}{3}=\frac{\frac{2}{5}}{\frac{2}{5}+\frac{5}{3}}=0.19$$

Solutions for 78 – 80

78. (1 + min(2*A*3, 1*C*2))B(max(1*A*2), 1*C*1)

=[1 + min(1, 2)]B max(1, 1)

(1 + 1)B(1^2) = 2B1 = [2 ÷ 1] = 2

79. max(7*A*3, 16*B*2) = max(4, 8) = 8^2 = 64

Now by checking the options we get only option (b) that gives us the correct value.

(32*B*2)*C*(min(4, 8))

|[32 ÷ 2] × 4| = |16 × 4| = 64

Hence option (*b*) is correct.

80. max(3, 4) ÷ min(8, 4) = 4^2 ÷ 4 = 4. Checking the options we see:

Option (*a*): 8*A*2 = |8 − 2| = 6

Option (*b*): 28*B*7 = 28 ÷ 7 = 4

Option (*c*): 4*C*2 = |4 × 2| = 8

Hence option (*b*) is correct.

Solution for 81-85:

81. $f(1,3,5,7)+g(2,4,6,8)=1+8=9$

$$h[aK,K]=\left[\frac{aK}{K}\right]=[a]=a,[a\in I]$$

$\Rightarrow a = 9$

82. $t(1,2,3,4)=1\times2\times3\times4=24$

$i(1,2,3,4)=1+2+3+4=10$

$f(t(1,2,3,4),i(1,2,3,4))=f(24,10)=10$

83. *f*(5,6,7,8) = 5, *i*(1,2,3,4,) = 1 + 2 + 3 + 4 = 10

$$h(5,10)=\left[\frac{5}{10}\right]=[0.5]=0$$

84. $P = f(2,3,4,6) = 2$

$Q = g(1,2,3,4) = 4$

$$R=h(8,4)=\left[\frac{8}{4}\right]=2$$

$S = t(1, 2, 3, 4) = 1 \times 2 \times 3 \times 4 = 24$

$T = i(4,5,6) = 4 + 5 + 6 = 15$

$\therefore P = R < Q < T < S.$

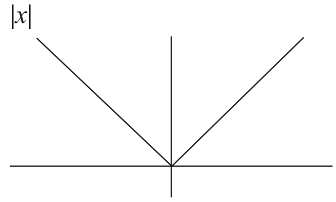
Option (*c*)is correct

85. $f(1,2,3)=1, g(2,3,4)=4, f(0,1,2)$

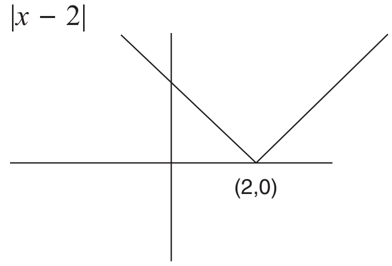
$=0, g(-3,-2)=-2$

$f(1,4,0,-2)=-2.$

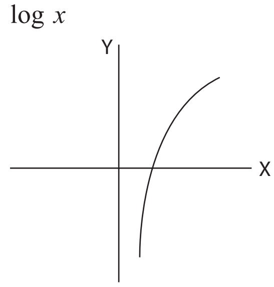
86. |*x*|



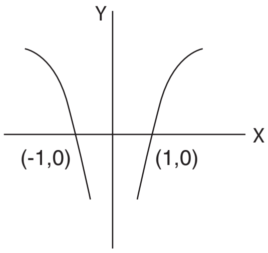
|*x* − 2|



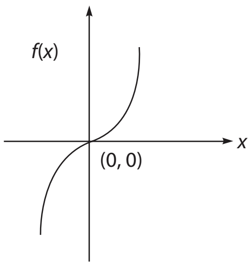
87. log *x*



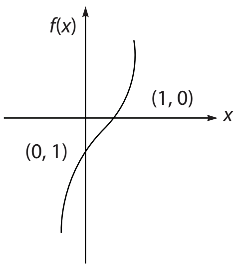
$f(x) \to f(|x|)$
Take mirror image about y-axis
$\log|x| \to$



88. $x^3 \to$

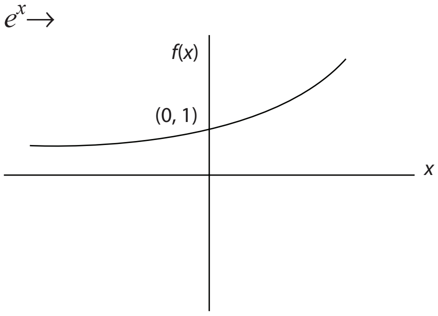


$(x-1)^3 \to$
[Shift curve one unit right]

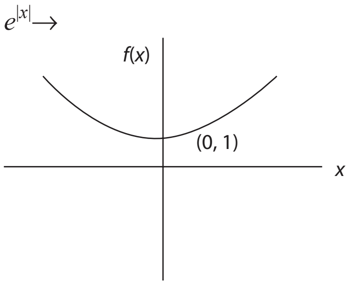


Hence option (a) is correct.

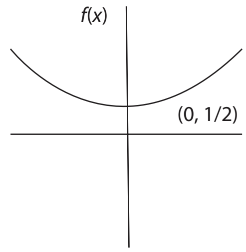
89. $e^x \to$



$e^{|x|} \to$



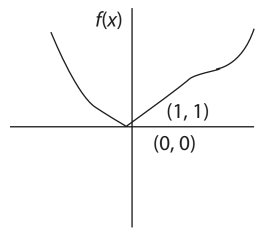
$\frac{e^{|x|}}{2} \to$



Option (b) is correct.

90. $\max\left(x, x^2\right) = \begin{cases} x^2, \text{for} -\infty < x \le 0 \\ x, \text{for } 0 \le x \le 1 \\ x^2, \text{for } x > 1 \end{cases}$

$\Rightarrow f(x) = \max(x, x^2) \Rightarrow$



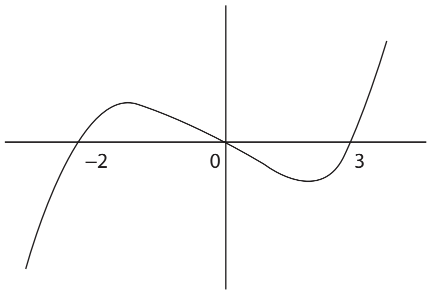
Option (b) is correct.

91. When $f(x)$ and $g(x)$ both are odd then $S(x) = f(x) + g(x)$
$S(-x) = f(-x) + g(-x) = -[f(x) + g(x)]$, $S(x)$ is an odd function. This conclusion rejects option (a).
Their product $P(x) = f(x).g(x)$
$P(-x) = f(-x).g(-x) = [-f(x)][-g(x)] = f(x)g(x)$. $P(x)$ is an even function. This is what is being said by the option (c). Hence, it is the correct answer.
If we check for option (b) we can see that: when $f(x)$ and $g(x)$ both are even then $S(x) = f(x) + g(x)$
$S(-x) = f(-x) + g(-x) = [f(x) + g(x)]$, $S(x)$ is an even function.
Hence only option (c) is true.

92. $f(g(-x)) = f(-g(x)) = f(g(x))$
$\therefore$ $f(g(x))$ is an even function.
$g(f(x)) = g(f(-x)) = g(f(x))$
$\therefore$ $g(f(x))$ is an even function.
Hence option (c) is true.

93. $f(x) = x^3 - x^2 - 6x$
$= (x + 2)\, x\, (x - 3)$
$\Rightarrow f(x) = 0$
$\Rightarrow (x + 2)\, x\, (x - 3) = 0$
$X = 0, 3, -2$. There are 3 such values.

94. Curve of $f(x)$ will look like this:



For interval (–2, 3), $f(x)$ will attain it's minima in the interval (0, 3).

95. Options (a), (b), (d) are undefined for $x = 0$
As $x^4 + 7$ is always positive for $x \in R$, therefore log $(x^4 + 7) \in R$ for all $x \in R$. Hence option (c) is true. Each of the other options have at least one value where $f(x)$ does not remain real.

Level of Difficulty (II)

1. For the function to be defined $4 - x^2 > 0$
This happens when $-2 < x < 2$.
Option (a) is correct.
2. For the function to be defined two things should happen
(a) $(1 - x) > 0 \Rightarrow x < 1$ and
(b) $(x + 2) \geq 0 \Rightarrow x \geq -2$. Also $x \neq 0$
Thus, option (d) is correct.
3. $\dfrac{5x - x^2}{4} \geq 1 \Rightarrow 1 \leq x \leq 4$.
4. Neither $2^{-x \cdot x}$ nor $2^{x - x \cdot x \cdot x \cdot x}$ is an odd function as for neither of them is $f(x) = -f(-x)$.
5. $1 - |x|$ should be non negative.
[–1, 1] would satisfy this.
6. $4 - x^2 \neq 0$ and $(x^3 - x) > 0 \Rightarrow (-1, 0) \cup (1, \infty)$ but not 2 or –2.
7. $f(0) = 1, f(1) = 2$ and $f(2) = 4$
Hence, they are in G.P.
8. x would become – 2 and $y = -3$.
9. $u(f(v(t))) = u\ (f(t^2)) = u\ (1/t^2) = \left(\dfrac{4}{t^2}\right) - 5$.
10. $g(f(h(t))) = g\ (f(4t - 8)) = g\left(\sqrt{4t - 8}\right)$
$= \dfrac{\sqrt{4t - 8}}{4}$
11. $h(g(f(t)) = h\ (g(\sqrt{t}\ )) = h\ (\sqrt{t}/4)$
$= \sqrt{t} - 8$
12. $f(h(g(t))) = f(h(t/4)) = f(t - 8) = \sqrt{t - 8}$.
13. All three functions would give the same values for $x > 0$. As $g(x)$ is not defined for negative x, and $h(x)$ is not defined for $x = 0$.
14. $e^x + e^{-x} = e^{-x} + e^x$
Hence, this is an even function.
15. $(x + 3)^3$ would be shifted 3 units to the left and hence $(x + 3)^3 + 1$ would shift 3 units to the left and 1 unit up. Option (c) is correct.
16. $f(x) \cdot g(x) = 15x^8$ which is an even function. Thus, option (a) is correct.
17. $(x^2 + \log_e x)$ would be neither odd nor even since it obeys neither of the rules for even function ($f(x) = f(-x)$) nor for odd functions ($f(x) = -f(-x)$).
18. $(x^3 - x^2/5) = f(x) - g(x)$ is neither even nor odd.
19. $y = 1/(x - 2) \Rightarrow (x - 2) = 1/y \Rightarrow x = 1/y + 2$.
Hence, $f^{-1}(x) = 1/x + 2$.
20. $y = e^x$
$\Rightarrow \log_e y = x$.
$\Rightarrow f^{-1}(x) = \log_e x$.
21. $y = x/(x - 1)$
$\Rightarrow (x - 1)/x = 1/y$
$\Rightarrow 1 - (1/x) = 1/y$
$\Rightarrow 1/x = 1 - 1/y \Rightarrow 1/x = (y - 1)/y$
$\Rightarrow x = y/(y - 1)$
Hence, $f^{-1}(x) = x/(x - 1)$.
22. If you differentiate each function with respect to x, and equate it to 0 you would see that for none of the three options will get you a value of $x = -3$ as its solution. Thus, option (d) viz. None of these is correct.

Directions for Questions 23 to 32: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between. This is seen in Questions 25, 26, 30 and 32.

We see even functions in Questions 24 and 28. [Symmetry about the y axis]. We see odd functions in Questions 23 and 31.

While the figures in Questions 27 and 29 are neither odd nor even.

Even ⇒ 24, 28,
Odd 23, 31.
Neither 27, 29,
doesn't exist: 25, 26, 30 and 32.

33. $-f(x)$ would be the mirror image of the function, about the 'x' axis which is seen in option (b).
34. $-f(x) + 1$ would be mirror image about the x axis and then shifted up by 1. Option (a) satisfies this.
35. $f(x) - 1$ would shift down by 1 unit. Thus option (c) is correct.
36. $f(x) + 1$ would shift up by 1 unit. Thus, option (d) is correct.
37. The given function would become $h[11, 80, 1] = 2640$.
38. The given function would become $g[0, 0, 3] = 0$.
39. The given function would become $f[3, 3, 3] = 27$.
40. $f(1, 2, 3) - g(1, 2, 3) + h\ (1, 2, 3) = 11 - 23 + 18 = 6$.

41. The number of g's and f's should be equal on the LHS and RHS since both these functions are essentially inverse of each other.
    Option (c) is correct.
42. The required minimum value would occur at $f(x) = g(x) = 1$.
43. $SQ\ [R[(a + b)/b]] = SQ\ [R[17/5]] \Rightarrow SQ\ [2] = 2$.
44. $Q\ [[SQ(63) + 7\ ]/9] = Q\ [[\ 8 + 7]/9] = Q\ [15/9] = 1$.
45. $Q\ [[SA\ (36) + R\ (18/7)]/2] = Q\ [(7 + 4)/2] = Q\ [11/2] = 5$.
46. $[x] - \{x\} = -1$
47. $[x] + \{x\}$ will always be odd as the values are consecutive integers.
48. At $x = 5.5$, the given equation can be seen to be satisfied as: $6 + 5 = 2 \times 5.5 = 11$.
49. $f(g(t)) - g(f(t)) = f(2.5) - g(6) = 8.25 - 2.166 = 6.0833$.
50. fog $= f(3t + 2) = k(3t + 2) + 1$
    gof $= g\ (kt + 1) = 3(\ kt + 1) + 2$
    $$k(3t+2)+1 = 3(kt+1)+2$$
    $$\Rightarrow \quad 2k+1 = 5$$
    $$\Rightarrow \quad k = 2.$$
51. When the value of $x = 81$ and 82 is substituted in the given expression, we get,
    $F(81)\ F(82) = -\ F(80)\ F(79)\ F(78)\ F(77)$ …(i)
    $F(82)\ F(83) = -\ F(81)\ F(80)\ F(79)\ F(78)$ …(ii)
    On dividing (i) by (ii), we get
    $$\frac{F(81)}{F(83)} = \frac{F(77)}{F(81)} \Rightarrow F(81) \times F(81) = 81 \times 9$$
    $\Rightarrow F(81) = 27$
    Option (a) is the correct answer.
52. In order to understand this question, you first need to develop your thought process about what the value of $h(x)$ is in various cases. A little bit of trial and error would show you that the value of $h(x)$ since it depends on the minimum of $f(x)$ and $g(x)$, would definitely be dependant on the value of $f(x)$ once x becomes greater than 11 or less than –11. Also, the value of $g(x)$ is fixed as an integer at 16, whenever x is between –8 to +8. Also, at $x = 9$, $x = 10$ and $x = -9$ and $x = -10$, the value of $h(x)$ would still be an integer.
    With this thought when you look at the expression of $f(x) = 121–x^2$, you realise that the value of x can be –10, –9, –8, –7, …0, 1, 2, 3,….8, 9, 10, i.e., 21 values of x when $h(x) = g(x)$. When we use $x = 11$ or $x = -11$, the value of $f(x) = 0$ and is not a positive integral value.
    Hence, the correct answer is Option (c).
53. Since, $R(x)$ is the maximum amongst the three given functions, its value would always be equal to the highest amongst the three. It is easy to imagine that $x^2 - 8$ and $3x$ are increasing functions, therefore the value of the function is continuously increasing as you increase the value of x. Similarly $x^2 - 8$ would be increasing continuously as you go farther and farther down on the negative side of the x-axis. Hence, the maximum value of $R(x)$ would be infinity. Option (c) is the correct answer.
54. In this case, the value of the function, is the minimum of the three values. If you visualise the graphs of the three functions (viz: $y = x^2 - 8$, $y = 3x$ and $y = 8$) you realise that the function $y = 3x$ (being a straight line) will keep going to negative infinity as you move to the left of zero on the negative side of the x-axis.
    Hence, the minimum value of the function $R(x)$ after a certain point (when x is negative) would get dictated by the value of $3x$. This point will be the intersection of the line $y = 3x$ and the function $y = x^2 - 8$ when x is negative.
    The two intersection points of the line ($3x$) and the quadratic curve ($x^2 - 8$) would be got by equating $3x = x^2 - 8$. Solving this equation tells us that the intersection points are:
    $$\frac{3-\sqrt{41}}{2} \text{ and } \frac{3+\sqrt{41}}{2}.$$
    $R(x)$ would depend on the following structures based on the value of x:
    (i) When x is smaller than $\frac{3-\sqrt{41}}{2}$, the value of the function $R(x)$ would be given by the value of $3x$.
    (ii) When x is between $\frac{3-\sqrt{41}}{2}$ and 4 the value of the function $R(x)$ would be given by the value of $x^2 - 8$, since that would be the least amongst the three functions.
    (iii) After $x = 4$, on the positive side of the x-axis, the value of the function would be defined by the third function viz: $y = 8$.
    A close look at these three ranges would give you that amongst these three ranges, the third range would yield the highest value of $R(x)$. Hence, the maximum possible value of $R(x) = 8$. Option (b) is correct.
55. The expression is $2x^2 - 5x + 4$, and its value at $x = 5$ would be equal to $50 - 25 + 4 = 29$. Option (b) is correct.
56. At $x = 0$, the value of the function is 20 and this value rejects the first option. Taking some higher values of x, we realise that on the positive side, the value of the function will become negative when we take x greater than 5 since the value of $(5 - x)$ would be negative. Also, the value of $f(x)$ would start tending to $-\infty$, as we take bigger values of x.

Similarly, on the negative side, when we take the value of x lower than –4, $f(x)$ becomes positive and when we take it farther away from 0 on the negative side, the value of $f(x)$ would continue tending to $+\infty$. Hence, Option (c) is the correct answer.

57. The remainder when $6^x + 4$ is divided by 2 would be 0 in every case (when x is odd)
Also, when x is even, we would get $6^x - 3$ as an odd number. In every case the remainder would be 1 (when it is divided by 2.)
Between $f(2), f(4), f(6),\ldots f(1000)$ there are 500 instances when x is even. In each of these instances the remainder would be 1 and hence the remainder would be 0 (in total). Option (b) is correct.

58. The product of p, q and r will be maximum if p, q and r are as symmetrical as possible. Therefore, the possible combination is (4, 3, 3).
Hence, maximum value of $pq + qr + pr + pqr = 4 \times 3 + 4 \times 3 + 3 \times 3 + 4 \times 3 \times 3 = 69$.
Hence, Option (c) is correct.

59. The equation given in the question is: $3\alpha(x) + 2\alpha(2-x) = (x + 3)^2$ ….(i)
Replacing x by $(2-x)$ in the above equation, we get
$3\alpha(2-x) + 2\alpha(x) = (5-x)^2$
Solving the above pairs of equation, we get
$5\alpha(x) = 3(x + 3)^2 - 2(5 - x)^2 = 3(x^2 + 6x + 9) - 2(25 - 10x + x^2) = 3x^2 + 18x + 27 - 50 + 20x - 2x^2 = x^2 + 38x - 23$
Thus, $\alpha(x) = (x^2 + 38x - 23)/5$
Thus, $\alpha(-5) = -188/5 = -37.6$. The value of $[-37.6] = -38$. Hence, option (b) is the correct answer.

