

1. Consider the set $S = \{2, 3, 4, \dots, 2n + 1\}$, where 'n' is a positive integer. Define

X as the average of the odd integers in S and

Y as the average of the even integers in S.

What is the value of $X - Y$?

- (1) 0 (2) 1 (3) $\frac{1}{2}n$ (4) $n + \frac{1}{2}n$ (5) 2008

2. If the product of n positive real numbers is unity, then their sum is necessarily

- a multiple of n
- equal to $n + \frac{1}{n}$
- never less than n
- a positive integer

3. The number of positive integers n in the range $12 \leq n \leq 40$ such that the product $(n-1)(n-2) \dots (n-11)$ is not divisible by n is

1. 5 2. 7 3. 13 4. 14

4. Let x, y and z be distinct integers. x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true?

- $y(x-z)^2$ is even
- $y^2(x-z)$ is odd
- $y(x-z)$ is odd
- $z(x-y)^2$ is even

5. P and Q are two positive integers such that $PQ = 64$. Which of the following cannot be the value of $P + Q$?

- a. 20 b. 65 c. 16 d. 35

Inequalities

6. What values of x satisfy

$$x^{2/3} + x^{1/3} - 2 \leq 0?$$

- (1) $-8 \leq x \leq 1$ (2) $-1 \leq x \leq 8$
 (3) $1 < x < 8$ (4) $1 \leq x \leq 8$

7. Find the range of real values of x satisfying the inequalities: $3x - 2 > 7$ and $4x - 13 > 15$

- (1) $x > 3$
 (2) $x > 7$
 (3) $x < 7$
 (4) $x < 3$

8. A real number x satisfying following inequality, for every positive integer n, is best described by

$$1 - \frac{1}{n} < x \leq 3 + \frac{1}{n}$$

1. $1 < x < 4$ 2. $1 < x \leq 3$
 3. $0 < x \leq 4$ 4. $1 \leq x \leq 3$

9. If n is such that $36 \leq n \leq 72$, then find x satisfying following inequality

$$x = \frac{n^2 + 2\sqrt{n(n+4)} + 16}{n + 4\sqrt{n} + 4}$$

1. $20 < x < 54$ 2. $23 < x < 58$
 3. $25 < x < 64$ 4. $28 < x < 60$

10. If $x > 2$ and $y > -1$, then which of the following statements is necessarily true?

- $xy > -2$
- $-x < 2y$
- $xy < -2$
- $-x > 2y$

Functions

11. Find $f \circ g(x)$

- X
- X^2
- $X+3$
- $2x+3$

12. If $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$

For what value of x; $f(x) = g(x-3)$

- 3
- $\frac{1}{4}$
- 4
- None of these

13. $le(x, y) =$ Least of (x, y)

$$mo(x) = |x|$$

$me(x, y) =$ Maximum of (x, y)

Find the value of $me[1, mo[le(a, b)]]$ at $a = -2$ and $b = -3$.

- a. 1 b. 0 c. 5 d. 3

14. The minimum value of is attained at

$$f(x) = x^8 + x^6 - x^4 - 2x^3 - x^2 - 2x + 9$$

- f(1)
- f(2)
- f(1/2)
- f(-1)

15. Let $g(x) = \max(5-x, x+2)$,

Then the minimum possible value of g(x) is:

- (1) 4 (2) 4.5
 (3) 1.5 (4) None of these

16. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number.

Then the minimum possible value of f(x) is:

- (1) $\frac{1}{3}$ (2) $\frac{1}{2}$
 (3) $\frac{2}{3}$ (4) $\frac{5}{3}$

17. Largest value of $\min(2 + x^2, 6 - 3x)$, when $x > 0$, is

- a. 1 b. 2 c. 3 d. 4

18. If $f(1) = 2$ and $f(n+1) = f(n) \times 5$. Find $f(5) = ?$

- a. 10 b. 50 c. 250 d. 1250

19. If $f(1) = 2$; $f(2) = 4$; $f(3) = 8$

$f(4) = 6$; $f(5) = 2$ and $f(6) = 4$

Find $f(2016)$

- a. 2 b. 1217 c. 6 d. 5

20. Two operations \oplus and $*$ are defined as per the following tables:

\oplus	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

$*$	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table $f \oplus g = a$, while according to the second table $g * h = f$, and so on. Also, let $f^2 = f * f$, $g^3 = g^2 * g$, and so on.

Upon simplification, $f \oplus [f * \{f \oplus (f * f)\}]$ equals

1. e 2. f 3. g 4. h

Graphs

21. When the curves

$y = \log_{10}x$ and $y = x-1$ are drawn in the x-y plane, how many times do they intersect for values $x \geq 1$?

1. Never 2. Once
3. Twice 4. More than twice

22. Consider the following two curves in the x-y plane:

$y = x^3 + x^2 + 5$ and $y = x^2 + x + 5$, where x is integer

Which of following statements is true for $-2 \leq x \leq 2$?

1. The two curves intersect once.
2. The two curves intersect twice.
3. The two curves do not intersect
4. The two curves intersect thrice.

23. What is the equation of the line that is parallel to the line $3x + 7y = 10$ and passes through the point (4, 8)?

- a) $7x - 3y = 46$
b) $3x + 7y = 44$
c) $9x + 21y + 184 = 0$
d) $3x + 7y = 68$

24. What is the equation of the line that is parallel to the line $3x = 2y + 4$ and passes through the point (3,1)?

- a) $6x = 4y - 5$
b) $3x + 7y = 44$
c) $3x = 2y - 1$
d) $3x + 7y = 68$

25. What is the equation of the line that is perpendicular to the line $2y + x = 3$?

- a) $x = -4y - 5$
b) $y = -x - 1$
c) $y = -2x - 7$
d) $3x - y = 8$

Polynomials

26. Consider a sequence where the n^{th} term,

$t_n = n/(n+2)$, $n = 1, 2, \dots$

The value of $t_3 \times t_4 \times t_5 \times \dots \times t_{53}$ equals:

- (1) $2/495$ (2) $2/477$
(3) $12/55$ (4) $1/1485$

27.

What is the sum of 'n' terms in the series

$$\log m + \log \left(\frac{m^2}{n} \right) + \log \left(\frac{m^3}{n^2} \right) + \log \left(\frac{m^4}{n^3} \right) + \dots ?$$

1. $\log \left[\frac{n^{(n-1)}}{m^{(n+1)}} \right]^{\frac{n}{2}}$ 2. $\log \left[\frac{m^m}{n^n} \right]^{\frac{n}{2}}$

3. $\log \left[\frac{m^{(1-n)}}{n^{(1-m)}} \right]^{\frac{n}{2}}$ 4. $\log \left[\frac{m^{(n+1)}}{n^{(n-1)}} \right]^{\frac{n}{2}}$

28. If $a_1 = 1$ and $a_n + 1 = 2a_n + 5$, $n = 1, 2, \dots$, then a_{100} is equal to

- a. $(5 \times 2^{99} - 6)$
b. $(5 \times 2^{99} + 6)$
c. $(6 \times 2^{99} + 5)$
d. $(6 \times 2^{99} - 5)$

29. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then

a_{100} equals? CAT 2005

1. $3^{99} - 200$ 2. $3^{99} + 200$
3. $3^{100} - 200$ 4. $3^{100} + 200$

30.

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{2007^2} + \frac{1}{2008^2}}$$

Find the sum

- (1) $2008 - 1/2008$ (2) $2007 - 1/2007$
(3) $2007 - 1/2008$ (4) $2008 - 1/2007$