60. The first thing you do in this question is to create the chain of values of $f(x)$ for $x = 1, 2, 3$ and so on. The chain of values would look something like this:

When x is odd			*When x is even*		
$f(1)$	Value is given	6	$f(2)$	Value is given	4
$f(3)$	$= 1 + f(1)$	7	$f(4)$	$= 3 + f(2)$	7
$f(5)$	$= 3 + f(3)$	10	$f(6)$	$= 3 + f(4)$	10
$f(7)$	$= 5 + f(5)$	15	$f(8)$	$= 3 + f(6)$	13
$f(9)$	$= 7 + f(7)$	22	$f(10)$	$= 3 + f(8)$	16
$f(11)$	$= 9 + f(9)$	31			

In order to evaluate the value of the embedded function represented by $(f(f(f(f(1)))))$, we can use the above values and think as follows:
$f(f(f(f(1)))) = f(f(f(6))) = f(f(10)) = f(16) = 25$
Also, $f(f(f(f(2)))) = f(f(f(4))) = f(f(7)) = f(15) = 55$
Hence, the product of the two values is $25 \times 55 = 1375$.
Option (a) is correct.

61. For $x > 0$. $x+\frac{1}{x}$ has a minimum value of 2, when x is taken as 1. Why we would need to minimise $x+\frac{1}{x}$ is because it is raised to the power 6 in the numerator, so allowing $x+\frac{1}{x}$ to become greater than its' minimum would increase the value of the expression. Also, the value of any expression of the form $x^n+\frac{1}{x^n}$ would also give us a value of 2. Hence, the value of the expression would be:
$$\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)}=\frac{2^6-2-2}{2^3+2}=6$$
Hence, (d) is the correct choice.

62. The function would be defined when the term $\frac{1}{\{\log_{10}(3-x)\}}$ is real, which will occur when $x < 3$. However, if $x = 2$, then the denominator of the term becomes 0, which should not be allowed. The other limit of the function gets defined by the constraint defined by the term $\sqrt{x+7}$. For $\sqrt{x+7}$ to be real, $x \geq -7$ is the requirement. Hence, the required domain is:
Required domain $= -7 \leq x < 3, x \neq 2$
i.e., $x \in [-7,3) - \{2\}$
Option (a) is correct.

63. $\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\left[\frac{1}{3}+\frac{65}{99}\right]=0$
$\left[\frac{1}{3}+\frac{66}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\ldots\left[\frac{1}{3}+\frac{98}{99}\right]=33$
$$\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\ldots\left[\frac{1}{3}+\frac{98}{99}\right]=0+33=33$$
Option (b) is correct.

64. $x^2 + 4xy + 6y^2 - 4y + 4$
$= x^2 + 4y^2 + 4xy + 2y^2 - 4y + 2 + 2$
$= (x + 2y)^2 + 2(y^2 - 2y + 1) + 2$
The above expression is minimum for $y = 1, x = -2$
So minimum value of the given expression
$= 0 + 0 + 2 = 2$.
Option (c) is correct.

65. Let $f(X) = 21 \sin X + 72 \cos X$
$\Rightarrow f'(X) = 21 \cos X - 72 \sin X$
If $f'(X) = 0$, $21 \cos X = 72 \sin X$.

$\therefore$ tan X = 21/72 therefore sin X = 21/75, cos X = 72/75 (Since, from the value of tan X we can think of a right angled triangle with the legs as 21 and 72 respectively. This would give us the hypotenuse length of the triangle as 75 – using the Pythagoras theorem).

Since $f''(x)$ = –21 sin X – 72 cos X < 0 therefore $f(X)$ has a maximum at $f'(X)$ = 0. Thus, we can use the values of sin X = 21/75 & cos X = 72/75.

$\therefore$ Maximum value of

$$f(x) = \frac{21.21}{75} + \frac{72.72}{75} = \frac{75^2}{75} = 75$$

Option (d) is correct.

66. For $x < -7$

$|x + 7| + |x - 8| = -(x + 7) - (x - 8)$

$-(x + 7) - (x - 8) = 16$

$-2x + 1 = 16$

$x = -7.5$

For $-7 \leq x \leq 8$

$|x + 7| + |x - 8| = x + 7 - x + 8 = 15 \neq 16$

Therefore the given equation has no solution in this range.

For $x \geq 8$

$|x + 7| + |x - 8| = x + 7 + x - 8 = 2x - 1$

$2x - 1 = 16$

$$\Rightarrow x = \frac{17}{2} = 8.5$$

So the required sum = –7.5 + 8.5 = 1

Hence option (b) is correct.

67. $|3x + 4| \leq 5$

$-5 \leq 3x + 4 \leq 5$

$-3 \leq x \leq 1/3$

$a = -3$, $b = 1/3$

$$a + b = -3 + \frac{1}{3}$$

$$= -\frac{8}{3}$$

68. $x^3 - 16x + x^2 + 20 \leq 0 = (x + 5)(x - 2)^2 \leq 0$

For any positive integer the given expression can never be less than 0. Therefore $x = 2$, is the only positive integer value of x for which the given inequality holds true. Alternately, you can also solve this question using trial and error, where you can start with $x = 1$ and then try to see the value of the expression at $x = 2$. At $x = 1$, the expression is positive, at $x = 2$ it is 0, while at $x = 3$ it again becomes positive. Once, x crosses 3, the term x^3 by itself would become so large that it would not be possible to pull the value of the expression into the non-positive territory because the magnitude of the negative term in the expression viz $16x$, would not be large enough to make the expression ≤ 0

69. Putting $x = 7$ in the given equation we get:

$3f(7) + 2f(11) = 70\ldots$ (1)

Similarly by putting $x = 11$ in the given equation we get:

$3f(11) + 2f(7) = 98\ldots$ (2)

Solving equation 1 and 2 we get

$$f(11) = \frac{154}{5} = 30.8$$

70. $q = p \times [p]$

When you start to think about the values of q from 8 onwards to 16, the first solution is quite evident at $q = 9$ and $p = 3$. At $q = 10$, p can be taken to be 10/3 to give us the expression of $p \times [p]$ equal to 10. Similarly

For $q = 11$, $a = 3$, $p = 11/3$.

For $q = 16$, $a = 4$, $p = 4$

So the required number of positive real values of p = 4.

71. Required product is = $3 \times \frac{10}{3} \times \frac{11}{3} \times 4 = \frac{440}{3}$

72. $f(3) = f(1) + 8(1 + 1)$

$= -1 + 16 = 15$

$f(5) = f(3) + 8(3 + 1)$

$= (15 + 32)$

$= 47$

$f(10) = 4.f(5) + 9$

$= 4 \times 47 + 9$

$= 197$

$f(20) = 4 \times 197 + 9$

$= 797$

$f(22) = f(20) + 8(20 + 1)$

$= 797 + 168$

$= 965$

$f(24) = 965 + 8(22 + 1) = 1149$

$f(7) = f(5) + 8(5 + 1) = 47 + 48 = 95$

Hence, $f(24) - f(7) = 1149 - 95 = 1054$

73. If we observe values of $f(x)$ for different values of x, then we can see that $f(x) = 2x^2 - 3$.

Hence, $f(1000) = 2(1000)^2 - 3$

$= 1999,997$

74. $f(x) = (x^2 + [x]^2 - 2x[x])^{1/2} = \left[(x - [x])^2\right]^{\frac{1}{2}} = x - [x]$

$f(x) = x - [x]$ represents the fractional part of x.

Hence $f(10.08) = 0.08$

$f(100.08) = 0.08$

$f(10.08) - f(100.08) = 0.08 - 0.08 = 0$.

75. Let $f(x) = (x - 4)^7 (x - 3)^4 (x - 5)^2$

$f(1) = (1 - 4)^7 (1 - 3)^4 (1 - 5)^2$

$= (-3)^7 (-2)^4 (-4)^2$

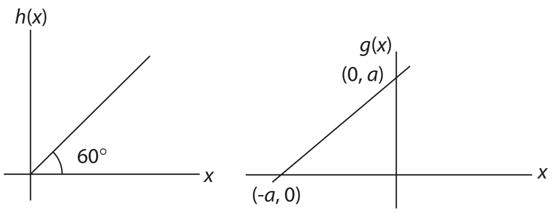
$= -2^8.3^7$

Option (c) is correct.

76. $f(x) = x - \dfrac{1}{3(3-x)} - 3 = (x-3) + \dfrac{1}{3(x-3)} \geq$

$\left[(x-3)\times\dfrac{1}{3(x-3)}\right]^{\frac{1}{2}}$. Hence, $(x-3)+\dfrac{1}{3(x-3)} \geq \dfrac{1}{\sqrt{3}}$

Option (b) is correct.

77. 

$h(x) = x\tan 60^{\circ} = x\sqrt{3}$

$x = \dfrac{h(x)}{\sqrt{3}}$

$\dfrac{x}{-a} + \dfrac{g(x)}{a} = 1$

$g(x) = \left[1 + \dfrac{x}{a}\right]a = a + x = a + \dfrac{h(x)}{\sqrt{3}}$

$\sqrt{3}g(x) = a\sqrt{3} + h(x)$

$\sqrt{3}g(x) - h(x) = a\sqrt{3}$

Option (a) is correct.

Alternately, you can also solve this by looking at the values of the graphs. At $x = 0$, $h(x) = 0$ and $g(x) = a$. At $x = 1$, $h(x) = \sqrt{3}$ (This can be visualised, since the triangle that is formed by the graph of $h(x)$ with the x axis is a 30-60,90 triangle. Hence, if we take the side opposite the 30° angle as 1, the height (side opposite the 60° angle) would be $\sqrt{3}$. Also, the value of $g(x)$ would be $a + 1$ (since the gradient of the $g(x)$ slope is 45°). The first option satisfies both these pairs of values. Hence, it is the correct answer.

78. $\dfrac{f(xy)}{f(x+y)} = 1$ or $f(xy) = f(x+y)$

Put $x = 0$: $f(0.y) = f(0 + y) \Rightarrow f(y) = f(0)$

Put $y = 0$: $f(x.0) = f(x + 0) \Rightarrow f(x) = f(0)$

Therefore function 'f' is a constant function. (This can also be interpreted since the function reads that the value of f when you put an argument equal to the product of x & y is the same as the value of f when you put the argument of the function as $x + y$).

$f(-10) = f(10) = f(6) = 7$

$f(-10) + f(10) = 7 + 7 = 14$

79. Putting $x = 9$, $y = 3$, in the above equation we get

$f\left(\dfrac{9}{3}\right) = \dfrac{f(9)}{f(3)}$

$f(3) = \dfrac{f(9)}{f(3)}$

$f(9) = [f(3)]^2 = 5^2 = 25$

Similarly $x = 81$, $y = 9$

$f\left(\dfrac{81}{9}\right) = \dfrac{f(81)}{f(9)}$

$f(9) = \dfrac{f(81)}{f(9)}$

$f(81) = [f(9)]^2 = 25^2 = 625$

80. We can find the sum of all coefficients of a polynomial by putting each of the variable equals to 1:

Therefore the required sum $= (1-4)^3(1-2)^{10}(1-3)^3$

$= -3^3 \times 1 \times (-2)^3$

$= 27 \times 8$

$= 216$

81. $f(a) = 3^a$ (If a is an odd number)

$f(a + 1) = 3^{a+1} + 4 = 3.3^a + 4$

$\dfrac{1}{4}[f(a) + f(a+1)] = \dfrac{3^a + 3.3^a + 4}{4}$

$= \dfrac{3^a.4 + 4}{4} = 3^a + 1$

$\Rightarrow \dfrac{1}{4}[f(1) + f(2)] + (f(3) + f(4))$

$+ \ldots + f(71) + f(72)]$

$= \dfrac{f(1)+f(2)}{4} + \dfrac{f(3)+f(4)}{4}$

$+ \ldots + \dfrac{f(71)+f(72)}{4}$

$= 3^1 + 1 + 3^3 + 1 + \ldots + 3^{71} + 1$

$= (3^1 + 3^3 + \ldots + 3^{71}) + 36$

$= \dfrac{3\left((3^2)^{36} - 1\right)}{3^2 - 1} + 36$

(using the formula for the sum of a geometric progression, since the series containing the powers of 3 is essentially a geometric progression).

$= \dfrac{3}{8}\left(3^{72} - 1\right) + 36$

82. Put $x = 0$ then $f(0 + y) = f(0) \rightarrow f(y) = p$

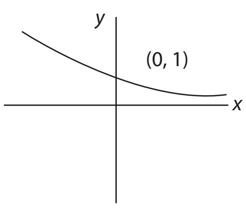
Put $y = 0$ then $f(x + 0) = f(0) \rightarrow f(x) = p$

Therefore 'f' is a constant function

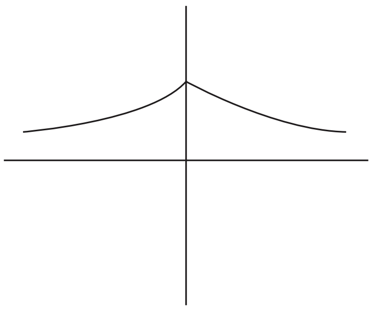
$f(7) = f(10) = f(5) = 12$

$[f(7)]^{143} - [f(11)]^{143} + f(5) = 12^{143} - 12^{143} + 12 = 12$

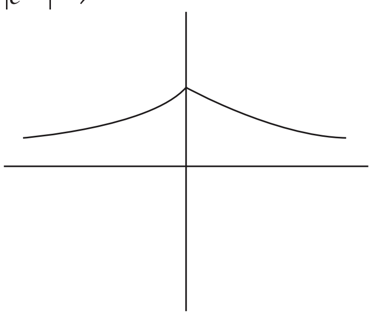
83. $e^{-x} \rightarrow$



$e^{-|x|} \rightarrow$

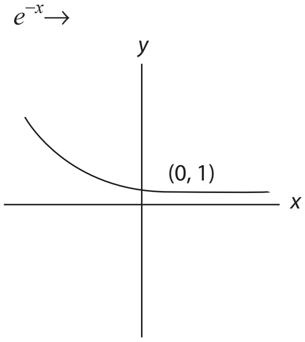


$|e^{-|x|}| \rightarrow$

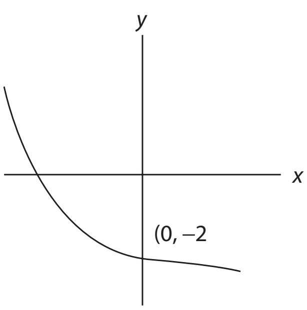


Hence option (c) is correct.

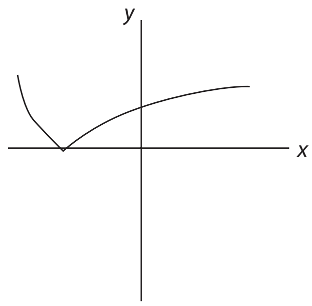
84. $e^{-x} \rightarrow$



$e^{-x} - 3 \rightarrow$

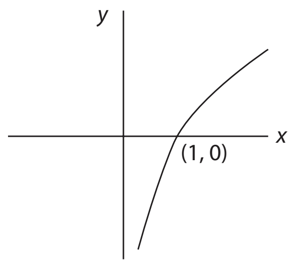


$|e^{-x} - 3| \rightarrow$

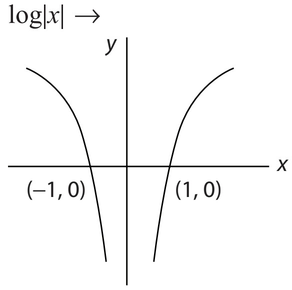


Hence option (a) is correct.

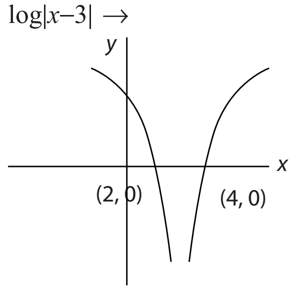
85. Log $x \rightarrow$



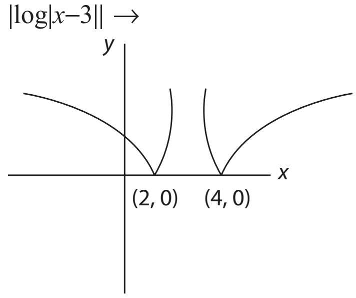
$\log|x| \rightarrow$



$\log|x-3| \rightarrow$



$|\log|x-3|| \rightarrow$



Option (d) is correct.

86. $f(x, y) = x^2 + y^2 - x - \frac{3y}{2} + 1$ can be split as:

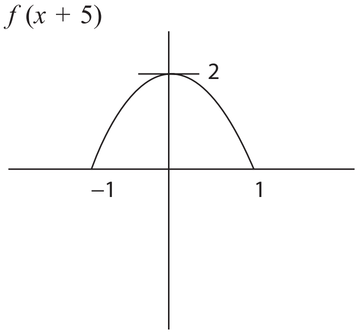
$$= x^2 - x + \frac{1}{4} + y^2 - \frac{3y}{2} + \frac{9}{16} + \frac{3}{16}$$

$$= \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{4}\right)^2 + \frac{3}{16}$$

$f(x, y)$ will be minimum when $x = ½$, $y = ¾$.

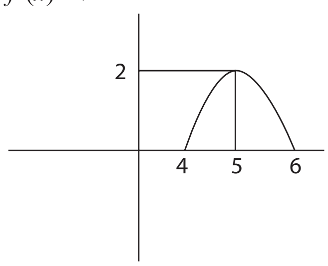
Therefore $x + y = \frac{1}{2} + \frac{3}{4} = \frac{5}{4} = 1.25$

87. $f(x, y)$ min = 3/16.
88. $f(x + 5)$

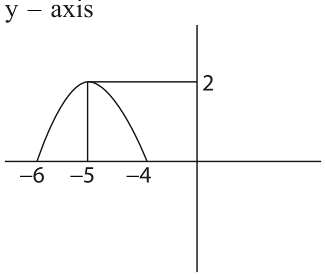


$f(x)$ can be obtained by shifting $f(x + 5)$ right by 5 units.

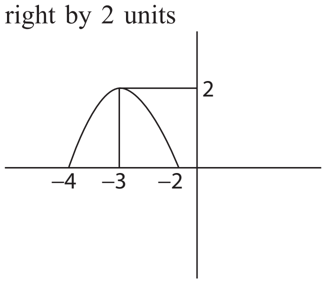
$f(x) \Rightarrow$



$f(-x)$can be got by reflecting the graph $f(x)$ about the y – axis

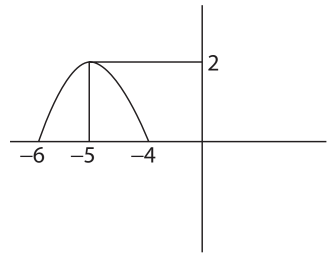


$f(-x - 2)$ can be got by shifting curve of $f(-x)$ to the right by 2 units

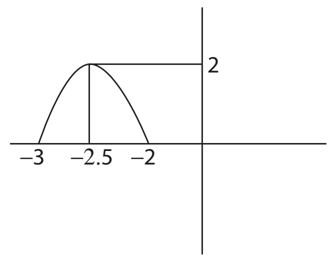


Option (d) is correct.

89. From our discussion of the previous question, we know that $f(-x)$ will look as below:



$f(-2x)$ would mean that the graph's value on the x-axis, would get halved at each of it's points



Option (c) is correct.

90. $\frac{g(x+y)+g(x-y)}{2} = g(x)g(y)$

$g(x + y) + g(x - y) = 2g(x)\,g(y)$ …(i)

By replacing y with x and x with y, we get

$g(x + y) + g(y - x) = 2g(x)\,g(y)$ …(ii)

From equation (i) & equation (ii)

$g(x + y) + g(x - y) = g(x + y) + g(y - x)$

$g(x - y) = g(y - x)$

By putting $y = 0$, we get $g(x) = g(-x)$

Therefore $g(x)$ must be an even function: therefore only option (c) satisfies because option c also represents an even function.

Level of Difficulty (III)

1. $x - |x|$ is either negative for $x < 0$ or 0 for $x \geq 0$. Thus, option (d) is correct.
2. The domain should simultaneously satisfy:

   $x - 1 \geq 0$, $(1 - x) \geq 0$ and $(x^2 + 3) \geq 0$.

   Gives us: $x \geq 1$ and $x \leq 1$

   The only value that satisfies these two simultaneously is $x = 1$.
3. For the function to exist, the argument of the logarithmic function should be positive. Also, $(x + 4) \geq 0$ should be obeyed simultaneously.

   For $\frac{(x-5)}{(x^2 - 10x + 24)}$ to be positive both numerator and denominator should have the same sign. Considering all this, we get:

   $4 < x < 5$ and $x > 6$.

Option (c) is correct.

4. Both the brackets should be non-negative and neither $(x + 3)$ nor $(1+ x)$ should be 0.

For $(x - 3)/(x + 3)$ to be non negative we have $x > 3$ or $x < -3$.

Also for $(1- x)/(1+ x)$ to be non-negative $-1 < x < 1$. Since there is no interference in the two ranges, Option (d) would be correct.

8. $f(f(t)) = f((t-1)/(t+1))$

$$= \left[\left(\frac{t-1}{t+1}\right)-1\right]\Big/\left[\left(\frac{t-1}{t+1}\right)+1\right] = \frac{t-1-t-1}{t-1+t+1}$$

$= -2/2t = -1/t.$

9. fog $= f(\log_e x) = e^{\log_e x} = x$.

10. gof $= g(e^x) = \log_e e^x = x$.

11. Looking at the options, one unit right means x is replaced by $(x - 1)$. Also, 1 unit down means -1 on the RHS.

Thus, $(y + 1) = 1/(x - 1)$

12. For option (c) we can see that $f(t) = f(-t)$. Hence, option (c) is correct.

13. Option (b) is odd because:

$$\frac{a^{-t}+a^t}{a^t-a^{-t}} = -1 \times \left(\frac{a^{-t}+a^t}{a^{-t}-a^t}\right)$$

Similarly option (c) is also representing an odd function. The function in option (a) is not odd.

14. $f(f(t)) = f[t/(1+t^2)^{1/2}] = t/(1+2t^2)^{1/2}$.

15. By trial and error it is clear that at $x = 3$, the value of the function is 19. At other values of 'x' the value of the function is less than 19.

17. Take different values of x to check each option. Each of Options (a), (b) and (c) can be ruled out. Hence, Option (d) is correct.

Solutions to 18 to 20:

$f(1) = 0, f(2) = 1,$

$f(3) = f(1) - f(2) = -1$

$f(4) = f(2) - f(3) = 2$

$f(5) = f(3) - f(4) = -3$

$f(6) = f(4) - f(5) = 5$

$f(7) = f(5) - f(6) = -8$

$f(8) = f(6) - f(7) = 13$

18. It can be seen that $f(x)$ is positive wherever x is even and negative whenever x is odd once x is greater than 2.

19. $f(f(6)) = f(5) = -3$.

20. $f(6) - f(8) = 5 - 13 = -8 = f(7)$.

21. Option (b) is not even since $e^x - e^{-x} \neq e^{-x} - e^x$.

22. We have $f(x) \cdot f(1/x) = f(x) + f(1/x)$

$\Rightarrow \quad f(1/x)\,[f(x) - 1] = f(x)$

For $x = 4$, we have $f(1/4)\,[f(4) - 1] = f(4)$

$\Rightarrow \quad f(1/4)\,[64] = 65$

$\Rightarrow \quad f(1/4) = 65/64 = 1/64 + 1$

This means $f(x) = x^3 + 1$

For $f(6)$ we have $f(6) = 216 + 1 = 217$.

Directions for Questions 23 to 34: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, Option (d) would occur if the function does not exist at, atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between (as in questions 26, 31 and 33).

We see even functions in Questions 23, 28, 30, 32 and 34 [Symmetry about the y axis]. We see odd functions in Questions 24, 25 and 27.

While the figure in Question 29 is neither odd nor even.

Solutions to 35–40:

In order to solve this set of questions first analyse each of the functions:

$A(x, y, z)$ = will always return the value of the highest between x and y.

$B(x, y, z)$ will return the value of the maximum amongst x, y and z.

$C(x, y, z)$ and $D(x, y, z)$ would return the second highest values in all cases while max (x, y, z) and min(x, y, z) would return the maximum and minimum values amongst x, y, and z respectively.

35. When either x or y is maximum.
36. This would never happen.
37. When z is maximum, A and B would give different values. Thus, option (c) is correct.
38. Never.
39. I and III are always true.
40. We cannot determine this because it would depend on whether the integers x, y, and z are positive or negative.

Solutions to 41 to 49:

$f(x, y)$ is always positive or zero

$F(f(x, y)$ is always negative or zero

$G(f(x, y)$ is always positive or zero

41. $F \times G$ would always be negative while $-F \times G$ would always be positive except when they are both equal to zero.

Hence, Option (b) $F \times G \le -F \times G$ is correct.

42. Option (b) can be seen to give us $4a^2/4 = a^2$.

43. $(5 - 1)/(1+ 3) = 4/4 = 1$.

44. The given expression $= (45 - 10)/(5 + 2) = 35/7 = 5$. Option (b) $= 20/4 = 5$.

Directions for Questions 45 to 49: Do the following analysis:

$A(f(x, y))$ is positive

B (*f*(*x*, *y*)) is negative
C(*f*(*x*, *y*)) is positive
D (*f*(*x*, *y*)) is negative
E(*f*(*x*, *y*)) is positive and so on.

45. 1 – 3 + 5 – 7 + 9 – 11 + … – 51
= (1 + 5 + 9 + 13 + … 49) – (3 + 7 + 11 … + 51)
= – 26
46. Verify each statement to see that (ii) and (iii) are true.
47. The given expression becomes:
Min (max [5, – 7, 9], min [3, – 1 , 1], max [7, 6, 10])
= Min [9, – 1, 10]
= – 1.
48. The given expression becomes:
Max [|*a* + *b*|, – |*b* + *c*|, |*c* + *d*|]
This would never be negative.
49. The respective values are:
–3/2, –7/12, – 8/15, and –5/6.
Option (b) is second lowest.
50. Let *s* = 1, *t* = 2 and *b* = 3
Then, *f*(*s* + *t*) + *f*(*s* – *t*)
$= f(3) + f(-1) = (3^3 + 3^{-3})/2 + (3^{-1} + 3^1)/2$
= [(27 + (1/27)]/2 + [3 + (1/3)]/2
= 730/54 + 10/6
= 820/54 = 410/27
Option (b) 2 *f*(*s*) × *f*(*t*) gives the same value.
51. This question is based on the logic of a chain function. Given the relationship
$A_t = (t + 1)A_{(t-1)} - tA_{(t-2)}$
We can clearly see that the value of A_2 would depend on the values of A_0 and A_1. Putting *t* = 2 in the expression, we get:
$A_2 = 3A_1 - 2A_0 = 7$; $A_3 = 19$; $A_4 = 67$ and $A_5 = 307$. Clearly, A6 onwards will be larger than 307 and hence none of the three conclusions are true. Option (e) is the correct answer.
52. In order to solve this question, we would need to check each of the value ranges given in the conclusions: Checking whether Conclusion I is possible
For B = 2, we get A + C = 4 (since A + B + C = 6). This transforms the second equation AB + BC + CA = 9 to:
2(A + C) + CA = 9 → CA = 1.
Solving CA = 1 and A + C = 4 we get: (4 – A)A = 1 → $A^2 - 4A + 1 = 0 \rightarrow A = 2 + 3^{1/2}$ and $C = 2 - 3^{1/2}$. Both these numbers are real and it satisfies A < B < C and hence, Conclusion I is true.
Checking Conclusion II: If we chose A = 2.5, the condition is not satisfied since we get the other two variables as $(3.5 + 11.25^{1/2}) \div 2 \approx 3.4$ and $(3.5 - 11.25^{1/2}) \div 2 \approx 0.1$. In this case, A is no longer the least value and hence Conclusion II is rejected.
Checking Conclusion III we can see that 0 < C < 1 cannot be possible since C being the largest of the three values has to be greater than 3 (the largest amongst A, B, and C would be greater than the average of A, B, C).
Option (a) is correct.
53. The number of ways of distributing *n* identical things to *r* people such that any person can get any number of things including 0 is always given by ${}^{n+r-1}C_{r-1}$. In the case of *F*(4,3), the value of *n* = 4 and *r* = 3 and hence the total number of ways without any constraints would be given by ${}^{4+3-1}C_{3-1} = {}^6C_2 = 15$. However, out of these 15 ways of distributing the toys, we cannot count any way in which more than 2 toys are given to any one child. Hence, we need to reduce as follows:
The distribution of 4 toys as (3, 1, 0) amongst three children A, B and C can be done in 3! = 6 ways.
Also, the distribution of 4 toys as (4, 0, 0) amongst three children A, B and C can be done in 3 ways.
Hence, the value of *F*(4, 3) = 15 – 6 – 3 = 6.
Option (b) is correct.
54. *f*(*f*(*x*)) = 15 when *f*(*x*) = 4 or *f*(*x*) = 12 in the given function. The graph given in the figure becomes equal to 4 at 4 points and it becomes equal to 12 at 3 points in the figure. This gives us 7 points in the given figure when *f*(*f*(*x*) =15. However, the given function is continuous beyond the part of it which is shown between –10 and +13 in the figure. Hence, we do not know how many more solutions to *f*(*f*(*x*) = 15 would be there. Hence, Option (e) is the correct answer.
55. The given function is a chain function where the value of A_{n+1} depends on the value A_n.
Thus for *n* = 0, $A_1 = A_0^2 + 1$.
For n = 1, $A_2 = A_1^2 + 1$ and so on.
In such functions, if you know the value of the function at any one point, the value of the function can be calculated for any value till infinity.
Hence, Statement I is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.
So also, the Statement II is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.
Hence, Option (d) is correct.
56. This question can be solved by first putting up the information in the form of a table as follows:

	Product A	*Product B*	*No of machines available*	*No of Hours/day per Machine.*	*Total Hrs. per day available for each activity*
Grinding	2 hr	3 hr	10	12	120
Polishing	3 hr	2 hr	15	10	150
Profit	₹ 5	₹ 7			

On the surface, the profit of Product B being higher, we can think about maximising the number of units of Product B. Grinding would be the constraint when we maximise Product B production and we can produce a maximum of 120 ÷ 3 = 40 units of Product B to get a profit of ₹ 280. The clue that this is not the correct answer comes from the fact that there is a lot of 'polishing' time left in this situation. In order to try to increase the profit we can check that if we reduce production of Product B and try to increase the production of Product A, does the profit go up?

When we reduce the production of Product B by 2 units, the production of Product A goes up by 3 units and the profit goes up by +1 (–2 × 7 + 3 × 5 gives a net effect of +1). In this case, the grinding time remains the same (as there is a reduction of 2 units × 3 hours/unit = 6 hours in grinding time due to the reduction in Product B's production, but there is also a simultaneous increase of 6 hours in the use of the grinders in producing 3 units of Product A). Given that a reduction in the production of Product B, with a simultaneous maximum possible increase in the production of Product A, results in an increase in the profit, we would like to do this as much as possible. To think about it from this point this situation can be tabulated as under for better understanding:

	Product A Production (A)	*Product B Production (B)*	*Grinding Machine Usage = 3A + 2B*	*Polishing Machine Usage = 2A + 3B*	*Time Left on Grinding Machine*	*Time Left on Polishing Machine*	*Profit = 7A+5B*
Case 1	40	0	120	80	0	70	280
Case 2	38	3	120	85	0	65	281
Case 3	36	6	120	90	0	60	282

The limiting case would occur when we reduce the time left on the polishing machine to 0. That would happen in the following case:

Optimal case	12	42	120	150	0	0	294

Hence, the answer would be 294.

57. The value of $f(x)$ as given is: $f(x) = x^4 + x^3 + x^2 + x + 1 = 1 + x + x^2 + x^3 + x^4 + x^5$. This can be visualised as a geometric progression with 5 terms with the first term 1 and common ratio x. The sum of the GP = $f(x) = \frac{x^5-1}{x-1}$

The value of $f(x^5) = x^{20} + x^{15} + x^{10} + x^5 + 1$ and this can be rewritten as:

$F(x^5) = (x^{20}–1) + (x^{15}–1) + (x^{10}–1) + (x^5–1) + 5$. When this expression is divided by $f(x) = \frac{x^5-1}{x-1}$ we get each of the first four terms of the expression would be divisible by it, i.e. $(x^{20}–1)$ would be divisible by $f(x) = \frac{x^5-1}{x-1}$ and would leave no remainder (because $x^{20}–1$ can be rewritten in the form $(x^5–1) \times (x^{15} + x^{10} + x^5 + 1)$ and when you divide this expression by $\left(\frac{x^5-1}{x-1}\right)$ we get the remainder as 0.)

A similar logic would also hold for the terms $(x^{15}–1)$, $(x^{10}–1)$ and $(x^5–1)$. The only term that would leave a remainder would be 5 when it is divided by $\left(\frac{x^5-1}{x-1}\right)$

Also, for $x \geq 2$ we can see the value of $\left(\frac{x^5-1}{x-1}\right)$ would be more than 5. Hence, the remainder would always be 5 and Option (c) is the correct answer.

58. Start by putting $\frac{x}{x-1} = (\text{cosec } \alpha)^2$ in the given expression

$F\left(\frac{x}{x-1}\right) = \frac{1}{x}$

Now for $0 < \alpha < 90°$

$\frac{x}{x-1} = (\text{cosec } \alpha)^2 \Rightarrow x = \frac{1}{1-\sin^2\alpha} \Rightarrow \frac{1}{x} = \cos^2\alpha$

Hence, Option (b) is correct.

59. Given that the roots of the equation $F(x) = 0$ are –2, –1, 1 and 2 respectively and the $F(x)$ is a polynomial with the highest power of x as x^4, we can create the value of

$F(x) = (x + 2)(x + 1)(x – 1)(x - 2)$

Hence, $F(p) = (P + 2)(P + 1)(P - 1)(P - 2)$

It is given to us that P is a prime number greater than 97. Hence, p would always be of the form $6n \pm 1$ where n is a natural number greater than or equal to 17.

Thus, we get two cases for $F(p)$.

Case 1: If $p = 6n + 1$.

$$F(6n + 1) = (6n + 3)(6n + 2)(6n)(6n - 1)$$
$$= 3(2n + 1)\cdot 2(3n + 1)(6n)(6n - 1)$$
$$= (36)(2n + 1)(3n + 1)(n)(6n - 1) \quad \ldots(i)$$

If you try to look for divisibility of this expression by numbers given in the options for various values of $n \geq 17$, we see that the for $n = 17$ and 18 both 360 divides the value of $F(p)$. However at $n = 19$, none of the values in the four options divides $36 \times 39 \times 58 \times 19 \times 113$. In this case however, at $n = 19$, $6n + 1$ is not a prime number hence, this case is not to be considered. Whenever we put a value of n as a value greater than 17, such that $6n+1$ becomes a prime number, we also see that the value of $F(p)$ is divisible by 360. This divisibility by 360 happens since the expression $(2n + 1)(3n + 1)(n)(6n - 1)$ … is always divisible by 10 in all such cases. A similar logic can be worked out when we take $p = 6n-1$. Hence, the Option (d) is the correct answer.

60. In order to solve this question, we start from the value of $x = (9 + 4\sqrt{5})^{48}$.

Let the value of $x(1-f) = xy$. (We are assuming $(1-f) = y$, which means that y is between 0 to 1).

The value of $x = (9 + 4\sqrt{5})^{48}$ can be rewritten as $[{}^{48}C_0\, 9^{48} + {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2\, 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}]$ using the binary theorem.

In this value, it is going to be all the odd powers of the $(4\sqrt{5})$ which would account for the value of 'f' in the value of x. Thus, for instance it can be seen that the terms ${}^{48}C_0\, 9^{48}, {}^{48}C_2\, 9^{46}(4\sqrt{5})^2, \ldots {}^{48}C_{48}\,(4\sqrt{5})^{48}$ would all be integers. It is only the terms: ${}^{48}C_1 9^{47}(4\sqrt{5}), {}^{48}C_3\, 9^{45}(4\sqrt{5})^3, \ldots {}^{48}C_{47}\,(9)(4\sqrt{5})^{47}$ which would give us the value of 'f' in the value of x.

Hence, $x(1-f) = x\,[1 - {}^{48}C_1\, 9^{47}\,(4\sqrt{5}) - {}^{48}C_3\, 9^{45}(4\sqrt{5})^3 - \ldots - {}^{48}C_{47}\,(9)(4\sqrt{5})^{47}]$

In order to think further from this point, you would need the following thought. Let $y = (9-4\sqrt{5})^{48}$.

Also, $x+y = \{{}^{48}C_0 9^{48} + {}^{48}C_1 9^{47}(4\sqrt{5}) + {}^{48}C_2 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}\,(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}\} + \{{}^{48}C_0\, 9^{48} - {}^{48}C_1\, 9^{47}\,(4\sqrt{5}) + {}^{48}C_2\, 9^{46}\,(4\sqrt{5})^2 + \ldots - {}^{48}C_{47}\,(9)\,(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}\} = 2\{{}^{48}C_0\, 9^{48} + {}^{48}C_2\, 9^{46}\,(4\sqrt{5})^2 + \ldots {}^{48}C_{48}\,(4\sqrt{5})^{48}\}$ – the bracket in this expression has only retained the even terms which are integral. Hence, the value of $x+y$ is an integer.

Further, $x + y = [x] + f + y$ and hence, if $x+y$ is an integer, $[x] + f + y$ would also be an integer. This automatically means that $f+y$ must be an integer (as $[x]$ is an integer).

Now, the value of y is between 0 to 1 and hence when we add the fractional part of x i.e. 'f' to y, and we need to make it an integer, the only possible integer that $f + y$ can be equal to is 1.

Thus, if $f + y = 1 \rightarrow y = (1 - f)$.

In order to find the value of $x(1 - f)$ we can find the value of $x \times y$.

Then, $x(1 - f) = x \times y = (9 + 4\sqrt{5})^{48} \times (9 - 4\sqrt{5})^{48} = (81-80)^{48} = 1$

$x(1 - f) = 1$

61. $3f(x + 2) + 4f\left(\dfrac{1}{x+2}\right) = 4x$

Let $x + 2 = t$

$3f(t) + 4f\left(\dfrac{1}{t}\right) = 4t - 8$ or $\dfrac{3}{4}f(t) + f\left(\dfrac{1}{t}\right) = t - 2 \quad \ldots(1)$

Now replacing t with $\dfrac{1}{t}$ in the above equation, we get

$3f\left(\dfrac{1}{t}\right) + 4f(t) = \dfrac{4}{t} - 8$ or $f\left(\dfrac{1}{t}\right) + \dfrac{4}{3}f(t) = \dfrac{4}{3t} - \dfrac{8}{3} \quad \ldots(2)$

From (1) and (2)

$$f(t) = \frac{12}{7}\left\{\frac{4}{3t} - \frac{8}{3} - t + 2\right\}$$

$$f(4) = \frac{12}{7}\left\{\frac{1}{3} - \frac{8}{3} - 4 + 2\right\} = \frac{-52}{7}$$

62. According to the graph, $f(4) = 15$ and $f(12) = 15$.

So $f(f(x)) = 15$ for $f(x) = 4, 12$.

According to the graph $f(x) = 4$ has four solutions.

According to the graph $f(x) = 12$ has three solutions.

Hence, the given equation has 7 solutions.

63. $[f(x)]^{g(x)} = 1$

Now three cases are possible:

Case I: $f(x) = 1$ and $g(x)$ may be anything.

$x - 6 = 1$ or $x = 7$

But for $x = 7$, $g(x)$ is not defined.

Case II: $f(x) = -1$ and $g(x)$ is an even exponent

$x - 6 = -1$

$x = 5$

For $x = 5$

$$g(x) = \frac{(5-9)(5-1)}{(5-7)(5-3)} = \frac{-4\times 4}{-2\times 2} = 4$$

So for $x = 5$, $g(x)$ is even, which satisfies the given equation.

Case III: $g(x) = 0$ and $f(x) \neq 0$

$$\frac{(x-9)(x-1)}{(x-7)(x-3)} = 0 \text{ for } x = 1, 9$$

For $x = 1$ & 9 $f(x) \neq 0$. So both of these values of x satisfy the given equation.

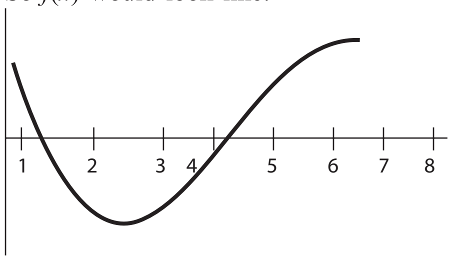
So the given equation is satisfied for three values of x.

64. $f(4) + f(6) = 0$ implies that $f(4)$ & $f(6)$ are of opposite sign but same absolute value. Hence one root of the equation lies between 4 and 6.

$f(1) > 0$ & $f(2) < 0$ implies that another root lies between 1 and 2.

$f(5).f(7) > 0$ implies that $f(5)$ & $f(7)$ are of same sign, so $f(4)$ & $f(5)$ must be of opposite sign. So the second root of $f(x) = 0$ must lie between $x = 4$ & $x = 5$.

So $f(x)$ would look like:



As $f(1) > 0$ & $f(2)$ & $f(4) < 0$

So $f(1)\, f(2)\, f(4) > 0$. Option (a) is incorrect.

As $f(5)$, $f(6)$ & $f(7)$ are greater than 0.

So $f(5)\, f(6)\, f(7) > 0$. So option (b) is wrong.

As $f(1) > 0$ & $f(3)$ & $f(4) < 0$. So $f(1)\, f(3)\, f(4) > 0$

So option (c) is true.

65. $f(x) = 12 + x$

$7[x] + 4\{x\} = 12 + x$

$3[x] + 4[[x] + \{x\}] = 12 + x$

$3[x] + 4x = 12 + x$

$3[x] + 3x = 12$

$[x] + x = 4$

Since 4 and $[x]$ are both integers, in the above equations x must also be an integer. This means that the value of $[x] = x$. So:

$2x = 4$

$x = 2$

Therefore only one value of x satisfies the given equation.

66. $x^2 - xy + y^2 = x + y$

Multiplying both sides by 2, we get:

$2x^2 - 2xy + 2y^2 = 2x + 2y$

$x^2 - 2xy + y^2 + x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$(x - y)^2 + (x - 1)^2 + (y - 1)^2 = 2$

In the question we are interested to find non-negative integer solutions therefore three cases are possible.

Case I: $x - y = 0$, $(x - 1)^2 = 1$, $(y - 1)^2 = 1$

Possible solutions (0, 0) & (2, 2)

Case II: $(x - y)^2 = 1$, $(x - 1)^2 = 1$, $(y - 1)^2 = 0$

Possible solutions: (2, 1), (0, 1).

Case III: $(x - y)^2 = 1$, $(y - 1)^2 = 1$, $(x - 1)^2 = 0$

Possible solutions: (1, 2) and (1, 0)

Possible solutions (x, y) such that $x \geq y$ are (0, 0), (2, 2), (1, 0), (2, 1). There are 4 such solutions.

67. $g(n) = \dfrac{n-1}{n} g(n-1)$

$$g(2) = \frac{1}{2} g(1)$$

$$g(3) = \frac{2}{3} g(2) = \frac{2}{3} \times \frac{1}{2} g(1) = \frac{1}{3} g(1).$$

Similarly:

$$g(4) = \frac{1}{4} g(1); g(5) = \frac{1}{5} g(1);$$

$$g(6) = \frac{1}{6} g(1); g(7) = \frac{1}{7} g(1); g(8) = \frac{1}{8} g(1)$$

Since $g(1) = 2$, the given expression would become:

$$\frac{\left[\frac{1}{2} \times \frac{2}{2} \times \frac{3}{2} \times \ldots \frac{8}{2}\right]}{\left[\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \ldots \frac{8}{2}\right]}$$

Required answer is $\dfrac{8!}{2^8} \times \dfrac{1}{18}$

68. Let $f(x) = a(x - 1)(x - 2)(x - 3)\ldots(x - 77) + x$

Where 'a' is any constant.

Now putting $x = 78$ in the above equation we get

$f(78) = a.77.76.75.74\ldots1 + 78 = a.77! + 78$

Similarly $f(0) = a.(-1)(-2)(-3)\ldots(-77) + 0$

$f(0) = a(-1)^{77} 77! = -a.77!$

$f(78) + f(0) = a.77! + 78 - a.77! = 78$

69. $f(n - 1)\,(2 - f(n)) = 1$

$$2 - f(n) = \frac{1}{f(n-1)}$$

$$f(n) = 2 - \frac{1}{f(n-1)}$$

$$f(2) = 2 - \frac{1}{f(1)} = 2 - \frac{1}{3} = \frac{5}{3}$$

$$f(3) = 2 - \frac{1}{f(2)} = 2 - \frac{3}{5} = \frac{7}{5}$$

$$f(4) = 2 - \frac{1}{f(3)} = 2 - \frac{5}{7} = \frac{9}{7}$$

Observing this pattern, we can see that:

$$f(n) = \frac{2n+1}{2n-1}$$

$$f(21) = \frac{2 \times 21 + 1}{2 \times 21 - 1} = \frac{43}{41}$$

70. Since: $0 \leq \{x\} < 1$

The expression: $10[x] + 22\{x\} = 250$ gives us the inequality: $228 < 10[x] \le 250$

$22.8 < [x] \le 25$

Possible values of $[x] = 23, 24, 25$

For $[x] = 23$, $\{x\} = \dfrac{250-230}{22} = \dfrac{20}{22} = \dfrac{10}{11}$

For $[x] = 24\{x\} = \dfrac{250-240}{22} = \dfrac{10}{22} = \dfrac{5}{11}$

For $[x] = 25$, $\{x\} = 0$

So the possible values of x are $23\frac{10}{11}, 24\frac{5}{11}, 25$.

So there are three possible values of x.

71. $23\frac{10}{11} + 24\frac{5}{11} + 25 = 73\frac{4}{11} \approx 73.36$

72. $f(x + 1) = f(x) - f(x - 1)$

$f(x) = f(x + 1) + f(x - 1)$

$f(17) = f(18) + f(16)$

$2f(16) = f(18) + f(16)$

$f(16) = f(18)$

Let $f(16) = f(18) = x$

$f(17) = 2x$

$f(16) = f(15) + f(17) \rightarrow f(15) = -x$;

$f(15) = f(14) + f(16) \rightarrow f(14) = -2x$;

$f(14) = f(13) + f(15) \rightarrow f(13) = -x$;

$f(13) = f(12) + f(14) \rightarrow f(12) = x$

$f(12) = f(11) + f(13) \rightarrow f(11) = 2x$

$f(11) = f(10) + f(12) \rightarrow f(10) = x$

$f(10) = f(9) + (f(11) \rightarrow f(9) = -x$

If we observe the above pattern of values that we are getting, we can observe that $f(18) = f(12)$; $f(17) = f(11)$; $f(16) = f(10)$ and $f(15) = f(9)$. Here we can easily observe that values repeat for every six terms. So $f(5) = f(11) = f(17) = 6$

Option (b) is correct.

73. $\dfrac{h(x)}{h(x-1)} = \dfrac{h(x-2)}{h(x+1)}$

On putting $x = 54$ we get:

$\dfrac{h(54)}{h(53)} = \dfrac{h(52)}{h(55)}$ (i)

On putting $x = 55$, we get:

$\dfrac{h(55)}{h(54)} = \dfrac{h(53)}{h(56)}$ (ii)

Equation (i) ÷ Equation (ii)

$\dfrac{[h(54)]^2}{h(53)\times h(55)} = \dfrac{h(52)\times h(56)}{h(55)\times h(53)}$

$[h(54)]^2 = 4 \times 16$

$h(54) = 8$

74. $f(x) = 1 - \dfrac{2}{x+1} = \dfrac{x+1-2}{x+1} = \dfrac{x-1}{x+1}$

$$f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$$

$$f^3(x) = f(f(f)) = -\frac{x+1}{x-1}$$

$$f^4(x) = f(f(f(f(x)))) = x$$

$$f^5(x) = f(x) = \frac{x-1}{x+1}$$

Here we can see that $f(x) = f^5(x)$ so the given function has a cyclicity of 4, therefore:

$f^n(x) = f^{n+4k}(x)$ where k is a whole number

$$f^{802} = f^{2+4\times 200}(x) = f^2(x) = -\frac{1}{x}$$

$$f^{802}(x)\, at\ x = -\frac{1}{2} = -\frac{1}{-\frac{1}{2}} = 2$$

75. $\log_3(x + y) + \log_3(x - y) = 3$

$\log_3(x^2 - y^2) = 3$

$x^2 - y^2 = 3^3 = 27$

$(x - y)(x + y) = 27$

Here both $(x + y)$ & $(x - y)$ are positive integers (since they have to be used as the arguments of the logarithmic functions. Hence, $(x - y) > 0$ or $x > y$. From this point, we need to think of factor pairs of 27, in order to find out the values that are possible for x and y.

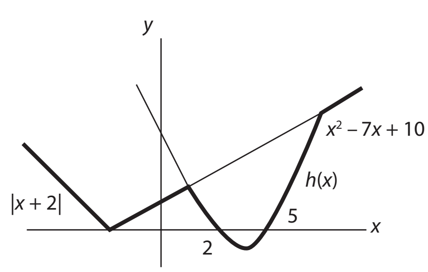
Case 1: $x + y = 9$, $x - y = 3$ or $x = 6$, $y = 3$

Case 2: when $x + y = 27$, $x - y = 1$ or $x = 14$, $y = 13$

Two pairs of (x, y) are possible.

76. The maximum value of $x + y = 14 + 13 = 27$

77. In the following figure, the bold portion shows the graph of $h(x)$.



Therefore $h(x) \le 0$ for $x = 2, 3, 4, 5$. There are 4 such values.

78. We can observe from the graph that $h(x) < 0$ only for two integer values (3, 4) of x. So the required sum $= 3 + 4 = 7$.

79. $[2p - 3]$ is an integer. Hence, $q + 7$ is also an integer or q must be an integer.

Similarly p is also an integer (since $[3q + 1]$ is an integer, hence $p + 6$ should also be an integer.)

$\Rightarrow [2p - 3] = 2p - 3 = q + 7$

$2p - q = 10$ (i)

$\Rightarrow 3q + 1 = p + 6$

$3q - p = 5$ (ii)

By solving equations (i) and (ii) we get the values of p and q as:

$p = 7, q = 4$

The required answer is then given by $7^2 \times 4^2 = 784$.

80. $f(a) = 3^a$ (If a is an odd number)

$f(a + 1) = 3^{(a+1)} + 4 = 3.3^a + 4$

$$\frac{1}{4}[f(a)+f(a+1)] = \frac{3^a+3.3^a+4}{4} = \frac{3^a.4+4}{4} = 3^a+1$$

$$\Rightarrow \frac{1}{4}[f(1)+f(2)+(f(3)+f(4))+\ldots+f(71)+f(72)]$$

$$= \frac{f(1)+f(2)}{4}+\frac{f(3)+f(4)}{4}+\ldots+\frac{f(71)+f(72)}{4}$$

$= 3^1 + 1 + 3^3 + 1 + \ldots + 3^{71} + 1$

$= (3^1 + 3^3 + \ldots + 3^{71}) + 36$

$$= \frac{3((3^2)^{36}-1)}{3^2-1}+36$$

$$= \frac{3}{8}(3^{72}-1)+36$$

81. $$g(f(x)) = 2.\frac{x^{2\left[\frac{3x^2}{4}\right]}}{4}+2$$

$$= \frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2$$

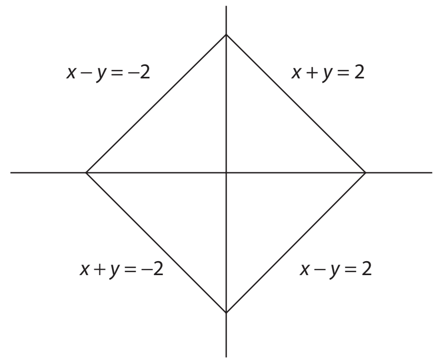
$g(f(x))$ is an even function, so option (a) is incorrect. As we increase the value of x, value of $g(f(x))$ will get increased. Therefore it will attain it's maxima at ∞. So option (b) is also incorrect.

$\frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2$ will attain it's minima when $0 \le \frac{3x^2}{4} \le 1$.

Since, the expression $\frac{3x^2}{4}$ is always going to be positive, hence we can say that the only constraint we need to match for the minima of the function is $\frac{3x^2}{4} \le 1$. Therefore, option(c) is true.

82. $\frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2 = 2^5 + 2 = 34$. Hence, option (b) is correct.

83.



As shown in the above diagram the region bounded by $|x + y| = 2$ and $|x - y| = 2$ is a square of side $\sqrt{2^2+2^2} = 2\sqrt{2}$

Required area = $\left(2\sqrt{2}\right)^2 = 8$

84. For $n = 8$

$f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 32|$

The minimum value of $f(x)$ will be when $x = -16$ when the middle term of this expression viz. $|x + 16|$ becomes 0. (i.e. it is minimized)

We have: $f(-16) = |-16| + |-12| + |-8| + |-4| + 0 + |4| + |8| + |12| + |16| = 80$

85. For $n = 7$, $f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 28|$. In this case there would be two middle terms in the expression viz. $|x + 12|$ and $|x + 16|$. The value of the expression would be minimized when the value of the sum of the middle terms is minimized.

We can see that $|x + 12| + |x + 16|$ gets minimized at $-16 \le x \le -12$; Note that the values of the sum of the remaining 6 terms of the expression would remain constant whenever we take the values of x between -12 and -16.

Thus, we have a total of 5 values at which the expression is minimised for $n = 7$.

86. For $n = 9$, $f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 36|$. The middle terms of this expression are $|x + 16|$ and $|x + 20|$. Hence, this expression would attain its' minimum value when

$-20 \le x \le -16$

Therefore $f(x)$ is minimum for a total of 5 values of x.

Minimum value of $f(x)$ can be seen at $x = -16 \rightarrow$ $f(-16) = |-16| + |-16 + 4| + | + |-16 + 8| + |-16 + 12| + |-16 + 16| + |-16 + 20| + |-16 + 24| + |-16 + 28| + |-16 + 32| + |-16 + 36|$

$= 16 + 12 + 8 + 4 + 0 + 4 + 8 + 12 + 16 + 20$

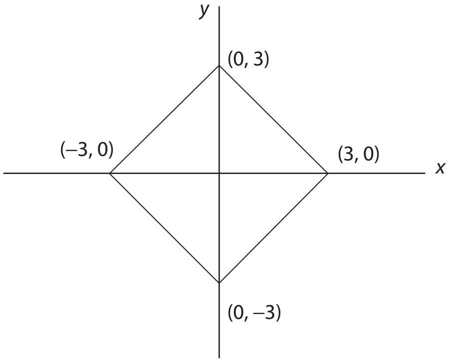
$= 100$

For $n = 9$, $f(x)$ will be minimum for $x = -16$ to -20

$\therefore f(-17) = f(-19) = 100$ minimum value of $f(x)$

Hence, option (d) is correct.

87. The Curve of $|x| + |y| = 3$ is shown below



The curve is a square of side of length $3\sqrt{2}$ units.

Therefore required area = $\left(3\sqrt{2}\right)^2$ = 18 square units.

88. In the previous question we found the area of curve $|x| + |y| = 3$,
$|x - a| + |y - b| = 3$ also has the same graph with the same shape and size only it's center shifted to a new point (a, b). (Previous center was (0, 0)).
Hence area enclosed by curve $|x - 2| + |y - 3| = 3$ is same as area enclosed by curve $|x| + |y| = 3$ = 18 square units

89. $8\{x\} = x + 2[x] \rightarrow 8\{x\} = [x] + \{x\} + 2[x] \rightarrow 7\{x\} = 3[x] \rightarrow [x] = \frac{7}{3}\{x\}$. This gives us the relationship between $[x]$ and $\{x\}$ and can also be expressed as $\{x\} = \frac{3}{7}[x]$.
Further, since $\{x\}$ is a fraction between 0 and 1 we get: $0 \le \frac{3}{7}[x] < 1 \rightarrow$

$0 \le 3[x] < 7 \rightarrow 0 \le [x] < \frac{7}{3}$

Thus, $[x]$ = 0, 1, 2 (three possible values between the limits we got).
Then using the relationship between $\{x\}$ and $[x]$ we get the possible values of $\{x\} = 0, \frac{3}{7}, \frac{6}{7}$ when $[x]$ is 0,1 and 2 respectively.
Since $x = [x] + \{x\}$ we get $x = 0, \frac{10}{7}, \frac{20}{7}$
Therefore, there are two positive values of x for which the given equation is true.

90. Difference between the greatest and least value of x
$= \frac{20}{7} - 0 = \frac{20}{7} = 2.85$

91. $f(x) = \frac{4^{x-1}}{4^{x-1}+1} = \frac{4^x}{4^x+4}$

$fog(x) = \frac{4^{2x}}{4^{2x}+4}$

$fog(1-x) = \frac{4^{2(1-x)}}{4^{2(1-x)}+4}$

$= \frac{4^2.4^{-2x}}{4^2.4^{-2x}+4}$

$= \frac{4^2}{4^2+4.4^{2x}}$

$= \frac{4}{4+4^{2x}}$

$fog(x) + fog(1-x) = \frac{4^{2x}}{4^{2x}+4} + \frac{4}{4+4^{2x}}$

$= \frac{4^{2x}+4}{4^{2x}+4} = 1$

put $x = \frac{1}{4}$

we get $fog\left(\frac{1}{4}\right) + fog\left(1-\frac{1}{4}\right) = fog\left(\frac{1}{4}\right) + fog\left(\frac{3}{4}\right) = 1$

$\Rightarrow fog\left(\frac{1}{4}\right) + fog\left(\frac{3}{4}\right) = 1$

92. put $x = ½$

$fog\left(\frac{1}{2}\right) + fog\left(1-\frac{1}{2}\right) = 1$

$2\,fog\left(\frac{1}{2}\right) = 1 \Rightarrow fog\left(\frac{1}{2}\right) = \frac{1}{2}$

$fog\left(\frac{1}{2}\right) + fog\left(\frac{1}{4}\right) + fog\left(\frac{3}{4}\right) + fog\left(\frac{1}{8}\right)$

$+ fog\left(\frac{7}{8}\right) + fog\left(\frac{1}{16}\right) + fog\left(\frac{15}{16}\right)$

$= \frac{1}{2} + 1 + 1 + 1 = 3\frac{1}{2} = 3.5$

93. $f(x + 2) = f(x) + 2(x + 1)$ when x is even.
$f(2) = 5$
$f(4) = f(2) + 2(2 + 1) = 5 + 6 = 11$
$f(6) = f(4) + 2(4 + 1) = 11 + 10 = 21$

Therefore for even values of x, $f(x) = \frac{x^2}{2} + 3$
$f(1) = 1$
$f(3) = 1 + 1 = 2$
$f(5) = 2 + 1 = 3$
$\Rightarrow$ For odd value of x, $f(x) = \frac{x+1}{2}$

$f(24) = \frac{(24)^2}{2} + 3$

= 291

94. $f(14) = \frac{(14)^2}{2} + 3 = 101$

$f(11)=\frac{11+1}{2}=6$

$\left[\frac{f(14)}{f(11)}\right]=\left[\frac{101}{6}\right]=[16.83]=16$

95. From the solution of question 93 it is clear that only option (c) is correct.

96. $f(f(f(f(3)))) + f(f(f(2)))$
$= f(f(f(2))) + f(f(5))$
$= f(f(5)) + f(3)$
$= f(3) + 2$
$= 2 + 2$
$= 4$

97. From the given information we can assume $F(x)$ as a sum of $P(x)$ and x, where
$P(x) = kx\,(x - 1)\,(x - 2)\,(x - 3)\,(x - 4)\,(x - 5)$, k is a constant.
$F(x) = kx\,(x - 1)\,(x - 2)\,(x - 3)\,(x - 4)\,(x - 5) + x$
$F(6) = k \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6$
It is given $F(6) = 7$
$\therefore\ k \times 6! + 6 = 7$
$k \times 6! = 1$
Hence, $k=\frac{1}{6!}$

Thus, $F(x)=\frac{x(x-1)(x-2)(x-3)(x-4)(x-5)}{6!}+x$

$F(8)=\frac{8\times7\times6\times5\times4\times3}{6!}+8$

$= 28 + 8 = 36$

98. By putting negative values of x, we can see that $F(x)$ is a decreasing function for negative integer values of x therefore $F(x)$ will be minimum for $x = -1$

Minimum value of $f(x)=\frac{-1\times-2\times-3\times-4\times-5\times-6}{6!}-1$
$\Rightarrow 1 - 1 = 0$

99. $g(x + y) = g(x)\ g(y)$
$g(1 + 1) = g(1)\,.g(1) = g(1)^2 = 5^2 = 25$
$g(2) = 5^2$
Similarly, $g(3) = 5^3$, $g(4) = 5^4$, $g(5) = 5^5$
$g(1) + g(2) + g(3) + g(4) + g(5) = 5 + 25 + 125 + 625 + 3125 = 3905$

100. $g(x) = 5^x$
If we put $n = 1$ in the given summation then
$g(q+1)=\frac{1}{4}(5^4-125)=\frac{500}{4}=125$
$5^{q+1} = 125 \Rightarrow q + 1 = 3$ or $q = 2$

14

Inequalities

This chapter will seem to be highly mathematical to you when you read the theory contained in the chapter and look at the solved examples. For students weak in Math, there is no need to be disheartened about the seemingly high mathematical content. I would advise you to go through this chapter and internalise the concepts. However, keep in mind the fact that in an aptitude test, the questions will have options, and with options all you will need to do will be check the validity of the inequality for the different options.

In fact, the questions in this chapter have options on both the levels and with option-based solutions, all these questions will seem easy to you.

However, I would advise students aiming to score high marks in Quantitative Ability to try to mathematically solve all the questions on all three levels in this chapter (even though option-based solution will be much easier.)

Two real numbers or two algebraic expressions related by the symbol > ("Greater Than") or < ("Less Than") (and also by the signs ≥ or ≤) form an inequality.

$A < B$, $A > B$ (are plain inequalities)

$A \geq B$, $A \leq B$ (are called as inequations)

The inequality consists of two sides—the left hand side, A and the right hand side, B. A and B can be algebraic expressions or they can be numbers.

An inequality with the < or > sign is called a *strict inequality* while an inequality having ≥ or ≤ sign is called a *slack inequality*. The expressions A and B have to be considered on the set where A and B have sense simultaneously. This set is called the set of permissible values of the inequality. If the terms on the LHS and the RHS are algebraic equations/identities, then the inequality may or may not hold true for a particular value of the variable/set of variables assumed.

The direction in which the inequality sign points is called *the sense of the inequality*. If two or several inequalities contain the same sign (< or >) then they are called *inequalities of the same sense*. Otherwise they are called *inequalities of the opposite sense*.

Now let us consider some basic *definitions* about inequalities.

For 2 real numbers a and b

The inequality $a > b$ means that the difference $a - b$ is positive.

The inequality $a < b$ means that the difference $a - b$ is negative.

PROPERTIES OF INEQUALITIES

For any two real numbers a and b, only one of the following restrictions can hold true:

$$a = b,\ a > b \text{ or } a < b$$

Definitions of Slack Inequalities

The inequality $a \geq b$ means that $a > b$ or $a = b$, that is, a is not less than b.

The inequality $a \leq b$ means that $a < b$ or $a = b$, that is, a is not greater than b.

We can also have the following double inequalities for simultaneous situations:

$$a < b < c,\ a < b \leq c,\ a \leq b < c,\ a \leq b \leq c$$

Properties of Inequalities

1. If $a > b$ then $b < a$ and vice versa.
2. If $a > b$ and $b > c$ then $a > c$.
3. If $a > b$ then for any c, $a + c > b + c$. In other words, an inequality remains true if the same number is added on both sides of the inequality.

Contd

Properties of Inequalities *(Contd)*

4. Any number can be transposed from one side of an inequality to the other side of the inequality with the sign of the number reversed. This does not change the sense of the inequality.
5. If $a > b$ and $c > 0$ then $ac > bc$. Both sides of an inequality may be multiplied (or divided) by the same positive number without changing the sense of the inequality.
6. If $a > b$ and $c < 0$ then $ac < bc$. That is, both sides of an inequality may be multiplied (or divided) by the same negative number but then the sense of the inequality is reversed.
7. If $a > b$ and $c > d$ then $a + c > b + d$. (Two inequalities having the same sense may be added termwise.)
8. If $a > b$ and $c < d$ then $a - c > b - d$
   From one inequality it is possible to subtract termwise another inequality of the opposite sense, retaining the sense of the inequality from which the other was subtracted.
9. If a, b, c, d are positive numbers such that $a > b$ and $c > d$ then $ac > bd$, that is, two inequalities of the same sense in which both sides are positive can be multiplied termwise, the resulting inequality having the same sense as the multiplied inequalities.
10. If a and b are positive numbers where $a > b$, then $a^n > b^n$ for any natural n.
11. If a and b are positive numbers where $a > b$ then $a^{1/n} > b^{1/n}$ for any natural $n \geq 2$.
12. Two inequalities are said to be equivalent if the correctness of one of them implies the correctness of the other, and vice versa.
    Students are advised to check these properties with values and form their own understanding and language of these rules.

Certain Important Inequalities

1. $a^2 + b^2 \geq 2ab$ (Equality for $a = b$)
2. $|a + b| \leq |a| + |b|$ (Equality reached if both a and b are of the same sign or if one of them is zero.)
   This can be generalised as $|a_1 + a_2 + a_3 + \ldots + a_n| \leq |a_1| + |a_2| + |a_3| + \ldots\ldots|a_n|$
3. $|a - b| \geq |a| - |b|$
4. $ax^2 + bx + c \geq 0$ if $a > 0$ and $D = b^2 - 4ac \leq 0$. The equality is achieved only if $D = 0$ and $x = -b/2a$.
5. Arithmetic mean $\geq$ Geometric mean. That is,
   $$\frac{(a+b)}{2} \geq ab$$
6. $a/b + b/a \geq 2$ if $a > 0$ and $b > 0$ or if $a < 0$ and $b < 0$, that is, both a and b have the same sign.

Contd

Certain Important Inequalities *(Contd)*

7. $a^3 + b^3 \geq ab(a + b)$ if $a > 0$ and $b > 0$, the equality being obtained only when $a = b$.
8. $a^2 + b^2 + c^2 \geq ab + ac + bc$
9. $(a + b)(b + c)(a + c) \geq 8\,abc$ if $a \geq 0$, $b \geq 0$ and $c \geq 0$, the equation being obtained when $a = b = c$
10. For any 4 numbers x_1, x_2, y_1, y_2 satisfying the conditions
    $$x_1^2 + x_2^2 = 1$$
    $$y_1^2 + y_2^2 = 1$$
    the inequality $|x_1y_1 + x_2y_2| \leq 1$ is true.
11. $\dfrac{a}{b^{1/2}} + \dfrac{b}{a^{1/2}} \geq a^{1/2} + b^{1/2}$ where $a \geq 0$ and $b \geq 0$
12. If $a + b = 2$, then $a^4 + b^4 \geq 2$
13. The inequality $|x| \leq a$, means that
    $$-a \leq x \leq a \text{ for } a > 0$$
14. $2^n > n^2$ for $n \geq 5$

Some Important Results

- If $a > b$, then it is evident that
  $a + c > b + c$
  $a - c > b - c$
  $ac > bc$
  $a/c > b/c$; that is,
  an inequality will still hold after each side has been increased, diminished, multiplied, or divided by the same positive quantity.
  If $a - c > b$,
  By adding c to each side,
  $a > b + c$; which shows that
  in an inequality any term may be transposed from one side to the other if its sign is changed.
- If $a > b$, then evidently $b < a$; that is,
  if the sides of an inequality be transposed, the sign of inequality must be reversed.
- If $a > b$, then $a - b$ is positive, and $b - a$ is negative; that is, $-a-(-b)$ is negative, and therefore $-a < -b$; hence,
  if the signs of all the terms of an inequality be changed, the sign of inequality must be reversed.
  Again, if $a > b$, then $-a < -b$ and, therefore, $-ac < -bc$; that is,
  if the sides of an inequality be multiplied by the same negative quantity, the sign of inequality must be reversed.

Contd

Some Important Results (*Contd*)

If $a_1 > b_1, a_2 > b_2, a_3 > b_3, \ldots a_m > b_m$, it is clear that $a_1 + a_2 + a_3 + \ldots + a_m > b_1 + b_2 + b_3 + \ldots + b_m$; and $a_1 a_2 a_3 \ldots a_m > b_1 b_2 b_3 \ldots b_m$.

- If $a > b$, and if p, q are positive integers, then or $a^{1/q} > b^{1/q}$ and, therefore, $a^{p/q} > b^{p/q}$; that is, $a^n > b^n$, where n is any positive quantity. Further,

$$1/a^n < 1/b^n; \text{ that is } a^{-n} < b^{-n}$$

The square of every real quantity is positive, and therefore greater than zero. Thus $(a - b)^2$ is positive.

Let a and b be two positive quantities, S their sum and P their product. Then from the identity

$$4ab = (a + b)^2 - (a - b)^2$$

we have $4P = S^2 - (a - b)^2$, and $S^2 = 4P + (a - b)^2$

Hence, if S is given, P is greatest when $a = b$; and if P is given, S is least when $a = b$;
That is, *if the sum of two positive quantities is given, their product is greatest when they are equal; and if the product of two positive quantities is given, their sum is least when they are equal.*

To Find the Greatest Value of a Product, the Sum of Whose Factors is Constant

Let there be n factors $a, b, c, \ldots n$, of a composite number and suppose that their sum is constant and equal to S.

Consider the product $abc\ldots.n$, and suppose that a and b are any two unequal factors. If we replace the two unequal factors a and b by the two equal factors $(a+b)/2$, and $(a+b)/2$, the product is increased while the sum remains unaltered. Hence, so *long as the product contains two unequal factors it can be increased altering the sum of the factors; therefore, the product is greatest when all the factors are equal.* In this case the value of each of the n factors is S/n, and the greatest value of the product is $(S/n)^n$, or $\{(a + b + c + \ldots.. + n/n\}^n$

This will be clearer through an example.

Let us define a number as $a \times b = c$ such that we restrict $a + b = 100$ (maximum).

Then, the maximum value of the product will be achieved if we take the value of a and b as 50 each.

Thus $50 \times 50 = 2500$ will be the highest number achieved for the restriction $a + b \leq 100$.

Further, you can also say that $50 \times 50 > 51 \times 49 > 52 \times 48 > 53 \times 47 > 54 \times 46 > \ldots > 98 \times 2 > 99 \times 1$

Thus if we have a larger multiplication as

$4 \times 6 \times 7 \times 8$ this will always be less than $5 \times 5 \times 7 \times 8$. [Holds true only for positive numbers.]

Corollary If $a, b, c\ldots k$, are unequal, $\{(a + b + c + \ldots + k)/n\}^n > abc \ldots k$;
that is , $(a + b + c + ,\ldots + k)/n > (abc\ldots k)^{1/n}$.

By an extension of the meaning of the arithmetic and geometric means this result is usually quoted as follows: *The arithmetic mean of any number of positive quantities is greater than the geometric mean.*

Definition of Solution of an Inequality

The solution of an inequality is the value of an unknown for which this inequality reduces to a true numerical identity. That is, to solve an inequality means to find all the values of the variable for which the given inequality is true.

An inequality has no solution if there is no such value for which the given inequality is true.

Equivalent Inequalities: Two inequalities are said to be equivalent if any solution of one is also a solution of the other and vice versa.

If both inequalities have no solution, then they are also regarded to be equivalent.

To solve an inequality we use the basic properties of an inequality which have been illustrated above.

Notation of Ranges

1. Ranges Where the Ends are Excluded If the value of x is denoted as (1, 2) it means $1 < x < 2$ i.e. x is greater than 1 but smaller than 2.

Similarly, if we denote the range of values of x as $-(7, -2)U(3, 21)$, this means that the value of x can be denoted as $-7 < x < -2$ and $3 < x < 21$. This would mean that the inequality is satisfied between the two ranges and is not satisfied outside these two ranges.

Based on this notation write the ranges of x for the following representations:

$$(1, +\infty)\ U\ (-\infty, -7)$$
$$(-\infty, 0)\ U\ (4, +\infty), (-\infty, 50)\ U\ (-50, +\infty)$$

2. Ranges where the Ends are Included

$$[2, 5] \text{ means } 2 \leq x \leq 5$$

3. Mixed Ranges

$$(3, 21] \text{ means } 3 < x \leq 21$$

Solving Linear Inequalities in one Unknown

A linear inequality is defined as an inequality of the form

$$ax + b\ I\ 0$$

where the symbol 'I' represents any of the inequalities $<, >, \geq, \leq$.

For instance if $ax + b \leq 0$, then $ax \leq -b$
$\rightarrow x \leq -b/a$ if $a > 0$ and $x \geq -b/a$ if $a < 0$

Example: Solve the inequality $2(x - 3) - 1 > 3(x - 2) - 4(x + 1)$

$\rightarrow 2x - 7 > 3x - 6 - 4x - 4 \rightarrow 3x > -3$. Hence, $x > -1$

This can be represented in mathematical terms as $(-1, +\infty)$

Example: Solve the inequality $2(x - 1) + 1 > 3 - (1 - 2x)$ $\rightarrow 2x - 1 > 2 + 2x \rightarrow 0.x > 3 \rightarrow$ This can never happen. Hence, no solution.

Example: Solve the inequality $2(x - 1) + 1 < 3 - (1 - 2x)$

Gives: $0.x < 3$.

This is true for all values of x

Example: Solve the inequality $ax > a$.

This inequality has the parametre a that needs to be investigated further.

If $a > 0$, then $x > 1$
If $a < 0$, then $x < 1$

Solving Quadratic Inequalities

A quadratic inequality is defined as an inequality of the form:

$$ax^2 + bx + c\ I\ 0\ (a \neq 0)$$

where the symbol I represents any of the inequalities $<, >, \geq, \leq$.

For a quadratic expression of the form $ax^2 + bx + c$, $(b^2 - 4ac)$ is defined as the discriminant of the expression and is often denoted as D. i.e. $D = b^2 - 4ac$

The following cases are possible for the value of the quadratic expression:

***Case 1:* If $D < 0$**

1. If $a < 0$ then $ax^2 + bx + c < 0$ for all x
2. If $a > 0$ then $ax^2 + bx + c > 0$ for all x.

In other words, we can say that if D is negative then the values of the quadratic expression takes the same sign as the coefficient of x^2.

This can also be said as

If $D < 0$ then all real values of x are solutions of the inequalities $ax^2 + bx + c > 0$ and $ax^2 + bx + c \geq 0$ for $a > 0$ and have no solution in case $a < 0$.

Also, for $D < 0$, all real values of x are solutions of the inequalities $ax^2 + bx + c < 0$ and $ax^2 + bx + c \leq 0$ if $a < 0$ and these inequalities will not give any solution for $a > 0$.

***Case 2:* $D = 0$**

If the discriminant of a quadratic expression is equal to zero, then the value of the quadratic expression takes the same sign as that of the coefficient of x^2 (except when $x = -b/2a$ at which point the value of the quadratic expression becomes 0).

We can also say the following for $D = 0$:

1. The inequality $ax^2 + bx + c > 0$ has as a solution any $x \neq -(b/2a)$ if $a > 0$ and has no solution if $a < 0$.
2. The inequality $ax^2 + bx + c < 0$ has as a solution any $x \neq -(b/2a)$ if $a < 0$ and has no solution if $a > 0$.
3. The inequality $ax^2 + bx + c \geq 0$ has as a solution any x if $a > 0$ and has a unique solution $x = -b/2a$ if $a < 0$.
4. The inequality $ax^2 + bx + c \leq 0$ has as a solution any x if $a < 0$ and has a unique solution $x = -b/2a$ for $a > 0$.

***Case 3:* $D > 0$**

If x_1 and x_2 are the roots of the quadratic expression then it can be said that:

1. For $a > 0$, $ax^2 + bx + c$ is positive for all values of x outside the interval $[x_1, x_2]$ and is negative for all values of x within the interval (x_1, x_2). Besides for values of $x = x_1$ or $x = x_2$, the value of the quadratic expression becomes zero (By definition of the root).
2. For $a < 0$, $ax^2 + bx + c$ is negative for all values of x outside the interval $[x_1, x_2]$ and is positive for all values of x within the interval (x_1, x_2). Besides for values of $x = x_1$ or $x = x_2$, the value of the quadratic expression becomes zero (By definition of the root).

Here are a few examples illustrating how quadratic inequalities are solved.

Solve the following inequalities.

Example 1: $x^2 - 5x + 6 > 0$

Solution: (a) The discriminant $D = 25 - 4 \times 6 > 0$ and a is positive (+1); the roots of the quadratic expression are real and distinct: $x_1 = 2$ and $x_2 = 3$.

By the property of quadratic inequalities, we get that the expression is positive outside the interval [2, 3]. Hence, the solution is $x < 2$ and $x > 3$.

We can also see it as $x^2 - 5x + 6 = (x - 2)(x - 3)$ and the given inequality takes the form $(x - 2)(x - 3) > 0$.

The solutions of the inequality are the numbers $x < 2$ (when both factors are negative and their product is positive) and also the numbers $x > 3$ (when both factors are positive and, hence, their product is also positive).

Answer: $x < 2$ and $x > 3$.

Example 2: $2x^2 + x + 1 \geq 0$

Solution: The discriminant $D = 1 - 4 \cdot (-2) = 9 > 0$; the roots of the quadratic expression are real and distinct:

$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2 \cdot (-2)} = \frac{-1 \pm 3}{-4}$$

hence, $x_1 = -1/2$ and $x_2 = 1$, and consequently, $-2x^2 + x + 1 = -2(x + 1/2) \times (x - 1)$. We have

$$-2(x + 1/2)(x - 1) \geq 0 \text{ or } (x + 1/2)(x - 1) \leq 0$$

(When dividing both sides of an inequality by a negative number, the sense of the inequality is reversed). The inequality is satisfied by all numbers from the interval

$$[-1/2, 1]$$

Please note that this can also be concluded from the property of quadratic expressions when $D > 0$ and a is negative.

Answer: $-1/2 \leq x \leq 1$.

Example 3: $2x^2 + x - 1 < 0$

Solution: $D = 1 - 4 \cdot(-2) \cdot (-1) < 0$, the coefficient of x^2 is negative. By the property of the quadratic expression when $D < 0$ and a is negative $-2x^2 + x - 1$ attains only negative values.

Answer: x can take any value.

Example 4: $3x^2 - 4x + 5 < 0$

Solution: $D = 16 - 4 \times 3 \times 5 < 0$, the coefficient of x^2 is positive. The quadratic expression $3x^2 - 4x + 5$ takes on only positive values.

Answer: There is no solution.

Example 5: $4x^2 + 4x + 1 > 0$.

Solution: $D = 16 - 4 \times 4 = 0$. The quadratic expression $4x^2 + 4x + 1$ is the square $(2x + 1)^2$, and the given inequality takes the form $(2x + 1)^2 > 0$. It follows that all real numbers x, except for $x = -1/2$, are solutions of the inequality.

Answer: $x \neq -1/2$.

Example 6: Solve the inequality $(a - 2)x^2 - x - 1 \geq 0$

Here, the value of the determinant $D = 1 - 4(-1)(a - 2) = 1 + 4(a - 2) = 4a - 7$

There can then be three cases:

Case 1: $D < 0 \rightarrow a < 7/4$

Then the coefficient of $x^2 \rightarrow a - 2$ is negative.

Hence, the inequality has no solution.

Case 2: $D = 0$

$a = 7/4$. Put $a = 7/4$ in the expression and then the inequality becomes

$-(x - 2)^2 \geq 0$. This can only happen when $x = 2$.

Case 3: $D > 0$

Then $a > 7/4$ and $a \neq 2$, then we find the roots x_1 and x_2 of the quadratic expression:

$$x_1 = \frac{1 + \sqrt{4a - 7}}{2(a - 2)} \quad \text{and} \quad x_2 = \frac{1 - \sqrt{4a - 7}}{2(a - 2)}$$

Using the property of quadratic expression's values for $D > 0$ we get

If $a - 2 < 0$, the quadratic expression takes negative values outside the interval $[x_1, x_2]$. Hence, it will take positive values inside the interval (x_1, x_2).

If $a - 2 > 0$, the quadratic expression takes positive values outside the interval $[x_1, x_2]$ and becomes zero for x_1 and x_2.

If $a - 2 = 0$, then we get a straight linear equation. $-x - 1 \geq 0 \rightarrow x \leq -1$.

System of Inequalities in One Unknown

Let there be given several inequalities in one unknown. If it is required to find the number that will be the solution of all the given equalities, then the set of these inequalities is called a *system of inequalities*.

The solution of a system of inequalities in one unknown is defined as the value of the unknown for which all the inequalities of the system reduce to true numerical inequalities.

To solve a system of inequalities means to find all the solutions of the system or to establish that there is none.

Two systems of inequalities are said to be *equivalent* if any solution of one of them is a solution of the other, and vice versa. If both the systems of inequalities have no solution, then they are also regarded to be equivalent.

Example 1: Solve the system of inequalities:

$$\begin{aligned} 3x - 4 &< 8x + 6 \\ 2x - 1 &> 5x - 4 \\ 11x - 9 &\leq 15x + 3 \end{aligned}$$

Solution: We solve the first inequality:

$$\begin{aligned} 3x - 4 &< 8x + 6 \\ -5x &< 10 \\ x &> -2 \end{aligned}$$

It is fulfilled for $x > -2$.

Then we solve the second inequality

$$\begin{aligned} 2x - 1 &> 5x - 4 \\ -3x &> -3 \\ x &< 1 \end{aligned}$$

It is fulfilled for $x < 1$.

And, finally, we solve the third inequality:

$$\begin{aligned} 11x - 9 &\leq 15x + 3 \\ -4x &\leq 12 \\ x &\geq -3 \end{aligned}$$

It is fulfilled for $x \geq -3$. All the given inequalities are true for $-2 < x < 1$.

Answer: $-2 < x < 1$.

Example 2: Solve the inequality $\frac{2x - 1}{x + 1} < 1$

We have $\frac{2x - 1}{x + 1} - 1 < 0 \rightarrow \frac{x - 2}{x + 1} < 0$

This means that the fraction above has to be negative. A fraction is negative only when the numerator and the denominator have opposite signs.

Hence, the above inequality is equivalent to the following set of 2 inequalities:

$$x - 2 > 0 \quad \text{and} \quad x - 2 < 0$$

and $$x + 1 < 0 \qquad\qquad x + 1 > 0$$

From the first system of inequalities, we get $x > 2$ or $x < -1$. This cannot happen simultaneously since these are inconsistent.

From the second system of inequalities we get

$$x < 2 \text{ or } x > -1 \text{ i.e } -1 < x < 2$$

Inequalities Containing a Modulus

Result:
$|x| \le a$, where $a > 0$ means the same as the double inequality

$$-a \le x \le a$$

This result is used in solving inequalities containing a modulus.

Example 1: $|2x - 3| \le 5$

This is equivalent to $-5 \le 2x - 3 \le 5$

i.e. $2x - 3 \ge -5$ and $2x - 3 \le 5$

$2x \ge -2$ $\qquad x \le 4$

$x \ge -1$

The solution is

$$-1 \le x \le 4$$

Example 2: $|1 - x| > 3$

$|1 - x| = |x - 1|$

Hence, $|x - 1| > 3 \rightarrow x - 1 > 3$ i.e $x > 4$

or $x - 1 < -3$ or $x < -2$

Answer: $x > 4$ or $x < -2$.

Space for Notes

WORKED-OUT PROBLEMS

Problem 14.1 Solve the inequality $\frac{1}{x} < 1$.

Solution $\frac{1}{x} < 1 \Leftrightarrow \frac{1}{x} - 1 < 0 \Leftrightarrow \frac{1-x}{x} < 0 \Leftrightarrow \frac{x-1}{x} > 0$.

This can happen only when both the numerator and denominator take the same sign (Why?)

Case 1: Both are positive: $x - 1 > 0$ and $x > 0$ i.e. $x > 1$.

Case 2: Both are negative: $x - 1 < 0$ and $x < 0$ i.e. $x < 0$.

Answer: **(– ∞, 0) *U* (1 + ∞)**

Problem 14.2 Solve the inequality $\frac{x}{x+2} \le \frac{1}{x}$.

Solution $\frac{x}{x+2} \le \frac{1}{x} \Leftrightarrow \frac{x}{x+2} - \frac{1}{x} \le 0 \Leftrightarrow \frac{x^2 - x - 2}{x(x+2)} \le 0 \Leftrightarrow \frac{(x-2)(x+1)}{x(x+2)} \le 0$.

The function $f(x) = \frac{(x-2)(x+1)}{x(x+2)}$ becomes negative when numerator and denominator are of opposite sign.

Case 1: Numerator positive and denominator negative: This occurs only between $-2 < x < -1$.

Case 2: Numerator negative and Denominator Positive: Numerator is negative when $(x - 2)$ and $(x + 1)$ take opposite signs. This can be got for:

Case A: $x - 2 < 0$ and $x + 1 > 0$ i.e. $x < 2$ and $x > -1$

Case B: $x - 2 > 0$ and $x + 1 < 0$ i.e. $x > 2$ and $x < -1$. Cannot happen.

Hence, the answer is $-2 < x \le 2$.

Problem 14.3 Solve the inequality $\frac{x}{x-3} \le \frac{1}{x}$.

Solution $\frac{x}{x-3} \le \frac{1}{x} \Leftrightarrow \frac{x}{x-3} - \frac{1}{x} \ge 0 \Leftrightarrow \frac{x^2 - x + 3}{x(x-3)} \le 0$. The function $f(x) = \frac{x^2 - x + 3}{x(x-3)}$

The numerator being a quadratic equation with $D < 0$ and $a > 0$, we can see that it will always be positive for all values of x. (From the property of quadratic inequalities).

Further, for the expression to be negative, the denominator should be negative.

That is, $x^2 - 3x < 0$. This will occur when $x < 3$ and x is positive.

Answer: **(0, 3)**

Suppose $F(x) = (x - x_1)^{k1} (x - x_2)^{k2} ... (x_1 - x_n)^{kn}$, where k_1, $k_2 \ldots, k_n$ are integers. If k_j is an even number, then the function $(x - x_j)^{kj}$ does not change sign when x passes through the point x_j and, consequently, the function $F(x)$ does not change sign. If k_p is an odd number, then the function $(x - x_p)^{kp}$ changes sign when x passes through the point x_p and, consequently, the function $F(x)$ also changes sign.

Problem 14.4 Solve the inequality $(x - 1)^2 (x + 1)^3 (x - 4) < 0$.

Solution The above inequality is valid for

Case 1: $x + 1 < 0$ and $x - 4 > 0$

That is, $x < -1$ and $x > 4$ simultaneously. This cannot happen together.

Case 2: $x + 1 > 0$ and $x - 4 < 0$

That is $x > -1$ or $x < 4$, i.e. $-1 < x < 4$ is the answer

Problem 14.5 Solve the inequality $\frac{(x-1)^2 (x+1)^3}{x^4 (x-2)} \le 0$

Solution The above expression becomes negative when the numerator and the denominator take opposite signs.

That means that the numerator is positive and the denominator is negative or vice versa.

Case 1: Numerator positive and denominator negative: The sign of the numerator is determined by the value of $x + 1$ and that of the denominator is determined by $x - 2$.

This condition happens when $x < 2$ and $x > -1$ simultaneously.

Case 2: Numerator negative and denominator positive: This happens when $x < -1$ and $x > 2$ simultaneously. This will never happen.

Hence, the answer is $-1 \le x < 2$.

Space for Rough Work

LEVEL OF DIFFICULTY (I)

Solve the following inequalities:

1. $3x^2 - 7x + 4 \le 0$
   (a) $x > 0$ (b) $x < 0$
   (c) All x (d) None of these
2. $3x^2 - 7x - 6 < 0$
   (a) $-0.66 < x < 3$ (b) $x < -0.66$ or $x > 3$
   (c) $3 < x < 7$ (d) $-2 < x < 2$
3. $3x^2 - 7x + 6 < 0$
   (a) $0.66 < x < 3$ (b) $-0.66 < x < 3$
   (c) $-1 < x < 3$ (d) None of these
4. $x^2 - 3x + 5 > 0$
   (a) $x > 0$ (b) $x < 0$
   (c) Both (a) and (b) (d) $-\infty < x < \infty$
5. $x^2 - 14x - 15 > 0$
   (a) $x < -1$ (b) $15 < x$
   (c) Both (a) and (b) (d) $-1 < x < 15$
6. $2 - x - x^2 \ge 0$
   (a) $-2 \le x \le 1$ (b) $-2 < x < 1$
   (c) $x < -2$ (d) $x > 1$
7. $|x^2 - 4x| < 5$
   (a) $-1 \le x \le 5$ (b) $1 \le x \le 5$
   (c) $-1 \le x \le 1$ (d) $-1 < x < 5$
8. $|x^2 + x| - 5 < 0$
   (a) $x < 0$ (b) $x > 0$
   (c) All values of x (d) None of these
9. $|x^2 - 5x| < 6$
   (a) $-1 < x < 2$ (b) $3 < x < 6$
   (c) Both (a) and (b) (d) $-1 < x < 6$
10. $|x^2 - 2x| < x$
    (a) $1 < x < 3$ (b) $-1 < x < 3$
    (c) $0 < x < 4$ (d) $x > 3$
11. $|x^2 - 2x - 3| < 3x - 3$
    (a) $1 < x < 3$ (b) $-2 < x < 5$
    (c) $x > 5$ (d) $2 < x < 5$
12. $|x^2 - 3x| + x - 2 < 0$
    (a) $(1 - \sqrt{3}) < x < (2 + \sqrt{2})$
    (b) $0 < x < 5$
    (c) $(1 - \sqrt{3}, 2 - \sqrt{2})$
    (d) $1 < x < 4$
13. $x^2 - 7x + 12 < |x - 4|$
    (a) $x < 2$ (b) $x > 4$
    (c) $2 < x < 4$ (d) $2 \le x \le 4$
14. $x^2 - |5x - 3| - x < 2$
    (a) $x > 3 + 2\sqrt{2}$ (b) $x < 3 + 2\sqrt{2}$
    (c) $x > -5$ (d) $-5 < x < 3 + 2\sqrt{2}$
15. $|x - 6| > x^2 - 5x + 9$
    (a) $1 \le x < 3$ (b) $1 < x < 3$
    (c) $2 < x < 5$ (d) $-3 < x < 1$
16. $|x - 6| < x^2 - 5x + 9$
    (a) $x < 1$ (b) $x > 3$
    (c) $1 < x < 3$ (d) Both (a) and (b)
17. $|x - 2| \le 2x^2 - 9x + 9$
    (a) $x > (4 - \sqrt{2})/2$ (b) $x < (5 + \sqrt{3})/2$
    (c) Both (a) and (b) (d) None of these
18. $3x^2 - |x - 3| > 9x - 2$
    (a) $x < (4 - \sqrt{19})/3$ (b) $x > (4 + \sqrt{19})/3$
    (c) Both (a) and (b) (d) $-2 < x < 2$
19. $x^2 - |5x + 8| > 0$
    (a) $x < (5 - \sqrt{57})/2$ (b) $x < (5 + \sqrt{57})/2$
    (c) $x > (5 + \sqrt{57})/2$ (d) Both (a) and (c)
20. $3|x - 1| + x^2 - 7 > 0$
    (a) $x > -1$ (b) $x < -1$
    (c) $x > 2$ (d) Both (b) and (c)
21. $|x - 6| > |x^2 - 5x + 9|$
    (a) $x < 1$ (b) $x > 3$
    (c) $(1 < x < 3)$ (d) both (a) and (b)
22. $(|x - 1| - 3)(|x + 2| - 5) < 0$
    (a) $-7 < x < -2$ and $3 < x < 4$
    (b) $x < -7$ and $x > 4$
    (c) $x < -2$ and $x > 3$
    (d) Any of these
23. $|x^2 - 2x - 8| > 2x$
    (a) $x < 2\sqrt{2}$ (b) $x < 3 + 3\sqrt{5}$
    (c) $x > 2 + 2\sqrt{3}$ (d) Both (a) and (c)
24. $(x - 1)\sqrt{x^2 - x - 2} \ge 0$
    (a) $x \le 2$ (b) $x \ge 2$
    (c) $x \le -2$ (d) $x \ge 0$
25. $(x^2 - 1)\sqrt{x^2 - x - 2} \ge 0$
    (a) $x \le -1$ (b) $x \ge -1$
    (c) $x \ge 2$ (d) (a) and (c)
26. $\sqrt{\dfrac{x-2}{1-2x}} > -1$
    (a) $0.5 > x$ (b) $x > 2$
    (c) Both (a) and (b) (d) $0.5 < x \le 2$

27. $\sqrt{\dfrac{3x-1}{2-x}} > 1$
(a) $0 < x < 2$ (b) $0.75 < x < 4$
(c) $0.75 < x < 2$ (d) $0 < x < 4$

28. $\sqrt{3x-10} > \sqrt{6-x}$
(a) $4 < x \le 6$ (b) $x < 4$ or $x > 6$
(c) $x < 4$ (d) $x > 8$

29. $\sqrt{x^2-2x-3} < 1$
(a) $(-1-\sqrt{5} < x < -3)$ (b) $1 \le x < (\sqrt{5}-1)$
(c) $x > 1$ (d) None of these

30. $\sqrt{1-\dfrac{x+2}{x^2}} < \dfrac{2}{3}$
(a) $(-6/5) < x \le -1$ or $2 \le x < 3$
(b) $(-6/5) \le x < -1$
(c) $2 \le x < 3$
(d) $(-6/5) \le x < 3$

31. $2\sqrt{x-1} < x$
(a) $x > 1$ (b) $x \ge 1, x \ne 2$
(c) $x < 1$ (d) $1 < x < 5$

32. $\sqrt{x+18} < 2-x$
(a) $x \le -18$ (b) $x < -2$
(c) $x > -2$ (d) $-18 \le x < -2$

33. $x > \sqrt{24+5x}$
(a) $x < 3$ (b) $3 < x \le 4.8$
(c) $x \ge 24/5$ (d) $x > 8$

34. $\sqrt{9x-20} < x$
(a) $4 < x < 5$ (b) $20/9 \le x < 4$
(c) $x > 5$ (d) Both (b) and (c)

35. $\sqrt{x+7} < x$
(a) $x > 2$ (b) $x > \sqrt{30}/2$
(c) $x > (1+\sqrt{29})/2$ (d) $x > 1+\sqrt{29}/2$

36. $\sqrt{2x-1} < x-2$
(a) $x < 5$ (b) $x > 5$
(c) $x > 5$ or $x < -5$ (d) $5 < x < 15$

37. $\sqrt{x+78} < x+6$
(a) $x < 3$ (b) $x > 3$ or $x < 2$
(c) $x > 3$ (d) $3 < x < 10$

38. $\sqrt{5-2x} < 6x-1$
(a) $0.5 < x$ (b) $x < 2.5$
(c) $0.5 < x < 2.5$ (d) $x > 2.5$

39. $\sqrt{x+61} < x+5$
(a) $x < 3$ (b) $x > 3$ or $x < 1$
(c) $x > 3$ (d) $3 < x < 15$

40. $x < \sqrt{2-x}$
(a) $x > 1$ (b) $x < 1$
(c) $-2 < x < 1$ (d) $-1 < x$

41. $x+3 < \sqrt{x+33}$
(a) $x > 3$ (b) $x < 3$
(c) $-3 < x < 3$ (d) $-33 < x < 3$

42. $\sqrt{2x+14} > x+3$
(a) $x < -7$ (b) $-7 \le x < 1$
(c) $x > 1$ (d) $-7 < x < 1$

43. $x-3 < \sqrt{x-2}$
(a) $2 \le x < (7+\sqrt{5})/2$ (b) $2 \le x$
(c) $x < (7+\sqrt{5})/2$ (d) $x \le 2$

44. $x+2 < \sqrt{x+14}$
(a) $-14 \le x < 2$ (b) $x > -14$
(c) $x < 2$ (d) $-11 < x < 2$

45. $x-1 < \sqrt{7-x}$
(a) $x > 3$ (b) $x < 3$
(c) $-53 < x < 3$ (d) $-103 < x < 3$

46. $\sqrt{9x-20} > x$
(a) $x < 4$ (b) $x > 5$
(c) $x \le 4$ or $x \ge 5$ (d) $4 < x < 5$

47. $\sqrt{11-5x} > x-1$
(a) $x > 2, x < 5$ (b) $-3 < x < 2$
(c) $-25 < x < 2$ (d) $x < 2$

48. $\sqrt{x+2} > x$
(a) $-2 \le x < 2$ (b) $-2 \le x$
(c) $x < 2$ (d) $x = -2$ or $x > 2$

Directions for Questions 49 to 53: Find the largest integral x that satisfies the following inequalities.

49. $\dfrac{x-2}{x^2-9} < 0$
(a) $x = -4$ (b) $x = -2$
(c) $x = 3$ (d) None of these

50. $\dfrac{1}{x+1} - \dfrac{2}{x^2-x+1} < \dfrac{1-2x}{x^3+1}$
(a) $x = 1$ (b) $x = 2$
(c) $x = -1$ (d) None of these

51. $\dfrac{x+4}{x^2-9} - \dfrac{2}{x+3} < \dfrac{4x}{3x-x^2}$
(a) $x = 1$ (b) $x = 2$
(c) $x = -1$ (d) None of these

52. $\dfrac{4x+19}{x+5} < \dfrac{4x-17}{x-3}$
(a) $x = 1$ (b) $x = 2$
(c) $x = -1$ (d) None of these

53. $(x+1)(x-3)^2(x-5)(x-4)^2(x-2) < 0$
(a) $x = 1$ (b) $x = -2$
(c) $x = -1$ (d) None of these

Directions for Questions 54 to 69: Solve the following inequalities:

54. $(x-1)(3-x)(x-2)^2 > 0$
(a) $1 < x < 3$ (b) $1 < x < 3$ but $x \neq 2$
(c) $0 < x < 2$ (d) $-1 < x < 3$

55. $\frac{6x-5}{4x+1} < 0$
(a) $-1/4 < x < 1$ (b) $-1/2 < x < 1$
(c) $-1 < x < 1$ (d) $-1/4 < x < 5/6$

56. $\frac{2x-3}{3x-7} > 0$
(a) $x < 3/2$ or $x > 7/3$ (b) $3/2 < x < 7/3$
(c) $x > 7/3$ (d) None of these

57. $\frac{3}{x-2} < 1$
(a) $2 < x < 5$ (b) $x < 2$
(c) $x > 5$ (d) $x < 2$ or $x > 5$

58. $\frac{1}{x-1} \leq 2$
(a) $x < 1$ (b) $x \geq 1.5$
(c) $-5 < x < 1$ (d) Both (a) and (b)
or $x \geq 1.5$

59. $\frac{4x+3}{2x-5} < 6$
(a) $x < 2.5$ (b) $x < 33/8$
(c) $x \geq 2.5$ (d) $x < 2.5$ or $x > 33/8$

60. $\frac{5x-6}{x+6} < 1$
(a) $-6 < x < 6$ (b) $-6 < x < 0$
(c) $-6 < x < 3$ (d) None of these

61. $\frac{5x+8}{4-x} < 2$
(a) $x < 0$ or $x > 4$ (b) $0 < x < 4$
(c) $0 \leq x < 4$ (d) $x > 4$

62. $\frac{x-1}{x+3} > 2$
(a) $x < -7$ (b) $x < -3$
(c) $-7 < x < -3$ (d) None of these

63. $\frac{7x-5}{8x+3} > 4$
(a) $-17/25 < x < -3/8$ (b) $x > -17/25$
(c) $0 < x < 3/8$ (d) $-17/25 < x < 0$

64. $\frac{x}{x-5} > \frac{1}{2}$
(a) $-5 < x < 5$ (b) $-5 < x < 0$
(c) $-5 \leq x \leq 5$ (d) $x < -5$ or $x > 5$

65. $x \leq \frac{6}{x-5}$
(a) $x < -1$ (b) $x > 5$
(c) $x < 6$ (d) $x \leq -1$ or $5 < x \leq 6$

66. $\frac{30x-9}{x-2} \geq 25(x+2)$
(a) $x < -1.4$ or $x > 2$
(b) $x < -1.4$ or $2 < x \leq 2.6$
(c) $x \leq -1.4$ or $2 < x \leq 2.6$
(d) None of these

67. $\frac{4}{x+2} > 3 - x$
(a) $-2 < x < -1$ or $x > 2$
(b) $-2 < x < 2$
(c) $-2 < x < -1$
(d) $0 < x < 3$

68. $x - 17 \geq \frac{60}{x}$
(a) $x < -3$
(b) $x < 20$
(c) $-3 \leq x < 0$ or $x \geq 20$
(d) $-3 < x \leq 0$ or $x \geq 20$

69. $\sqrt{x^2} < x + 1$
(a) $x > 0.5$ (b) $x > 0$
(c) All x (d) $x > -0.5$

70. Find the smallest integral x satisfying the inequality
$\frac{x-5}{x^2+5x-14} > 0$
(a) $x = -6$ (b) $x = -3$
(c) $x = -7$ (d) None of these

71. Find the maximum value of x for which $\frac{x+2}{x} \geq x$

Directions for 72 and 73: If $f(x) = |2x - 4|$ and x is an integer. Then answer the following questions:

72. Find the maximum value of x for which $f(x) \leq 5$.
73. Find minimum value of x for which $f(x) \leq 5$.
74. For how many integer values of x, is the expression: $(x-1)(4-x)(x-2)^2 > 0$

Directions for Question numbers 75 and 76:
If $\frac{x^2-5x+6}{|x|+5} \leq 0$

75. Find the minimum value of x, for which the above inequality is true.
76. For how many integer values of x, the above inequality is true.
77. Find the minimum value of x for which $\frac{1}{x-0.5} < 2$ where $x \in I^+$
78. For how many integer values of x is:
$\frac{x^2+6x-7}{x^2+1} > 2$
79. Maximum value of x, for which $\frac{x^2-9}{x^2+x+1} \leq 0$

Directions for Question numbers 80 and 81:
$\frac{x^2-7|x|+10}{x^2-8x+16} < 0$

80. For how many negative integral values of x, is the above inequality true?
81. Find the sum of all integer values for which the above inequality is true.

Direction for Question numbers 82 and 83:

$$f(x) = \frac{x^2 - 4x + 5}{x^2 + 7x + 12}$$

82. For how many positive integer values of x is $f(x) \leq 0$
83. For how many negative integer values of x is $f(x) \leq 0$
84. If $f(x) = x^2 + 2|x| + 1$, then for how many real values of x is: $f(x) \leq 0$.
85. If $\frac{1}{|x| - 2} > \frac{1}{3}$ then the least positive integer value of x, for which this inequality is true?

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. $x^2 - 5|x| + 6 < 0$
   (a) $-3 < x < -2$ (b) $2 < x < 3$
   (c) Both (a) and (b) (d) $-3 < x < 3$
2. $x^2 - |x| - 2 \geq 0$
   (a) $-2 < x < 2$ (b) $x \leq -2$ or $x \geq 2$
   (c) $x < -2$ or $x > 1$ (d) $-2 < x < 1$

Directions for Questions 3 to 16: Solve the following polynomial inequalities

3. $(x - 1)(3 - x)(x - 2)^2 > 0$
   (a) $1 < x < 2$ (b) $-1 < x < 3$
   (c) $-3 < x < -1$ (d) $1 < x < 3, x \neq 2$
4. $\dfrac{0.5}{x - x^2 - 1} < 0$
   (a) $x > 0$ (b) $x \leq 0$
   (c) $x \geq 0$ (d) For all real x
5. $\dfrac{x^2 - 5x + 6}{x^2 + x + 1} < 0$
   (a) $x < 2$ (b) $x > 3$
   (c) $2 < x < 3$ (d) $x < 2$ or $x > 3$
6. $\dfrac{x^2 + 2x - 3}{x^2 + 1} < 0$
   (a) $x < -3$ (b) $-7 < x < -3$
   (c) $-3 < x < 1$ (d) $-7 < x < 1$
7. $\dfrac{(x - 1)(x + 2)^2}{-1 - x} < 0$
   (a) $x < -1$
   (b) $x < -1$ or $x > 1$
   (c) $x < -1$ and $x \neq 2$
   (d) $x < -1$ or $x > 1$ and $x \neq -2$
8. $\dfrac{x^2 + 4x + 4}{2x^2 - x - 1} > 0$
   (a) $x < -2$ (b) $x > 1$
   (c) $x \neq 2$ (d) None of these
9. $x^4 - 5x^2 + 4 < 0$
   (a) $-2 < x < 1$
   (b) $-2 < x < 2$
   (c) $-2 < x < -1$ or $1 < x < 2$
   (d) $1 < x < 2$
10. $x^4 - 2x^2 - 63 \leq 0$
   (a) $x \leq -3$ or $x \geq 3$ (b) $-3 \leq x \leq 0$
   (c) $0 \leq x \leq 3$ (d) $-3 \leq x \leq 3$

Directions for Questions 11 to 67: Solve the following polynomial and quadratic inequalities

11. $\dfrac{5x - 1}{x^2 + 3} < 1$
   (a) $x < 4$ (b) $1 < x < 4$
   (c) $x < 1$ or $x > 4$ (d) $1 < x < 3$
12. $\dfrac{x - 2}{x^2 + 1} < -\dfrac{1}{2}$
   (a) $-3 < x < 3$ (b) $x < -3$
   (c) $-3 < x < 6$ (d) $-3 < x < 1$
13. $\dfrac{x + 1}{(x - 1)^2} < 1$
   (a) $x > 3$ or x is negative
   (b) $x > 3$
   (c) $x > 3$ or $-23 < x < 0$
   (d) x is negative and $x > 2$
14. $\dfrac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0$
   (a) $x < 3$ or $x > 4$ (b) $3 < x < 4$
   (c) $4 < x < 24$ (d) $0 < x < 3$
15. $\dfrac{x^2 + 6x - 7}{x^2 + 1} \leq 2$
   (a) x is negative (b) $x \geq 0$
   (c) $x > 0$ or $x < 0$ (d) Always
16. $\dfrac{x^4 + x^2 + 1}{x^2 - 4x - 5} < 0$
   (a) $x < -1$ or $x > 5$ (b) $-1 < x < 5$
   (c) $x > 5$ (d) $-5 < x < -1$
17. $\dfrac{1 + 3x^2}{2x^2 - 21x + 40} < 0$
   (a) $0 < x < 8$ (b) $2.5 < x < 8$
   (c) $-8 < x < 8$ (d) $3 < x < 8$
18. $\dfrac{1 + x^2}{x^2 - 5x + 6} < 0$
   (a) $x < 2$ (b) $x > 3$
   (c) Both a and b (d) $2 < x < 3$
19. $\dfrac{x^4 + x^2 + 1}{x^2 - 4x - 5} > 0$
   (a) $-1 < x < 5$ (b) $x < -1$ or $x > 5$
   (c) $x \leq -1$ or $x > 5$ (d) $-1 < x < 1$
20. $\dfrac{1 - 2x - 3x^2}{3x - x^2 - 5} > 0$

(a) $x < -1$ or $x > 1/3$ (b) $x < -1$ or $x = 1/3$
(c) $-1 < x < 1/3$ (d) $x < 1/3$

21. $\frac{x^2 - 5x + 7}{-2x^2 + 3x + 2} > 0$

(a) $x > 0.5$ (b) $x > -0.5$
(c) $-0.5 < x < 5$ (d) $-0.5 < x < 2$

22. $\frac{2x^2 - 3x - 459}{x^2 + 1} > 1$

(a) $x > -20$ (b) $x < 0$
(c) $x < -20$ (d) $-20 < x < 20$

23. $\frac{x^2 - 1}{x^2 + x + 1} < 1$

(a) $x > -2$ (b) $x > 2$
(c) $-2 < x < 2$ (d) $x < 2$

24. $\frac{1 - 2x - 3x^2}{3x - x^2 - 5} > 0$

(a) $1 < x < 3$ (b) $1 < x < 7$
(c) $-3 < x < 3$ (d) None of these

25. $\frac{x}{x^2 - 3x - 4} > 0$

(a) $-1 < x < 0$ (b) $4 < x$
(c) both (a) and (b) (d) $-1 < x < 4$

26. $\frac{x^2 + 7x + 10}{x + 2/3} > 0$

(a) $-5 < x < -2$ or $\frac{-2}{3} < x < \infty$
(b) $-5 < x < 8$
(c) $x < -2$
(d) $x > -2$

27. $\frac{3x^2 - 4x - 6}{2x - 5} < 0$

(a) $x < (2 - \sqrt{22})/3$
(b) $x > (2 + \sqrt{22})/3$
(c) $(2 - \sqrt{22})/3 < x < (2 + \sqrt{22})/3$
(d) None of these

28. $\frac{17 - 15x - 2x^2}{x + 3} < 0$

(a) $-8.5 < x \le -3$
(b) $-17 < x < -3$
(c) $-8.5 < x < -3$ or $x > 1$
(d) $-8.5 < x < 1$

29. $\frac{x^2 - 9}{3x - x^2 - 24} < 0$

(a) $-3 < x < 3$ (b) $x < -3$ or $x > 3$
(c) $x < -5$ or $x > 5$ (d) $x < -7$ or $x > 7$

30. $\frac{x + 7}{x - 5} + \frac{3x + 1}{2} \ge 0$

(a) $1 < x < 5$ (b) $-1 < x < 5$
(c) $1 \le x \le 3$ or $x > 5$ (d) $-1 < x < 3$

31. $2x^2 + \frac{1}{x} > 0$

(a) $x > 0$ (b) $x < -1/2$
(c) Both (a) and (b) (d) None of these

32. $\frac{x^2 - x - 6}{x^2 + 6x} \ge 0$

(a) $x < -6$ (b) $-2 \le x < 0$
(c) $x > 3$ (d) All of these

33. $\frac{x^2 - 5x + 6}{x^2 - 11x + 30} < 0$

(a) $x < 3$ or $x > 5$
(b) $2 < x < 4$ or $5 < x < 7$
(c) $2 < x < 3$ or $5 < x < 6$
(d) $2 < x < 3$ or $5 < x < 7$

34. $\frac{x^2 - 8x + 7}{4x^2 - 4x + 1} < 0$

(a) $x < 1$ or $x > 7$ (b) $1 < x < 7$
(c) $-7 < x < 1$ (d) $-7 < x < 7$

35. $\frac{x^2 - 36}{x^2 - 9x + 18} < 0$

(a) $-6 < x < 3$ (b) $-6 < x < 6$
(c) $x < -6$ or $x > 3$ (d) $-3 < x < 3$

36. $\frac{x^2 - 6x + 9}{5 - 4x - x^2} \ge 0$

(a) $-5 < x < 1$ or $x = 3$ (b) $-5 \le x < 1$ or $x = 3$
(c) $-5 < x \le 1$ or $x = 3$ (d) $-5 \le x \le -1$

37. $\frac{x - 1}{x + 1} < x$

(a) $x < -1$ (b) $x > -1$
(c) $-1 < x < 1$ (d) For all real values of x

38. $\frac{1}{x + 2} < \frac{3}{x - 3}$

(a) $-4.5 < x < -2$ (b) $-4.5 < x < -2$ or $3 < x$
(c) $-4.5 < x < -2, x > 3$ (d) (b) or (c)

39. $\frac{14x}{x + 1} - \frac{9x - 30}{x - 4} < 0$

(a) $-1 < x < 1$ or $4 < x < 6$
(b) $-1 < x < 4, 5 < x < 7$
(c) $1 < x < 4$ or $5 < x < 7$
(d) $-1 < x < 1$ or $5 < x < 7$

40. $\frac{5x^2 - 2}{4x^2 - x + 3} < 1$

(a) $x < 1$
(b) $-2 < x < 2$
(c) $-2.7 < x < 1.75$
(d) $(-(1 + \sqrt{21})/2 < x < (\sqrt{21} - 1)/2$

41. $\dfrac{x^2 - 5x + 12}{x^2 - 4x + 5} > 3$

(a) $x > 0.5$ (b) $1/2 < x < 3$
(c) $x < 0.5, x > 3$ (d) $1 < x < 3$

42. $\dfrac{x^2 - 3x + 24}{x^2 - 3x + 3} < 4$

(a) $x < -1$ (b) $4 < x < 8$
(c) $x < 4$ or $x > 8$ (d) None of these

43. $\dfrac{x^2 - 1}{2x + 5} < 3$

(a) $x < -2.5$ or $-2 < x < 8$
(b) $-2.5 < x < -2$
(c) $-2.5 < x < 8$
(d) Both (a) and (b)

44. $\dfrac{x^2 + 1}{4x - 3} > 2$

(a) $x > 7$ (b) $x > 7, x < 87$
(c) $0.75 < x < 1, x > 7$ (d) $0.25 < x < 1, x > 7$

45. $\dfrac{x^2 + 2}{x^2 - 1} < -2$

(a) $-1 < x < 2$
(b) $-1 < x < 1,$
(c) $-1 < x < 0, 0 < x < 1$
(d) $-2 < x < 2$

46. $\dfrac{3x - 5}{x^2 + 4x - 5} > \dfrac{1}{2}$

(a) $x < -5$ (b) $x > 1$
(c) $-5 < x < 1$ (d) $-5 < x < 5$

47. $\dfrac{2x + 3}{x^2 + x - 12} \leq \dfrac{1}{2}$

(a) $-4 < x < -3, 3 < x < 6$
(b) $-4 < x < -3, 0 < x < 6$
(c) $x < -4, -3 \leq x < 3, x > 6$
(d) $x < -4, x > 6$

48. $\dfrac{5 - 2x}{3x^2 - 2x - 16} < 1$

(a) $x < -\sqrt{7}$ (b) $-2 < x < \sqrt{7}$
(c) $8/3 \leq x$ (d) All of these

49. $\dfrac{15 - 4x}{x^2 - x - 12} < 4$

(a) $x < -\sqrt{63}/2, -3 < x < \sqrt{63}/2$
(b) $x > 4$
(c) Both (a) and (b)
(d) $x > 4, x < -63/2$
(e) None of these

50. $\dfrac{1}{x^2 - 5x + 6} > 1/2$

(a) $1 < x < 2, 3 < x < 4$ (b) $1 < x < 4$
(c) $x < 1, x > 3$ (d) None of these

51. $\dfrac{5 - 4x}{3x^2 - x - 4} < 4$

(a) $x < -\dfrac{\sqrt{7}}{2}$ (b) $-1 < x < \dfrac{\sqrt{7}}{2}$
(c) $x > 4/3$ (d) All of these

52. $\dfrac{(x + 2)(x^2 - 2x + 1)}{4 + 3x - x^2} \geq 0$

(a) $x < -2$ or $-1 < x < 4$
(b) $-2 < x < 4$ or $x > 6$
(c) $-2 < x < -1$ or $x > 4$
(d) None of these

53. $\dfrac{4}{1 + x} + \dfrac{2}{1 - x} < 1$

(a) $-1 < x < 1$ (b) $x < -1$
(c) $x > 1$ (d) both (b) and (c)

54. $2 + 3/(x + 1) > 2/x$

(a) $x < -2$ (b) $-1 < x < 0$
(c) $1/2 < x$ (d) All of these

55. $1 + \dfrac{2}{x - 1} > \dfrac{6}{x}$

(a) $0 \leq x \leq 1$ (b) $2 \leq x \leq 3$
(c) $-\infty < x < 1$ (d) Always except (a) and (b)

56. $\dfrac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} > 0$

(a) $x < -5$ (b) $1 < x < 2$
(c) $x > 6$ (d) Both (b) and (c)

57. $\dfrac{x - 1}{x} - \dfrac{x + 1}{x - 1} < 2$

(a) $-1 \leq x \leq 0$ (b) $1/2 \leq x \leq 1$
(c) $0 < x < 1/2$ (d) Always except (a) and (b)

58. $\dfrac{2(x - 3)}{x(x - 6)} \leq \dfrac{1}{x - 1}$

(a) $x < 0$ (b) $1 < x < 6$
(c) Both (a) and (b) (d) Always except (a) and (b)

59. $\dfrac{2(x - 4)}{(x - 1)(x - 7)} \geq \dfrac{1}{x - 2}$

(a) $1 < x < 2$ or $7 < x$ (b) $2 < x$
(c) $2 < x < 7$ (d) Both (a) and (c)

60. $\frac{2x}{x^2-9} \le \frac{1}{x+2}$

(a) $x < -3$ (b) $-2 < x < 3$
(c) All except (a) and (b) (d) Both (a) and (b)

61. $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$

(a) $-\sqrt{2} < x < 0$ or $2 < x$
(b) $\sqrt{2} < x$
(c) $1 < x < \sqrt{2}$
(d) Both (a) and (c)

62. $\frac{7}{(x-2)(x-3)} + \frac{9}{x-3} + 1 < 0$

(a) $-5 < x < 4$
(b) $-5 < x < 1$ and $1 < x < 3$ and x is not 2
(c) $-5 < x < 1$ and $2 < x < 3$
(d) $x < 1$

63. $\frac{20}{(x-3)(x-4)} + \frac{10}{x-4} + 1 > 0$

(a) $x < -2$ (b) $-1 < x < 3$ and $4 < x$
(c) All except (a) and (b) (d) Both (a) and (b)

64. $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$

(a) $x < -7$ (b) $x < -7$ and $-4 < x < -2$
(c) $-4 < x < 2$ (d) None of these

65. $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1$

(a) $-3 \le x \le -2$ (b) $x < -3$
(c) $-2 < x < -1$ (d) None of these

66. $(x^2 + 3x + 1)(x^2 + 3x - 3) \ge 5$

(a) $x < -4$ or $-2 < x$
(b) $-2 < x < -1$ or $1 < x$
(c) $x \le -4$; $-2 \le x \le -1$; $1 \le x$
(d) $x < -4$ or $1 < x$

67. $(x^2 - x - 1)(x^2 - x - 7) < -5$

(a) $-2 < x$
(b) $-2 < x < -1$ and $1 < x < 4$
(c) $-2 < x < -1$ and $2 < x < 3$
(d) $-2 < x < 0$ and $2 < x < 3$

Directions for Questions 68 to 92: Solve inequalities based on modulus

68. $|x^3 - 1| \ge 1 - x$

(a) $-1 < x < 0$ (b) $x < -1$
(c) $0 < x$ (d) Always except (a)

69. $\frac{x^2 - 5x + 6}{|x| + 7} < 0$

(a) $2 \le x \le 3$ (b) $2 < x$
(c) $1 < x < 3$ (d) $2 < x < 3$

70. $\frac{x^2 + 6x - 7}{|x+4|} < 0$

(a) $-7 < x < -5$ and $-4 < x < 1$
(b) $-7 < x < -5$ and $-4 < x < 0$
(c) $-7 < x < -4$ and $-4 < x < 1$
(d) None of these

71. $\frac{|x-2|}{x-2} > 0$

(a) $2 < x < 10$ (b) $3 \le x$
(c) $2 \le x$ (d) $2 < x$

72. $\left|\frac{2}{x-4}\right| > 1$

(a) $2 < x < 4$; $4 \le x \le 5$ (b) $2 < x \le 4$; $4 \le x \le 5$
(c) $2 < x < 4$; $4 < x < 6$ (d) $2 < x < 4$; $4 \le x \le 6$

73. $\left|\frac{2x-1}{x-1}\right| > 2$

(a) $2 < x$ (b) $1 < x$
(c) $3/4 < x < 1$ (d) Both (b) and (c)

74. $\left|\frac{x^2 - 3x - 1}{x^2 + x + 1}\right| < 3$

(a) $x < -2$ (b) $-1 < x$
(c) Always except (b) (d) Both (a) and (b)

75. $\frac{x^2 - 7|x| + 10}{x^2 - 6x + 9} < 0$

(a) $-5 \le x \le -2$; $2 < x < 5$
(b) $-5 < x < -2$; $2 < x < 5$
(c) $-5 < x < -2$; $2 < x < 3$; $3 < x < 5$
(d) $-5 < x < -2$; $3 < x < 5$

76. $\frac{|x+3| + x}{x+2} > 1$

(a) $-5 < x \le -2$ (b) $-2 \le x \le -1$
(c) $-1 < x$ (d) Always except (b)

77. $\frac{|x-1|}{x+2} < 1$

(a) $-8 < x \le -3$
(b) $-3 < x \le -2$
(c) Always except $x = -2$
(d) Both (a) and (b)

78. $\frac{|x+2| - x}{x} < 2$

(a) $-5 \le x < 0$ (b) $0 \le x \le 1$
(c) Both (a) and (b) (d) Always except (b)

79. $\frac{1}{|x| - 3} < \frac{1}{2}$

(a) $x < -5$ and $-3 < x < 3$
(b) $3 \le x \le 5$

(c) $-5 \le x \le -3$
(d) Always except (b) and (c)

80. $\left|\dfrac{3x}{x^2-4}\right| \le 1$
(a) $x \le -4$ and $-1 \le x \le 1$
(b) $4 \le x$
(c) Both of these
(d) None of these

81. $\left|\dfrac{x^2-5x+4}{x^2-4}\right| \le 1$
(a) $[0 < x < 8/5]\ U\ [5/2 < x < +\infty]$
(b) $[0, 5/2]\ U\ [16/5, +\infty]$
(c) $[0, 8/5]\ U\ [5/2, +\infty]$
(d) $[0, 8/5]\ U\ [5/2, +\infty]$

82. $\dfrac{|x-3|}{x^2-5x+6} \ge 2$
(a) [3/2, 1] (b) [1, 2]
(c) [1.5, 2] (d) None of these

83. $\dfrac{x^2-|x|-12}{x-3} \ge 2x$
(a) $-101 < x < 25$ (b) $[-\infty, 3]$
(c) $x \le 3$ (d) $x < 3$

84. $|x| < \dfrac{9}{x}$
(a) $x < -1$ (b) $0 < x < 3$
(c) $1 < x < 3; x < -1$ (d) $-\infty < x < 3$

85. $1 + \dfrac{12}{x^2} < \dfrac{7}{x}$
(a) $x < -2; 2 < x < 3$ (b) $3 \le x < 4$
(c) Both (a) and (b) (d) None of these

86. $\dfrac{(x^2-4x+5)}{(x^2+5x+6)} \ge 0$
(a) $-\infty < x < \infty$ (b) $x < -3$
(c) $x > -2$ (d) Both (b) and (c)

87. $\dfrac{x+1}{x-1} \ge \dfrac{x+5}{x+1}$
(a) $x < -1$ (b) $0 < x < 3$
(c) $1 < x \le 3, x < -1$ (d) $-\infty < x < 3$

88. $\dfrac{x-1}{x^2-x-12} \le 0$
(a) $x < -3; 2 < x < 3$ (b) $3 \le x < 4$
(c) Both (a) and (b) (d) None of these

89. $1 < (3x^2 - 7x + 8)/(x^2 + 1) \le 2$
(a) $1 < x < 6$ (b) $1 \le x < 6$
(c) $1 < x \le 6$ (d) $1 \le x \le 6$

90. If $f'(x) \ge g(x)$, where $f(x) = 5 - 3x + \dfrac{5}{2}x^2 - \dfrac{x^3}{3}$, $g(x) = 3x - 7$
(a) [2, 3] (b) [2, 4]
(c) $x = 2.5$ (d) None of these

91. $f'(x) \ge g'(x)$, if $f(x) = 10x^3 - 13x^2 + 7x$, $g(x) = 11x^3 - 15x^2 - 3$
(a) [–1, 7/3] (b) [–1, 3.5]
(c) [–1, 9/3] (d) [1, 7/3]

92. $\dfrac{1}{x-2} - \dfrac{1}{x} \le \dfrac{2}{x+2}$
(a) $(-2, -1)\ U\ (2, +\infty)$ (b) $-2 < x < 1$
(c) Both (a) and (b) (d) None of these

Directions for Questions 93 to 95: Solve the following irrational inequalities.

93. $(x-1)\sqrt{x^2-x-2} \ge 0$
(a) $x < 2$ (b) $3 \le x < \infty$
(c) Always except (a) (d) Both (a) and (b)

94. $(x^2-1)\sqrt{x^2-x-2} \ge 0$
(a) $x < -1$ (b) $2 \le x$
(c) Both (a) and (b) (d) None of these

95. $\dfrac{\sqrt{x-3}}{x-2} > 0$
(a) $0 \le x < 2$ (b) $x > 3$
(c) $0 < x < 1$ (d) Both (b) and (c)

96. If x satisfies the inequality $|x-1| + |x-2| + |x-3| \ge 6$, then: **IIFT 2010**
(a) $0 \le x \le 4$ (b) $x \le 0$ or $x \ge 4$
(c) $x \le -2$ or $x \ge 3$ (d) $x \ge 3$

Direction 97 and 98: If $f(x) = |x+4| - |x-4|$ and $|f(x)| < 8$, then answer the following questions:

97. How many integer values of x satisfies the above inequality?
(a) 6 (b) 7
(c) 8 (d) 9

98. What is the sum of all positive integer values of x satisfying the above inequality?
(a) 5 (b) 6
(c) 8 (d) 10

Directions for question number 99-100: If $x > -6$ and $\dfrac{1}{x-5} + \dfrac{1}{x+3} < 0$, then answer the following questions:

99. Find the number of positive integer values of x, which satisfy the given inequality.

100. Find the sum of all positive integer values of x which satisfy the given inequality.

Direction for question 101-102:

$|4x - 3| \le 8$ and $|3y + 4| \le 17$ then answer the following questions:

101. Minimum value of $|x|+|y|$ =

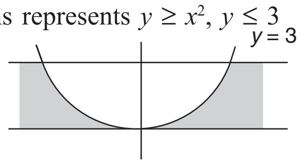
102. Maximum value of $|x| - |y|$ =

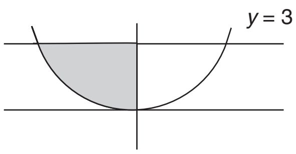
103. If $f(x) = \dfrac{x}{2x^2+5x+8}$ for all $x > 0$ what is the greatest value of $f(x)$?

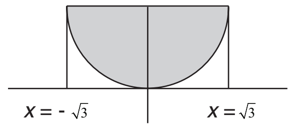
(a) 1/4 (b) 1/8

(c) 1/13 (d) 1/5

104. The Shaded portion of which of the following options represents $y \geq x^2$, $y \leq 3$

(a) 

(b) 

(c) 

(d) None of these

105. Find the range of values of x for which $x^4 + 8x < 8x^3 + x^2$

(a) $(-1,0) \cup (1,8)$ (b) $(0,1) \cup (8,\infty)$

(c) $(-\infty,-8) \cup (1, 8)$ (d) $(-1, 0) \cup (8, \infty)$

106. If $\dfrac{45}{25x^{14} - 8x^7 + 1} \leq p$

Then minimum value of p =

107. If $0 \leq x \leq 13$ for how many integer values of x, $7^{x-1} + 11^{x-1} > 170$

108. If $f(x) = \min(3x + 4, 6 - 2x)$ and $f(x) < p$ where p is an integer then the minimum possible value of p = ?

109. For how many non-negative integer values of 'x' is $||||||x - 1| - 2| - 3| - 4|-5|-6|-7| < 9$

(a) 35 (b) 36

(c) 37 (d) 38

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (d)	2. (a)	3. (d)	4. (d)
5. (c)	6. (a)	7. (d)	8. (d)
9. (c)	10. (a)	11. (d)	12. (c)
13. (c)	14. (d)	15. (b)	16. (d)
17. (d)	18. (c)	19. (d)	20. (d)
21. (c)	22. (a)	23. (d)	24. (b)
25. (d)	26. (d)	27. (c)	28. (a)
29. (d)	30. (a)	31. (b)	32. (d)
33. (d)	34. (d)	35. (c)	36. (b)
37. (c)	38. (c)	39. (c)	40. (b)
41. (d)	42. (b)	43. (a)	44. (a)
45. (b)	46. (d)	47. (d)	48. (a)
49. (a)	50. (a)	51. (b)	52. (b)
53. (b)	54. (b)	55. (d)	56. (a)
57. (d)	58. (d)	59. (d)	60. (c)
61. (a)	62. (c)	63. (a)	64. (d)
65. (d)	66. (c)	67. (a)	68. (c)
69. (b)	70. (a)	71. 2	72. 4
73. 0	74. 1	75. 2	76. 2
77. 2	78. 0	79. 3	80. 2
81. –4	82. 5	83. 1	84. 0
85. 3			

Level of Difficulty (II)

1. (c)	2. (b)	3. (d)	4. (d)
5. (c)	6. (c)	7. (d)	8. (d)
9. (c)	10. (d)	11. (c)	12. (d)
13. (a)	14. (a)	15. (d)	16. (b)
17. (b)	18. (d)	19. (b)	20. (a)
21. (d)	22. (c)	23. (a)	24. (d)
25. (c)	26. (a)	27. (d)	28. (c)
29. (b)	30. (c)	31. (d)	32. (d)
33. (c)	34. (b)	35. (a)	36. (a)
37. (b)	38. (b)	39. (a)	40. (d)
41. (b)	42. (d)	43. (a)	44. (c)
45. (c)	46. (c)	47. (c)	48. (d)
49. (c)	50. (a)	51. (d)	52. (a)
53. (d)	54. (d)	55. (d)	56. (d)
57. (d)	58. (c)	59. (a)	60. (d)
61. (d)	62. (b)	63. (d)	64. (b)
65. (d)	66. (c)	67. (c)	68. (d)
69. (d)	70. (c)	71. (d)	72. (c)
73. (d)	74. (d)	75. (c)	76. (d)
77. (c)	78. (d)	79. (d)	80. (c)
81. (a)	82. (d)	83. (d)	84. (b)
85. (d)	86. (d)	87. (c)	88. (d)
89. (d)	90. (d)	91. (a)	92. (d)
93. (c)	94. (c)	95. (b)	96 (b)
97 (b)	98 (b)	99 (3)	100 9
101 0	102 2.75	103 (c)	104 (c)
105 (a)	106 125	107 10	108 6
109 (c)			

Solutions and Shortcuts

While practically solving inequalities remember the following:

1. The answer to an inequality question is always in the form of a range and represents the range of values where the inequality is satisfied.
2. In the cases of all continuous functions, the point at which the range of the correct answer will start, will always be a solution of the same function if written as an equation.

   This rule is only broken for non-continuous functions.

   Hence, if you judge that a function is continuous always check the options for LHS = RHS at the starting point of the option.
3. The correct range has to have two essential properties if it has to be the correct answer:
   (a) The inequality should be satisfied for each and every value of the range.
   (b) There should be no value satisfying the inequality outside the range of the correct option.

   Questions on inequalities are always solved using options and based on (3) (a) and (3) (b) above we would reject an option as the correct answer if:
   (i) we find even a single value not satisfying the inequality within the range of a single option.
   (ii) we can reject given option, even if we find a single value satisfying the inequality but not lying within the range of the option under check.
   I will now show you certain solved questions on this pattern of thinking.

Level of Difficulty (I)

1. At $x = 0$, inequality is not satisfied. Thus, option (c) is rejected. Also $x = 0$ is not a solution of the equation. Since, this is a continuous function, the solution cannot start from 0. Thus options (a) and (b) are not right. Further, we see that the given function is quadratic with real roots. Hence, option (d) is also rejected.
2. At $x = 0$, inequality is satisfied. Hence, options (b) and (c) are rejected. $x = 3$ gives LHS = RHS. and $x = -0.66$ also does the same. Hence. roots of the equation are 3 and – 0.66.

   Thus, option (a) is correct.
3. At $x = 0$, inequality is not satisfied.
   Hence, options (b), (c) are rejected. At $x = 2$, inequality is not satisfied. Hence, option (a) is rejected.

   Thus, option (d) is correct.
4. The given quadratic equation has imaginary roots and is hence always positive.

   Thus, option (d) is correct
5. At $x = 0$ inequality is not satisfied. Thus option (d) is rejected.

$x = -1$ and $x = 15$ are the roots of the quadratic equation. Thus, option (c) is correct.

6. At $x = 0$, inequality is satisfied.
   Thus, options, (c) and (d) are rejected.
   At $x = 1$, inequality is satisfied
   Hence, we choose option (a).
7. At $x = 0$ inequality is satisfied, option (b) is rejected.
   At $x = 2$, inequality is satisfied, option (c) is rejected.
   At $x = 5$, LHS = RHS.
   At $x = -1$, LHS = RHS.
   Thus, option (d) is correct.
8. At $x = 0$ inequality is satisfied.
   Thus, options (a), (b), are rejected. Option (c) is obviously not true, as there will be values of x at which the inequality would not be satisfied.
   Option (d) is correct.
10. At $x = 1$ and $x = 3$ LHS = RHS.
    At $x = 2$ inequality is satisfied.
    At $x = 0.1$ inequality is not satisfied.
    At $x = 2.9$ inequality is satisfied.
    At $x = 3.1$ inequality is not satisfied.
    Thus, option (a) is correct.
12. The options need to be converted to approximate values before you judge the answer. At $x = 0$, inequality is satisfied.
    Thus, option (b) and (d) are rejected.
    Option (c) is correct.
13. At $x = 0$, inequality is not satisfied, option (a) is rejected.
    At $x = 5$, inequality is not satisfied, option (b) is rejected.
    At $x = 2$ inequality is not satisfied.
    Option (d) is rejected.
    Option (c) is correct.
14. At $x = 0$, inequality is satisfied, option (a) rejected.
    At $x = 10$, inequality is not satisfied, option (c) rejected.
    At $x = -5$, LHS = RHS.
    Also at $x = 5$, inequality is satisfied and at $x = 6$, inequality is not satisfied.
    Thus, option (d) is correct.
15. At $x = 2$, inequality is satisfied.
    At $x = 0$, inequality is not satisfied.
    At $x = 1$, inequality is not satisfied but LHS = RHS.
    At $x = 3$, inequality is not satisfied but LHS = RHS.
    Thus, option (b) is correct.

Solve other questions of LOD I and LOD II in the same fashion.

71. $\frac{x+2}{x} - x \geq 0$

$$\frac{x+2-x^2}{x} \geq 0$$

$$\frac{x^2-x-2}{x} \leq 0$$

$$\frac{(x-2)(x+1)}{x} \leq 0$$

Case 1: $(x - 2)(x + 1) \geq 0$ and $x < 0$
This occurs only for $x \leq -1$
Case 2: $(x - 2)(x + 1) \leq 0$ and x positive.
This occurs when $0 < x \leq 2$
Therefore maximum value of x which satisfies the condition is at $x = 2$.

Solution for 72 & 73:

x is an integer.
$|2x - 4| \leq 5$
$-5 \leq 2x - 4 \leq 5$
$-1 \leq 2x \leq 9$
$-\frac{1}{2} < x \leq \frac{9}{2}$.

72. Maximum value of $x = 4$.
73. Minimum value of $x = 0$.
74. $(x - 1)(4 - x)(x - 2)^2 > 0$
    As $(x - 2)^2$ is always non-negative
    This means that the product of the first two brackets would be positive. This also means that $(x - 1)(x - 4) < 0$
    $\Rightarrow 1 < x < 4$
    x has two integer values 2 and 3 between 1 and 4. But for $x = 2$, $(x - 1)(4 - x)(x - 2)^2 = 0$
    $\Rightarrow$ Therefore the given inequality is true only for one integer value of x.

Solution for 75 and 76:

$$\frac{x^2-5x+6}{|x|+5} \leq 0$$

$$\frac{(x-2)(x-3)}{|x|+5} \leq 0$$

Case I: $x > 0$
In this case the expression would become: $\Rightarrow \frac{(x-2)(x-3)}{x+5} \leq 0$
$\Rightarrow 2 \leq x \leq 3$
Case II: $x < 0$
In this case the expression would become:

$$\frac{(x-2)(x-3)}{-x+5} \leq 0$$

$$\frac{(x-2)(x-3)}{x-5} \geq 0$$

The above inequality is not satisfied for any negative value of x.